

Development of a Stellar Model-Fitting Pipeline for Asteroseismic Data from the TESS Mission

PI: Travis Metcalfe / Co-I: Rich Townsend
Collaborators: David Guenther, Dennis Stello

The launch of NASA's Kepler space telescope in 2009 revolutionized the quality and quantity of observational data available for asteroseismic analysis. Prior to the Kepler mission, solar-like oscillations were extremely difficult to observe, and data only existed for a handful of the brightest stars in the sky. With the necessity of studying one star at a time, the traditional approach to extracting the physical properties of the star from the observations was an uncomfortably subjective process. A variety of experts could use similar tools but come up with significantly different answers. Not only did this subjectivity have the potential to undermine the credibility of the technique, it also hindered the compilation of a uniform sample that could be used to draw broader physical conclusions from the ensemble of results. During a previous award from NASA, we addressed these issues by developing an automated and objective stellar model-fitting pipeline for Kepler data, and making it available through the Asteroseismic Modeling Portal (AMP). This community modeling tool has allowed us to derive reliable asteroseismic radii, masses and ages for large samples of stars (Metcalfe et al., 2014), but the most recent observations are so precise that we are now limited by systematic uncertainties associated with our stellar models. With a huge archive of Kepler data available for model validation, and the next planet-hunting satellite already approved for an expected launch in 2017, now is the time to incorporate what we have learned into the next generation of AMP.

We propose to improve the reliability of our estimates of stellar properties over the next 4 years by collaborating with two open-source development projects that will augment and ultimately replace the stellar evolution and pulsation models that we now use in AMP. Our current treatment of the oscillations does not include the effects of radiative or convective heat-exchange, nor does it account for the influence of turbulent pressure on the equilibrium structure of the star. The GYRE pulsation code (Townsend & Teitler, 2013) already includes a treatment of radiative heat-exchange, and its flexibility will allow us to incorporate additional contributions as they are quantified. We will also take advantage of the numerical stability and modular architecture of the MESA stellar evolution code (Paxton et al., 2013) to evaluate additional sources of bias that arise from the adopted input physics, and to incorporate heavy element diffusion and settling, which is not stable for all types of stars with our current models. GYRE was designed to interface with MESA models, and MESA includes modules to run our current models for easy comparison. Both projects have active developer communities, and are designed to run in parallel on multi-core architectures. The existing sample of targets with Kepler observations spanning more than 3 years will provide a rich data set to validate our new models and methods.

The golden age of asteroseismology for main-sequence and subgiant stars owes a great debt to the Kepler mission, but it promises to continue with the expected 2017 launch of NASA's Transiting Exoplanet Survey Satellite (TESS). While Kepler was able to provide asteroseismic data for hundreds of targets and could simultaneously monitor 512 stars with 1-minute sampling, TESS will observe the brightest Sun-like stars in the sky at a cadence sufficient to detect solar-like oscillations in thousands of targets. These bright stars are much better characterized than Kepler targets, with known distances and reliable constraints from ground-based observations, making asteroseismic analysis even more accurate. With several years of development time available, AMP promises to be ready to convert the avalanche of data from TESS into reliable estimates of the properties of the solar system's nearest neighbors.

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1 Significance & Objectives

The physical properties of stars are fundamental to our understanding of exoplanets and the Galaxy. The space-based photometry revolution now allows us to probe the interiors of stars and to characterize their planetary systems with high precision. The outer layers of stars like the Sun are convective, with highly turbulent motions carrying heat energy out to the surface where it is radiated away. This churning creates low-frequency sound waves that travel deep into the stellar interior and bring information to the surface in the form of periodic brightness variations. Like a giant musical instrument, the star can resonate not just with one musical note but with an entire symphony of discrete harmonics across a wide range of frequencies. Just as the human ear can easily distinguish between the sound of a violin and a cello from the timbre of their notes, the frequencies exhibited by a star are fundamentally an indication of its size and structure. By passing the signal through the equivalent of a stereo equalizer, we can separate it into the constituent harmonics to reveal more subtle information about the star including its density, composition, and age. Recently the number of main-sequence and subgiant stars known to exhibit these *solar-like oscillations* has increased dramatically. While only a few such data sets were available for detailed modeling just a decade ago, NASA’s *Kepler* mission has produced suitable observations for hundreds of new targets, and the recently-approved Transiting Exoplanet Survey Satellite (*TESS*, Ricker et al., 2014) promises to push the sample into the thousands. This rapid expansion in observational capacity demands a shift in analysis and modeling strategies to yield uniform sets of derived stellar properties more quickly and easily, which is the main thrust of this proposal.

Solar-like oscillations exhibit a broad envelope of power with a peak frequency ν_{\max} that scales approximately with the acoustic cutoff frequency (Brown et al., 1991; Belkacem et al., 2011). Within this envelope, the geometry of each oscillation mode is characterized by a radial order n and spherical degree ℓ , and only the low-degree ($\ell \leq 3$) modes are generally detectable without spatial resolution across the surface (see Figure 1). Consecutive radial orders define the so-called large frequency separation $\Delta\nu_0$, which reflects the mean stellar density (Tassoul, 1980), while the small frequency separation between adjacent radial ($\ell = 0$) and quadrupole ($\ell = 2$) modes is sensitive to chemical gradients in the core that reflect the stellar age (see Brown & Gilliland, 1994). The technique of *asteroseismology* attempts to determine the stellar structure and dynamics by interpreting these global oscillations.

For many purposes, the most interesting quantities to emerge from asteroseismic analysis are the stellar radius, mass, and age. For stars with planetary companions, the stellar radius is needed to establish the absolute planetary radius from transit photometry. The mass provides the absolute scale of the orbit, and when combined with radial velocity measurements yields the absolute mass of the planet. The age is important for assessing the dynamical stability of the system and establishing its chronology with respect to other planetary systems. For relatively faint stars, where only ν_{\max} and $\Delta\nu_0$ can be determined from the observations, empirical scaling relations can be used in conjunction with the effective temperature (T_{eff}) to estimate the stellar radius and mass. Comparisons with stellar models can use additional information from ground-based spectroscopy ($\log g$, $[M/H]$) to provide more precise estimates of the radius and mass, along with some information about the age of the star (and its planetary system). The most precise constraints on all of these properties—as well as information about the interior composition—come from models that match the dozens of individual oscillation frequencies.

Anticipating the flood of observations that *Kepler* would produce for stars like the Sun, PI Metcalfe began in 2004 developing a method for the automated interpretation of solar-like oscillations. The idea was to teach a supercomputer how to model the observations as well as the experts, but to do it automatically and consistently using state-of-the-art tools. After more than five years

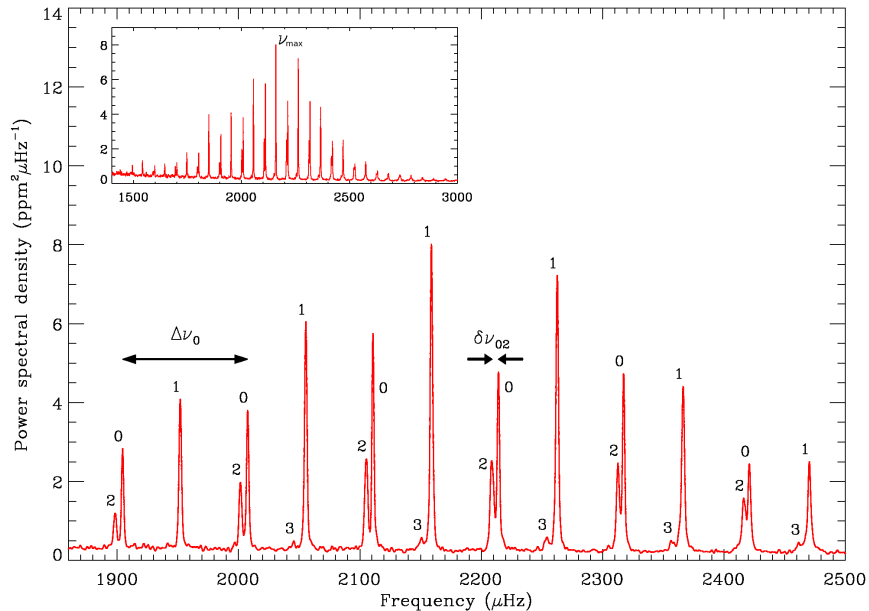


Figure 1: Solar-like oscillations in the bright star 16 Cyg A as observed by *Kepler*, showing the characteristic large and small frequency separations $\Delta\nu_0$ and $\delta\nu_{02}$. The frequency of maximum oscillation power ν_{\max} is indicated in the inset (adapted from Chaplin & Miglio, 2013).

of development, Metcalfe et al. (2009) demonstrated the effectiveness of the technique using observations of the Sun, and it was made available on a community website called the Asteroseismic Modeling Portal (AMP, Woitaszek et al., 2009). The approach was subsequently validated using some of the best asteroseismic observations to emerge from *Kepler* (Metcalfe et al., 2010, 2012), and it has now been applied to dozens of other stars (Mathur et al., 2012; Metcalfe et al., 2014) including many with planetary systems (Howell et al., 2012; Borucki et al., 2012; Carter et al., 2012; Chaplin et al., 2013; Gilliland et al., 2013; Ballard et al., 2014; Silva Aguirre et al., 2014). The dominant source of bias in our estimates of stellar properties from asteroseismology arises from incomplete modeling of the near-surface regions of the star. In the past it was sufficient to apply an empirical correction to the calculated oscillation frequencies based on the size of the discrepancy observed for the Sun (Kjeldsen et al., 2008), but more precise observations are beginning to reveal the limitations of this approach. *Now is the time to address the underlying sources of systematic error in the stellar models, before the next deluge of observations from the TESS mission.*

TESS will do for the brightest stars in the sky what *Kepler* did for 100 square degrees in the summer Milky Way. Already approved by NASA for an expected launch in late 2017, *TESS* is an all-sky planet search that will monitor the brightness of more than 200,000 stars at a cadence sufficient to detect solar-like oscillations in thousands of targets. The baseline mission will observe the northern ecliptic hemisphere in a “step and stare” mode during the first year, followed by the southern ecliptic hemisphere during the second year. Fields near the ecliptic plane will be observed continuously for 27 days, while those near the poles (around the continuous viewing zone of JWST) will overlap for extended time series of up to 1 year duration. The eccentric lunar resonance orbit will provide a >95% duty cycle, with data downlink near perigee for 4 hours every 13.7 days. Asteroseismic detections are expected for 6000 nearby stars, including FGK dwarfs and subgiants with visual magnitudes below $V \sim 7.5$ (Ricker et al., 2014). These brighter stars will generally be much better characterized than the *Kepler* targets—with parallaxes from *Hipparcos* and ultimately

Gaia (Perryman et al., 2001), as well as additional constraints from ground-based spectroscopy and in some cases interferometry—making asteroseismic analysis potentially even more precise and accurate (e.g., Silva Aguirre et al., 2012; Huber et al., 2012).

The primary objective of this proposal is to improve the reliability of our estimates of stellar properties from asteroseismology in preparation for the TESS mission. To realize this broad goal, we will collaborate with two open-source development projects (MESA and GYRE) that will augment and ultimately replace the stellar evolution and pulsation models that we now use in AMP (**section 2**). We propose to couple a powerful Bayesian analysis method to our existing parallel genetic algorithm, allowing us to marginalize over the dominant source of systematic error in our stellar models and providing a quantitative basis for minimizing other sources that arise from our choice of input physics (**section 3.1**). The majority of our development work will involve adapting the MESA stellar evolution code (Paxton et al., 2013) to interface with our existing optimization method, incorporating heavy element diffusion and settling into the AMP model-fitting approach, and augmenting the non-adiabatic treatment of oscillations within GYRE (Townsend & Teitler, 2013) to include additional terms. We will also extend the parallel capabilities of both codes to enhance performance on the XSEDE supercomputers that support AMP (**section 3.2**). Finally, we will validate the reliability, precision, and accuracy of the improved models and methods by applying the updated stellar model-fitting pipeline to synthetic data, Sun-as-a-star observations, and a sample of *Kepler* targets with independently constrained stellar properties (**section 3.3**). The proposed work will clearly contribute to the strategic objectives of the *TESS* mission by facilitating the interpretation of asteroseismic data for thousands of targets including exoplanetary systems (**section 4**), and it will leverage and augment the capabilities of the *Kepler* Asteroseismic Science Consortium (KASC) for the benefit of the broader community (**section 5**).

2 Technical Approach

In the past, ground-based data on solar-like oscillations emerged slowly enough that we could try to model one star at a time. The *Kepler* mission has now produced asteroseismic data for hundreds of solar-type targets, and the *TESS* mission will soon yield similar observations for thousands of nearby stars. With previous support from NASA (NNX09AE59G, PI: Metcalfe, 2009–2011), we developed a stellar model-fitting pipeline to match the asteroseismic observations from *Kepler*, and we made it available to the broader community through the Asteroseismic Modeling Portal (AMP, **section 2.1**), a Science Gateway website tied to XSEDE supercomputing resources. The space-based observations are of such high quality that the uncertainties in stellar properties derived by AMP are now dominated by the limitations of our statistical analysis methods (**section 2.2**) as well as the adopted stellar evolution models and our adiabatic treatment of the oscillations. To address these limitations and prepare for the next wave of data from the *TESS* mission, we will adapt the MESA stellar evolution code (**section 2.3**) to interface with our current model-fitting method. We will also extend the non-adiabatic treatment of the oscillations in the GYRE pulsation code (**section 2.4**), which is already included as a module within MESA.

2.1 Asteroseismic Modeling Portal

The Asteroseismic Modeling Portal (AMP, Woitaszek et al., 2009) is a web-based interface to the stellar model-fitting pipeline described in detail by Metcalfe et al. (2009, 2014). The underlying science code uses a parallel genetic algorithm (GA, Metcalfe & Charbonneau, 2003) on XSEDE supercomputing resources to optimize the match between asteroseismic models produced by the Aarhus stellar evolution code (ASTEAC, Christensen-Dalsgaard, 2008b) and adiabatic pulsation

code (ADIPLS, Christensen-Dalsgaard, 2008a) and a given set of observational constraints. The models are configured to use the Grevesse & Noels (1993) solar mixture with the OPAL 2005 equation of state (see Rogers & Nayfonov, 2002) and the most recent OPAL opacities (see Iglesias & Rogers, 1996), supplemented by Ferguson et al. (2005) opacities at low temperatures. The nuclear reaction rates come from the NACRE collaboration (Angulo et al., 1999), convection is described by the mixing-length theory from Böhm-Vitense (1958) without overshoot, and the effects of helium diffusion and settling are included following the prescription of Michaud & Proffitt (1993). To correct the calculated frequencies for so-called *surface effects* due to incomplete modeling of the near-surface layers, AMP uses the empirical correction proposed by Kjeldsen et al. (2008).

Mathur et al. (2012) were the first to apply AMP to a large sample of *Kepler* targets. They simultaneously optimized the match between the models and two sets of constraints: [1] the individual oscillation frequencies and [2] the atmospheric parameters derived from spectroscopy. This procedure generally yielded stellar radii, masses and ages that were consistent with empirical scaling relations and grid-based modeling of the global oscillation properties (ν_{\max} and $\Delta\nu_0$)—but with significantly improved precision. However, the optimal models for the 22 targets included six stars with an initial helium mass fraction Y_i significantly below the primordial value from standard Big Bang nucleosynthesis ($Y_P = 0.2482 \pm 0.0007$, Steigman, 2010), and four additional stars that were marginally below Y_P . The original motivation for including these sub-primordial values in the search was a recognition that there could be systematic errors in the determination of Y_i , but the source of the bias was not identified at that time.

As part of a study of convective cores in two *Kepler* targets, AMP was compared to several other fitting methods by Silva Aguirre et al. (2013). In addition to the individual frequencies and spectroscopic constraints, some of these methods also used sets of frequency ratios that eliminated the need to correct the model frequencies for surface effects (Roxburgh & Vorontsov, 2003). A comparison of the AMP results with models that used the frequency ratios as additional constraints revealed systematic differences in the interior structure that were correlated with the initial helium abundance. Metcalfe et al. (2014) subsequently modified the AMP optimization procedure to mitigate this bias, by adopting the frequency ratios as additional constraints and by reducing the weight at higher frequency, where the correction for surface effects is larger. Although the frequency ratios help to discriminate between families of models that provide comparable matches to the other sets of constraints, the individual frequencies contain additional information that we would like to exploit. The primary difficulty is that the Kjeldsen et al. (2008) correction for surface effects injects a bias into the determination of some stellar properties.

In the Kjeldsen et al. (2008) prescription, the differences between the observed and calculated frequencies are parametrized by a function of the form $a_0[\nu/\nu_0]^b$ with the amplitude a_0 determined from the data and the exponent b fixed at a solar-calibrated value near 4.9. The actual solar surface effect appears more linear at high frequencies (Christensen-Dalsgaard et al., 1996), so assuming any fixed exponent will tend to over-correct the highest-order modes. This tendency interacts with intrinsic parameter correlations—in particular, the well-known correlation between mass and initial helium abundance in stellar models—to favor higher-mass low-helium models that fit the frequencies better while getting the interior structure wrong. Including the frequency ratios as additional constraints favors the lower-mass higher-helium models, but it does not entirely eliminate the bias caused by the high-frequency modes. To help mitigate this bias, AMP now adopts an uncertainty for each frequency that is the quadratic sum of the statistical error and half the surface correction. This procedure implicitly acknowledges that surface effects represent a systematic error in the models (Guenther & Brown, 2004). Without a precise constraint on the luminosity and/or radius, this approach is required even to recover accurate solar properties from Sun-as-a-star helioseismic data (Metcalfe et al., 2009)—but we identify an alternative in the next section.

2.2 Bayesian Parallel Genetic Algorithm

Since we want to create a general-purpose model-fitting pipeline for asteroseismic data, our first task is to select a global method for optimizing the match between our model output and the available observations of any given star. Using only observations and the constitutive physics of the model to restrict the range of possible values for each parameter, a genetic algorithm (GA, Charbonneau, 1995; Metcalfe & Charbonneau, 2003) provides a relatively efficient means of searching globally for the optimal model. Although it is more difficult for a GA to find exact values for the optimal set of parameters efficiently, it is well suited to search for the region of parameter space that contains the global solution. In this sense, the GA is an objective means of obtaining a good starting point for a more traditional local analysis method, which can narrow in on the precise values and uncertainties of the optimal model parameters. After experimenting with several algorithms that employ different strategies for the global search (e.g., simulated annealing, Markov Chain Monte Carlo), we found the GA to be the most efficient choice in terms of minimizing the number of model evaluations required to obtain the solution, while also being less sensitive to the values of internal parameters that control the convergence behavior of the algorithm.

Our implementation of the GA optimizes four adjustable model parameters, including the stellar mass (M) from 0.75 to 1.75 M_{\odot} , the metallicity (Z) from 0.002 to 0.05 (equally spaced in $\log Z$), the initial helium mass fraction (Y_i) from 0.22 to 0.32, and the mixing-length parameter (α) from 1.0 to 3.0. The stellar age (t) is optimized internally during each model evaluation by matching the observed value of $\Delta\nu_0$, which decreases almost monotonically from the zero age main-sequence (ZAMS) to the base of the red giant branch (Christensen-Dalsgaard, 1993). The GA uses two-digit decimal encoding, so there are 100 possible values for each parameter within the ranges specified above. Each run of the GA evolves a population of 128 models in parallel through 200 generations to find the optimal set of parameters, and we execute four independent runs in parallel with different random initialization to ensure that the best model identified is truly the global solution. This method requires about 10^5 model evaluations, compared to 10^8 models for a complete grid at the same sampling density, making the GA nearly 1000 times more efficient than a complete grid (currently several days of computing time, compared to many years for a grid). Of course, a grid can in principle be applied to hundreds of observational data sets without calculating additional models—but the GA approach also gives us the flexibility to improve the physical ingredients over time, while the physics of a grid are fixed.

The metric that we currently pass to the GA for each model is constructed from up to four normalized χ^2 values that reflect the quality of the match to the individual frequencies, two sets of frequency ratios, and any additional constraints from photometry, spectroscopy, astrometry, and interferometry. This procedure recognizes that each oscillation frequency is not completely independent, but it uses the information content in several different ways to create sub-metrics that can be traded off against each other during the optimization. Using a composite metric like this prevents any one set of constraints from dominating the solution, but it also compromises the statistical interpretation of the results. An alternative has been proposed in the context of grid-based modeling by Gruberbauer et al. (2012), who devised a Bayesian analysis method that implements a full treatment of systematic errors like surface effects. It eliminates the need to apply an empirical correction to the model frequencies prior to the comparison with observations, and it results in a consistent set of probabilities that allow different model physics to be meaningfully compared. Gruberbauer & Guenther (2013) demonstrated the benefits of this Bayesian analysis method using solar observations, and Gruberbauer et al. (2013) extended this success to a sample of 23 *Kepler* targets. Although the GA does not produce a complete grid of models during the process of finding the optimal match to the observations, we can exploit this new approach to define

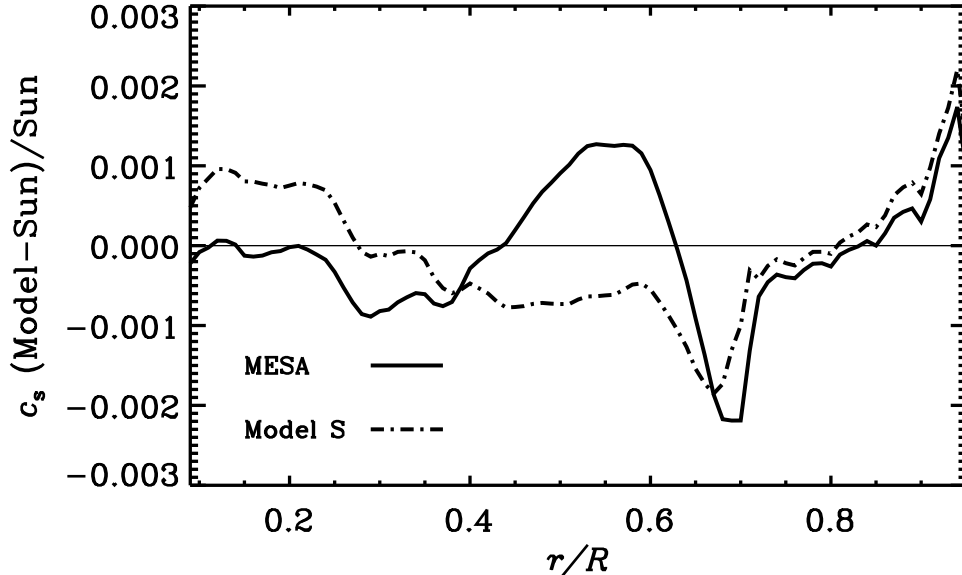


Figure 2: Deviations from the helioseismically inferred sound speed profile (Bahcall et al., 1998) for a MESA model and for the benchmark “Model S” from Christensen-Dalsgaard et al. (1996). [Figure reproduced from Paxton et al. (2013)].

a Bayesian metric for the quality of each model and pass it to the GA for optimization instead of the composite metric that we developed for *Kepler* data. This will allow us to deal effectively with the dominant source of systematic error in a statistically rigorous way, and it will help us quantify the more subtle biases due to our choice of model input physics. We provide additional details about our plans to use this method for the proposed work in section 3.1.

2.3 MESA Stellar Evolution Models

Modules for Experiments in Stellar Astrophysics (MESA) is an open-source stellar evolution code developed over six years by Paxton et al. (2011), and recently extended with new asteroseismic capabilities by Paxton et al. (2013). It employs modern numerical methods that make it robust and efficient, and it was designed from the beginning for parallel processing in multi-core shared memory environments using OpenMP. The input physics are modular, with an independent Fortran 95 library for each of the equations of state, opacity tables, nuclear reaction rates, diffusion and settling routines, and atmospheric boundary conditions. The stellar evolution module solves the fully-coupled structure and composition equations simultaneously, and it operates reliably over a wide range of stellar masses and evolutionary states. To provide one example of MESA’s bona fides, Figure 2 shows the difference between the sound speed profile predicted by MESA’s Standard Solar Model (distributed as one of the code’s validation tests), and the corresponding profile inferred from helioseismic inversion. Also shown is the profile produced by “Model S” from Christensen-Dalsgaard et al. (1996), long regarded as a gold-standard evolutionary model for the Sun. Both models employ comparable input physics and assume solar abundances from Grevesse & Sauval (1998) and Grevesse & Noels (1993), respectively. Clearly, the MESA model fits the observations at a level comparable to Model S, and is particularly effective at small fractional radii (important for correctly reproducing small frequency separations).

The asteroseismic module offers a variety of workflows for analysis of MESA models. At the simplest level, models can be written to disk in several standardized formats (e.g., `fgong`), for

subsequent analysis by a stand-alone oscillation code. Greater flexibility comes through MESA’s ability to pass models internally to the asteroseismic module *after each evolutionary timestep*, for processing with compiled-in versions of the ADIPLS or GYRE oscillation codes. The calculated frequencies are then passed back to MESA, which compares them with a set of user-supplied observations to produce a χ^2 metric. This allows MESA to optimize the stellar age along the track in real time, potentially avoiding more advanced stages of stellar evolution that can be computationally expensive. For further efficiency gains, a hierarchical approach can be adopted: calculation of radial-mode frequencies only takes place once the large frequency separation (estimated from the mean density of the model) matches the observations reasonably well, and calculation of non-radial frequencies only occurs once the radial frequencies provide a good fit to the observations.

In addition to optimizing the stellar age, MESA’s asteroseismic module can fine-tune parameters governing a star’s evolutionary trajectory, using local optimization methods for χ^2 minimization such as the simplex algorithm (Nelder & Mead, 1965) or the “Bound Optimization BY Quadratic Approximation” technique recently described by Powell (2009). These methods are controlled through a number of parameter bounds and step sizes, and users have full control over the relative weight assigned to the different sets of observables. Using the optimal set of parameters identified by the GA as a starting point, the local approach in MESA will evolve a pre-main-sequence (PMS) model and find the best match along that single evolutionary track. The code will then recalculate the track, again initiated at the PMS, with slightly different parameters such as mass, composition, mixing-length and overshoot, and repeat this process until the locally optimal solution has been found. Thus, MESA already includes some of the key capabilities that will allow it to be substituted for ASTEC in the current version of AMP.

2.4 GYRE Oscillation Calculations

GYRE is a new open-source code, developed by Co-I Townsend (see Townsend & Teitler, 2013), that calculates the oscillation frequencies of an input stellar model. As with all oscillation codes, these frequencies are defined as the eigenvalues of a two-point boundary eigenvalue problem formed by considering small-amplitude perturbations to the fluid conservation equations (mass, momentum, and energy), the radiative diffusion equation and Poisson’s equation (see Cox, 1980; Unno, 1989; Aerts et al., 2010). To isolate the eigenvalues GYRE employs a novel ‘Magnus Multiple Shooting’ (MMS) numerical scheme, which draws on the respective strengths of the traditionally-adopted relaxation and shooting schemes while avoiding their known weaknesses. This scheme allows GYRE to achieve outstanding accuracy (with a frequency error that scales as the sixth power of the grid spacing), while remaining remarkably robust. To take advantage of multiple processor cores and/or cluster nodes, GYRE implements the MMS scheme using a combination of OpenMP and MPI.

A key difference between GYRE and other publicly available oscillation codes (e.g., ADIPLS) is that GYRE includes non-adiabatic effects arising from thermal transfers between adjacent fluid elements. In their asteroseismic analysis of 23 *Kepler* targets, Gruberbauer et al. (2013) found that the Bayesian probabilities were uniformly higher when non-adiabatic frequencies were used for the model-fitting, and that for most stars the resulting amplitude of the surface effect was reduced. As with the current implementation of non-adiabatic effects in GYRE, their model accounted for radiative heat-exchange but neglected perturbations to the convective flux and turbulent pressure (Guenther, 1994). In the case of the Sun, the stability and frequency of the oscillation modes depends substantially on turbulent pressure and the inclusion of non-local effects in the treatment of convection (Balmforth, 1992b,a; Houdek, 2010), so these are the leading candidates to explain the residual surface effects in other stars. We outline our plans for further development of the non-adiabatic capabilities of GYRE in section 3.2.

3 Proposed Research

The development of AMP and its application to a wide variety of *Kepler* targets has demonstrated the potential of asteroseismology to yield precise stellar properties. This is particularly important for exoplanet host stars, because the derived asteroseismic radius, mass, age and composition can provide useful constraints on the properties of the associated planetary systems. The focus on matching individual oscillation frequencies typically improves the precision by a factor of two or more over pipelines that only use the global oscillation properties (Metcalf et al., 2014), and the uniform analysis method produces an ensemble of results that can probe broader questions about the formation and evolution of stellar and planetary systems. As the quality of asteroseismic data has steadily improved over the past few years, we have identified several limitations of the approach used by AMP that currently hinder the transition from *precision* to *accuracy* in our asteroseismic inferences. With the *TESS* mission poised to expand the sample of asteroseismic targets from the hundreds into the thousands, the next few years represent an important window of opportunity to update our models and methods. We propose to improve the reliability of our estimates of stellar properties from AMP over the next 4 years, which will involve:

- **Phase 1:** Implementing a Bayesian treatment of the dominant source of systematic error common to all 1D stellar evolution codes, which arises from incomplete modeling of the near-surface layers. This approach will eliminate the need to apply an empirical correction to the calculated oscillation frequencies, and it will provide a quantitative basis for meaningful comparisons of different input physics and other model assumptions.
- **Phase 2:** Adapting the MESA stellar evolution code to interface with the parallel genetic algorithm that runs behind AMP, and including additional non-adiabatic terms in the GYRE pulsation module. The numerical stability of MESA will permit a full treatment of heavy element diffusion and settling to alleviate biases in the derived stellar composition, while the non-adiabatic work will reduce the propagated uncertainties from surface effects.
- **Phase 3:** Validating the reliability of the improved stellar model-fitting pipeline using synthetic data, optimizing the precision of the derived stellar properties using solar observations with a variety of input physics, and verifying the accuracy of the results using a sample of *Kepler* targets with independently determined luminosities from space-based parallaxes and/or radii from ground-based interferometry.

Our collective experience developing the AMP pipeline for *Kepler* data and contributing to the development of MESA and GYRE suggests that we will be able to complete this work during the proposed timeline. We provide additional details for each of these phases of the project in the three subsections below.

3.1 Implementing the Bayesian Analysis Method

Any attempt to match the solar-like oscillation frequencies observed in a star with those calculated from 1D stellar models must adopt a method of dealing with surface effects. Mixing-length theory fails to describe accurately the thermodynamic structure of the near-surface layers, leading to significant errors in the calculated oscillation frequencies. This systematic error is almost negligible at the lowest frequencies, but it grows to a few parts per thousand at the highest observed frequencies which probe the near-surface structure more substantially. To avoid having the surface effect markedly bias the model-fitting, it is common to apply an empirical correction

to the calculated frequencies, as suggested by Kjeldsen et al. (2008), which is essentially a solar-calibrated power law. This is a good first approximation, but it still introduces some biases in the derived stellar properties (Mathur et al., 2012; Metcalfe et al., 2014). Another approach is to match carefully constructed ratios of the observed frequencies that are designed to be insensitive to the near-surface regions (Roxburgh & Vorontsov, 2003), but this method injects correlations into the problem that complicate the error analysis. An alternative pioneered by Gruberbauer et al. (2012) is to perform a Bayesian analysis that accounts for systematic errors and propagates the resulting uncertainty into the derived stellar properties. *In the first phase of this project, we will incorporate this Bayesian analysis method into the AMP parallel genetic algorithm, allowing us to circumvent the dominant source of bias in our current approach and facilitating the assessment of other systematic uncertainties due to our adopted input physics and other model assumptions.*

3.1.1 Marginalizing over the Unknown Surface Effect

Gruberbauer et al. (2012) describe a Bayesian analysis method for treating unknown systematic errors in the models without applying any *ad hoc* correction to the calculated frequencies. The basic idea is to calculate the Bayesian evidence consistently for each model in a dense grid, allowing for the possibility that the calculated frequencies may be systematically higher or lower than the observations. The possible magnitude of the error Δ_i on each frequency is constrained by adopting a maximum range and a suitable prior. To avoid ambiguity in the radial mode identification of each frequency, Δ_i should not exceed the large frequency separation $\Delta\nu$, so this defines the maximum range. A slightly more prescriptive alternative is to impose this maximum range on the highest observed radial order and assume a power law similar to that of Kjeldsen et al. (2008) to define the maximum range for the other frequencies. This makes the maximum systematic error smaller at low frequencies, without imposing a functional form on the actual surface effect (just the maximum). Within the specified range, the most general approach is to use a flat uniform prior on the possible systematic error, but Gruberbauer et al. (2012) argue for adopting a so-called *beta* prior [their Eq.(19)] to assign more weight to models that minimize the unknown errors (see Figure 3). This is the only prior that allows for a linearly decreasing probability density, is properly normalized, and reaches zero at $\Delta_{i,\max}$. By applying this type of analysis to each model (whether in a pre-calculated grid or evaluated on the fly by the GA) with the data fixed, it is possible to marginalize over the surface effects (i.e. integrate over the probabilities due to all possible corrections for each model) and determine them empirically for each star, given a set of input physics and other model assumptions. *We will use this Bayesian analysis method to replace the quality metric that is currently passed to the parallel genetic algorithm to identify the optimal model within AMP.*

3.1.2 Assessing other Systematic Uncertainties

A key advantage of Bayesian analysis is that it provides a framework for evaluating the systematic uncertainties due to our choice of input physics and other model assumptions. For example, Gruberbauer & Guenther (2013) applied a Bayesian analysis method to Sun-as-a-star helioseismic observations from the BiSON network (Broomhall et al., 2009) to compare the evidence for different solar mixtures (Grevesse & Sauval, 1998; Asplund et al., 2005, 2009). Model grids were constructed for each mixture, the Bayesian probabilities for each model in each grid were calculated from a comparison with identical data, and the properly normalized sum yielded a Bayesian evidence for each grid as a whole. Following the convention proposed by Jeffreys (1961), logarithmic differences in the evidence smaller than 0.5 are considered insignificant, those between 0.5 and 1.0 are marginal, and only differences above 1.0 imply decisive evidence for one alternative over an-

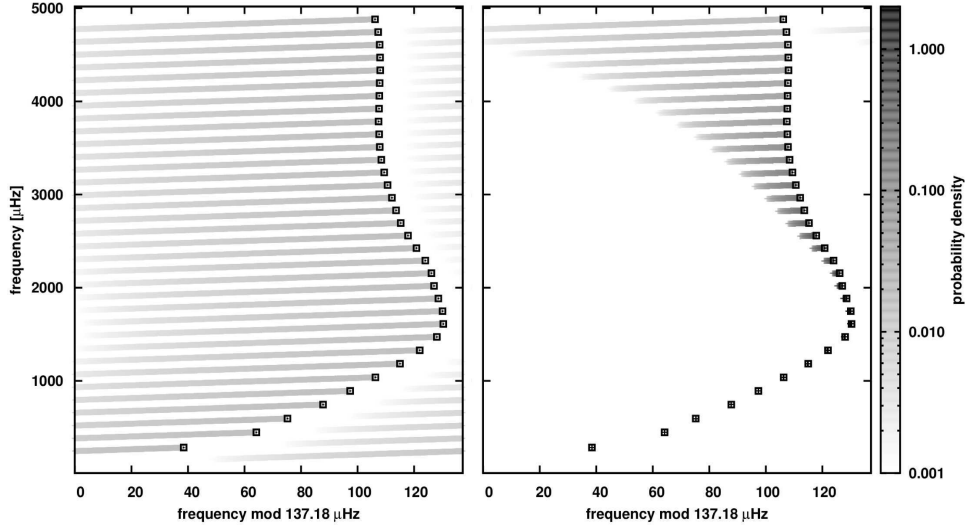


Figure 3: Behavior of the *beta* prior for systematic errors in an échelle diagram, where oscillation modes with the same geometry form a nearly vertical ridge at high frequency. The squares represent model frequencies, while the shaded trails indicate the prior probability density for varying Δ_i . The left panel uses a constant range of $\Delta\nu$ at each frequency, whereas the right panel uses a power law to prescribe smaller errors at the lowest frequencies (adapted from Gruberbauer et al., 2012).

other. The revised solar abundances were found to be more probable in general, but the evidence was marginal—suggesting that systematic errors in the models are currently too large to resolve the controversy. Although the parallel genetic algorithm in AMP does not calculate a complete grid, it does evaluate about 10^5 models in the vicinity of the best fit during the search for the optimal solution. Extending the grid or even calculating a full grid would have no effect on the unweighted probabilities because they quickly drop to zero. If we record the Bayesian probability for each evaluated model in a logfile, then we can calculate the evidence for the grid as a whole provided that we normalize properly. Furthermore, as long as the data do not change from one run to the next, we can meaningfully compare these grid-level evidence values to determine which particular choice of input physics (equations of state, opacities, nuclear reaction rates) and other model assumptions (treatment of surface effects, non-adiabatic terms included) are more probable. This approach can be applied immediately with the current version of AMP that uses ASTEC and ADIPLS, but it will be even more powerful when we begin using MESA and GYRE, which are modular by design. *We will use the Bayesian evidence to optimize the input physics and other model assumptions, and to maximize the precision of the resulting asteroseismic inferences.*

3.2 Adapting MESA and GYRE to Interface with AMP

From our experience adapting ASTEC and ADIPLS to interface with the parallel genetic algorithm, we will need to configure the codes to suppress verbose diagnostic output and excessive file input/output, both of which can significantly degrade performance when running hundreds of models in parallel. We also plan to extend the parallel capabilities of the codes to exploit the Xeon Phi coprocessors that are available on the XSEDE supercomputing facilities that we use for AMP. Aside from the obvious advantages to using modern, open-source codes with large developer communities, our primary motivations for replacing ASTEC and ADIPLS come from their numerical and physical limitations. The stellar models from ASTEC have the option to include heavy

element diffusion and settling, but the code is not numerically stable in this configuration for all types of stars—leading to subtle biases in the derived composition and inducing small systematic errors in the other stellar properties. The oscillation frequencies produced by ADIPLS do not account for *any* non-adiabatic terms, yielding a larger surface effect and inflating the uncertainties on the stellar properties even with the Bayesian analysis method described above. *In the second phase of this project, we will adapt the MESA and GYRE codes to interface with AMP, alleviating additional sources of bias by including heavy element diffusion and settling in the evolution models and accounting for more non-adiabatic terms in the pulsation calculations.*

3.2.1 Including Heavy Element Diffusion and Settling with MESA

Diffusion and settling of helium and heavy elements certainly operates in the envelopes of real stars, leading to changes in the composition and structure of the outer layers that can be probed with asteroseismology. If we choose to neglect some of these processes in our models, it will yield systematic errors in the derived stellar properties. For example, in the early development of the pipeline for *Kepler* data, we initially neglected all diffusion processes and found an optimal model for the Sun with a mass of $1.05 M_{\odot}$. By including helium diffusion and settling in the models, we brought this systematic error down to about 1% and found acceptable agreement with the solar composition and age (Metcalf et al., 2009). Extending this approach to larger samples of stars has shown that we might be able to probe Galactic chemical enrichment from the asteroseismic bulk composition (Metcalf et al., 2014), but it would be difficult to trust such a result without a proper treatment of heavy element diffusion and settling in the models. MESA treats chemical diffusion and gravitational settling by solving Burger’s equations using the method and diffusion coefficients of Thoul et al. (1994). The transport of material is calculated using the semi-implicit, finite difference scheme described by Iben & MacDonald (1985). *We will use a configuration of the MESA code that treats diffusion and settling of helium and heavy elements to allow an unbiased determination of the asteroseismic bulk composition.*

3.2.2 Augmenting the Non-Adiabatic Treatment of Oscillations in GYRE

GYRE currently neglects perturbations to the turbulent pressure and to convective energy transport (the latter through a variety of approaches to so-called *convective freezing*; e.g., see Pesnell, 1990). As we discuss in section 2.4, these phenomena have a significant impact on the stability and frequency of oscillation modes in the Sun, and are leading candidates for explaining the residual surface effects in other stars after accounting for radiative non-adiabatic processes. Accordingly, we will further develop GYRE to include these phenomena. The first step will be to modify the oscillation equations to incorporate perturbations to the turbulent pressure, guided by the earlier work of Mihalas & Toomre (1981) and Christensen-Dalsgaard & Frandsen (1983). Assessing the impact on mode frequencies will give us a preliminary handle on the extent to which purely mechanical processes can explain the residual surface effects. Next, we will implement a more self-consistent treatment based on the time-dependent convection approach pioneered by Unno (1967) and more recently expanded upon by Grigahcène et al. (2005), which includes perturbations to the convective flux, the turbulent Reynolds stress and the turbulent kinetic energy dissipation.

We also plan to improve other aspects of GYRE’s non-adiabatic treatment. We will replace the diffusion equation for radiative energy transport with the Eddington approximation—as discussed by Ando & Osaki (1975), the latter correctly models the optically thin limit and eliminates spurious excitation that the former may produce in this limit. We will moreover improve the algorithm used by GYRE to hunt down modes, by implementing better initial guesses for their complex

eigenfrequencies based on integral expressions (Cox, 1980), and by further developing a complex Ridders-based root solver that Co-I Townsend has recently prototyped.

3.3 Validating the Stellar Model-Fitting Pipeline

The development of our stellar model-fitting pipeline is not complete until the functionality has been validated with synthetic data, and the performance has been optimized using actual observations. The validation procedures are well established from our previous experience developing a similar pipeline for *Kepler* data, but the sample of stars with independently determined properties is now much larger. *In the third phase of this project, we will verify the reliability, precision and accuracy of the updated models and methods within AMP, ensuring that our tool is prepared for the dramatically expanded sample of asteroseismic targets expected from the TESS mission.*

3.3.1 Reliability: Initial Tests with Synthetic Data

Beyond the mechanics of modifying the AMP code to employ a Bayesian approach to optimization and to interface with MESA and GYRE, we will need to pass synthetic data through the updated fitting procedure to optimize the efficiency of the method. This will involve one team member calculating the theoretical oscillation frequencies for a specific set of model parameters, and then giving a subset typical of what is available from actual observations to another team member who does not know the source parameters. The frequencies are then passed through the complete optimization method in an attempt to recover the source parameters without any additional information. The results of such *hare & hound* (H&H) exercises are used to quantify the success rate of the optimization method (the fraction of independent runs that lead to the known source parameters), and to improve its efficiency (minimize the number of model evaluations) if possible.

We will conduct a series of H&H exercises spanning a representative range of model parameters. We will begin by using the exact frequencies from the model, and then add various levels of random Gaussian noise from the $\sim 0.1 \mu\text{Hz}$ that is found for the longest *Kepler* observations up to the $\sim 0.4 \mu\text{Hz}$ expected for the shortest *TESS* time series. We will include additional observables such as T_{eff} , $[\text{M}/\text{H}]$, and luminosity with typical random and systematic errors, allowing us to determine an optimal balance between the asteroseismic and other constraints at various signal-to-noise levels before we deal with the additional complications inherent in real observations.

3.3.2 Precision: Optimal Configuration from Solar Observations

Ultimately, our model-fitting pipeline can only be judged a success if it leads to precise estimates of the stellar properties for the star that we know best: the Sun. There are many ingredients in our models that could in principle be insufficient descriptions of the actual conditions inside of real stars—deficiencies that could easily lead to large uncertainties in our determinations of the optimal model parameters for a given set of oscillation data. Although it is natural to focus on using solar data to test the *accuracy* of our pipeline results, we will instead use the Sun to optimize the *precision* and use a larger ensemble of stars to validate the accuracy. This provides a hedge against the possibility that the Sun might be peculiar in some way (e.g., Böhm-Vitense, 2007), such that fine-tuning our models to reproduce it would actually hinder our ability to match other stars. For this work, we will adopt the Sun-as-a-star helioseismic data from Broomhall et al. (2009). Exploiting the modular architecture of the MESA and GYRE codes, and using the Bayesian analysis method to evaluate different model configurations following Gruberbauer & Guenther (2013), *we will determine the combination of input physics and other model assumptions that yields the most precise values of the solar radius, mass, age and composition within a reasonable accuracy tolerance.*

3.3.3 Accuracy: Kepler Targets with Independent Properties

In addition to the Sun, where the absolute properties are known with unusually high precision, we can validate the accuracy of our updated stellar model-fitting pipeline using some of the best-characterized samples of asteroseismic targets in the *Kepler* field. Silva Aguirre et al. (2012) demonstrated how the global oscillation properties even from relatively short time series could be combined with broadband photometry to yield the radius, mass, T_{eff} , and bolometric flux in a self-consistent manner—resulting in direct determinations of angular diameters and distances. For the 22 targets with precise *Hipparcos* parallaxes, the asteroseismic distances agreed to better than 4%. For the small subsample of stars with interferometrically determined angular diameters, Huber et al. (2012) found agreement with the asteroseismic results to 5%. Much more stringent tests should now be possible, using the full length data sets from *Kepler* (up to 3+ years for many targets) to model the individual frequencies and determine the asteroseismic properties with higher precision. The available samples are also likely to grow, with *K2* (Howell et al., 2014) now observing additional nearby stars with *Hipparcos* parallaxes and interferometric radii, and new parallaxes expected from the early data releases of *Gaia* in October 2015 and March 2016. *We will use the available samples of Kepler asteroseismic targets with precise parallaxes and/or interferometric radius measurements to quantify the absolute accuracy of our updated stellar model-fitting pipeline.*

4 Impact & Relevance

By the end of this project, we will have developed and validated an optimal, efficient, robust and unbiased model-fitting pipeline to address the vast quantities of asteroseismic data that will emerge from NASA’s *TESS* mission. The likelihood of success is enhanced substantially by drawing upon more than 7 years of experience creating a similar pipeline for *Kepler* data, and by collaborating with established open-source development projects that both have active developer communities. These advantages will also ensure that the final product of our efforts will be both accessible and fully exploited by a large and growing user base. Our timeline is designed to maximize the science return from NASA’s *TESS* mission, though our pipeline will also be useful for analyses of archival *Kepler* data and forthcoming observations from *K2*. It is primarily this large quantity of data that necessitates an automated approach to model-fitting, in particular to avoid the limitations and computational inefficiency of simply calculating a huge grid of models.

The work outlined in this proposal addresses NASA’s Strategic Goal 1, as defined in the 2014 Strategic Plan: “Expand the frontiers of knowledge, capability, and opportunity in space”, including several specific Objectives:

- **Objective 1.4:** *Understand the Sun* and its interactions with Earth and the solar system, including space weather. (Asteroseismic analysis of many stars will probe the fundamental physical processes that operate in the Sun under a broader range of physical conditions).
- **Objective 1.5:** Ascertain the content, origin, and evolution of the solar system and *the potential for life elsewhere*. (Measurement of stellar properties across the H-R diagram, including exoplanet host stars, will help assess the potential for life in distant solar systems).
- **Objective 1.6:** Discover how the universe works, *explore how it began and evolved*, and search for life on planets around other stars. (Characterization of stellar and planetary systems, including ages, will improve our understanding of how these systems form and evolve).

The *TESS* mission is designed to discover nearby planetary systems around some of the brightest stars in the sky. Using the techniques outlined in this proposal, the AMP stellar model-fitting

pipeline will enable the characterization of many exoplanet host stars with asteroseismology. This is essential to convert precise transit photometry into an absolute radius for the planetary body. In addition, accurate ages will provide important clues about the formation and evolution of the planetary systems. The determination of accurate stellar properties for a broad array of solar-type stars will stimulate new insights about stellar structure and evolution, and will provide a broader context for our understanding of the Sun and our own solar system.

5 Plan of Work

The work outlined in this proposal will comprise the primary research effort for PI Metcalfe, who relies entirely on grants to support his position as a Research Scientist at SSI. The AMP Science Gateway is supported by computational resources from XSEDE (formerly the TeraGrid). More than 7 million CPU-hours of computing time for the asteroseismic analysis of *Kepler* targets has already been allocated by XSEDE/TeraGrid, with 1 million more CPU-hours expected for our annual allocation in 2014-15. Annual renewal allocations are requested each April for time beginning in July, with supplement requests considered every three months.

5.1 Key Milestones

We expect the first phase of this project (Bayesian GA) to complete within 12 months. The second phase (MESA and GYRE code adaptation and extension) and third phase (code validation) will require approximately 18 months each, including publication of the results in a refereed journal and presentation at a scientific meeting. The key milestones during each year will be:

- **Year 1:** Finish incorporating the Bayesian analysis method into the existing parallel genetic algorithm and compare its performance to the traditional composite χ^2 metric. During the first summer, include perturbations to the turbulent pressure (diagonal elements of the turbulent Reynolds stress) in the adiabatic GYRE code, and update the non-adiabatic code to replace the diffusion approximation with the Eddington approximation.
- **Year 2:** Complete the interface between MESA/GYRE and AMP, using a configuration that includes heavy element diffusion and settling. During the second summer, begin including perturbations to the convective flux, the off-diagonal elements of the turbulent Reynolds stress, and the turbulent kinetic energy dissipation in the non-adiabatic GYRE code.
- **Year 3:** Complete the initial validation tests with synthetic data, verifying the functionality of the updated models and methods. Begin optimizing the MESA input physics from a Bayesian analysis of solar data. During the third summer, finish including the additional convective contributions to the non-adiabatic GYRE code.
- **Year 4:** Finish optimizing the precision of the pipeline and quantify the absolute accuracy using the sample of *Kepler* targets with independently determined luminosities and/or radii. During the final summer, complete the extension of parallel capabilities in MESA and GYRE to take full advantage of Xeon Phi coprocessors.

Our experience developing a similar pipeline for *Kepler* data suggests this is a reasonable timeline. Although the original pipeline required more than 7 years to develop, many of the methods are transferable and some of the required functionality already exists in the MESA and GYRE codes. The milestones involving validation only rely on data that are currently available in public archives.

5.2 Management Structure

Metcalfe will fully coordinate his activities with Townsend, consulting the external collaborators Guenther and Stello on projects relevant to their expertise. The collaboration will be conducted largely by email and teleconferences, with occasional gatherings at scientific meetings.

- **Travis Metcalfe (PI):** is the primary developer of the science code that runs behind the AMP Science Gateway. He is a member of the KASC steering committee and co-leads the modeling activities of the *Kepler* working group on solar-like oscillations. He will be responsible for implementing the Bayesian parallel genetic algorithm (with Guenther), adapting the MESA/GYRE code to interface with AMP (with Stello and Townsend), as well as project management and the interpretation, publication and presentation of results.
- **Rich Townsend (Co-I):** is the primary developer of the GYRE oscillation code, co-developer of the asteroseismic module in MESA, and he serves on the MESA Council—a steering committee charged with shaping a strategic vision for the code. He will be responsible for further development of the adiabatic and non-adiabatic capabilities (with Guenther), facilitating the modifications to MESA (with Metcalfe and Stello), and supporting the interpretation, publication and presentation of results.
- **David Guenther (Collaborator):** is co-developer (with Michael Gruberbauer) of the Bayesian analysis method for asteroseismology, and primary developer of a non-adiabatic oscillation code. He will advise Metcalfe on implementing the Bayesian parallel genetic algorithm, and advise Townsend on developing the non-adiabatic capabilities in GYRE.
- **Dennis Stello (Collaborator):** is the primary driver behind the development of asteroseismic capabilities in MESA and is an active member of KASC with overarching expertise linking data analysis and stellar modeling. He will advise Metcalfe and Townsend on the modifications to MESA and GYRE, and serve as a liaison between the project and primary MESA developer Bill Paxton.

5.3 Data Sharing Plan

All of the analysis tools and data products resulting from this proposal will be made available through the AMP website (<http://amp.phys.au.dk/>). The source code for the *Kepler* stellar model-fitting pipeline, based on ASTEC and ADIPLS, is already available through a subversion repository (<https://svn.nfit.au.dk/svn/AMP/>), and we anticipate updating this archive with the latest stable version as we complete the development outlined in this proposal. MESA and GYRE have both been released under the open-source GNU Public License, so all required modifications to those codes will become part of their respective archives.

Each AMP run conducted for the validation activities described in section 3.3 will be associated with a specific star, and the results will be available on the AMP website after publication. Each page of results (e.g., <http://amp.phys.au.dk/browse/simulation/191> for 16 Cyg A) is linked to a SIMBAD record and includes: notes from the researcher, input observational constraints, optimal model parameters and other observable properties, an échelle diagram and H-R diagram for the optimal model, and an archive of detailed model output files.

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