

RESONANCE PASSAGE VIA COLLISIONS

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When multiple particles are present at a Lindblad resonance, those librating are susceptible to collisions and gravitational scattering that may eject them into circulating orbits. Should an impact/scattering event jump a particle's Jacobi integral across the separatrix that divides libration from circulation, the particle is dislodged from resonance. Below we quantify the minimum impulse necessary to knock a particle from resonance. As a simple illustration of our results, we consider the impact event that may have formed the Pluto–Charon system, and show that a Charon-sized impactor would have ejected Pluto from the 3:2 resonance with Neptune over a large portion of Pluto's orbit.

First we characterize the pre-encounter system in the context of the circular restricted three-body problem: a massless test particle orbits exterior to a perturber having a disturbing function dominated by a single term of the form $F(a, e) \cos \phi$ associated with a $j + 1 : j$ resonance. Here, a and e are the particle's semi-major axis and eccentricity, and the resonance angle is $\phi = j\lambda_p - (j + 1)\lambda + \tilde{\omega}$, where λ_p and λ are the perturber's and test particle's mean longitude, and $\tilde{\omega}$ is the particle's perihelion longitude. This system admits two constants of the motion—the usual Jacobi integral $J(a, e, \phi)$, and $\beta(a, e) = a[j - (j + 1)\sqrt{1 - e^2}]^2$, where a is in units of the resonance radius. Replacing a appearing in J (subsequently written in dimensionless form) with the parameter β and expanding to fourth order in e and first order in the perturber's mass μ (in solar mass units) yields $J(e, \phi; \beta) \simeq Ae^4 - Be^2 + Ce \cos \phi + D$, which has constant coefficients $A \simeq 3(j + 1)^2/4$, $B(\beta) \simeq 3(j + 1)(1 - \beta)/2$, $C = 8j\mu/5$, and $D \simeq 3 + 2/j$, all written to lowest order in the small quantity $|1 - \beta| \ll 1$.

A graphical analysis of the $J(e, \phi)$ level curves [1] reveals the range of J values for which a particle at resonance librates rather than circulates. A particle with zero free eccentricity has a forced eccentricity that minimizes J , so $\phi = \pi$ and $e_- \simeq \sqrt{B/2A} + \mathcal{O}(C/B)$. The particle's level curve is thus a point in a $J(e, \phi)$ phase space diagram, denoted as $J(e_-, \pi; \beta) \equiv J_-$. A particle on the level curve separating a librating orbit from a circulating one (the separatrix) now has some additional free eccentricity, and exhibits a local minima in $J(e)$ when $\phi = 0$, denoted as $J(e_+, 0; \beta) \equiv J_+$, where $e_+ \simeq e_-$.

An impactor striking a librating particle may be treated as an instantaneous, non-adiabatic change in the particle's orbit elements: $e_- \rightarrow e_- + \delta e$, $\beta \rightarrow \beta + \delta\beta$, and $\tilde{\omega} \rightarrow \tilde{\omega} + \delta\tilde{\omega}$. The particle's post-encounter Jacobi integral is, to second order, $J' \simeq J_- + [4A(\delta e)^2 - \Delta B]e_-^2 - 2\delta e\Delta B e_- + e_-C(\delta\tilde{\omega})^2/2$. Similarly, the new separatrix level is $J'_+ \simeq J_+ + [4A(\delta e)^2 - \Delta B]e_+^2 - 2\delta e_+\Delta B e_+$, where $\Delta B = -3(j + 1)\delta\beta/2$ and $\delta e_+/e_+ = \Delta B/2B$. An encounter that knocks the particle out of libration is one that promotes J above the separatrix of the perturbed state, *ie.*, $J' > J'_+$. Noting that $J_+ - J_- \simeq 2Ce_- \gg e_-C(\delta\tilde{\omega})^2$ and $\delta\beta \simeq \delta a/a - 2(j + 1)e\delta e$, we find that all the remaining perturbations in δe and $\delta\beta$ vanish to second order, and the criterion $J' > J'_+$ greatly simplifies to

$$|\delta a| > 8a\sqrt{j\mu e/15}. \quad (1)$$

Generally, an impact acting in the radial direction will most strongly affect the particle's eccentricity, but since the above expression is insensitive to δe , we conclude that a librating particle is much more stable against purely radial impacts. This has been demonstrated in a numerical experiment by R. Malhotra [2], who also derived the above expression by considering tangential impacts. One of the aims of this communication is to show that Eq. (1) is in fact more general.

A change in the particle's semi-major axis, δa , is related to radial and tangential velocity impulses δv_r and δv_θ , acting over a short time interval Δt , through the Lagrange equation $\delta a = \dot{a}\Delta t \simeq 2[e\delta v_r \sin f + \delta v_\theta(1 + e \cos f)]/\Omega$, where f is the particle's true anomaly and Ω is its mean motion. As a simple application, consider a librating particle of mass M_p and eccentricity e struck by an impactor of mass m_i in a circular orbit. The impactor has a relative approach velocity $\Delta \mathbf{v} \simeq -eV(\sin f \hat{\mathbf{r}} + \cos f \hat{\boldsymbol{\theta}}/2)$, where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors in the radial and tangential directions. Further, assume the impact is sufficiently inelastic such that all the target and impactor mass remains bound as a binary pair, such as might occur during moon formation via a giant impact. Conservation of linear momentum requires the binary to recoil with velocity $\delta \mathbf{v} = m_i \Delta \mathbf{v} / (M_p + m_i)$. Inserting this expression into the above Lagrange equation which is then inserted into Eq. (1) yields the impactor threshold mass m_i that dislodges the target mass M_p from libration,

$$\frac{m_i}{M_p + m_i} > \frac{\sqrt{16j\mu/15e}}{|e \sin^2 f + \cos f(1 + e \cos f)/2|}, \quad (2)$$

as a function of the target particle's phase f . This result should be regarded a conservative estimate, for it assumes the target had no free eccentricity, whereas one with nonzero free eccentricity has J closer to the separatrix and requires a smaller impactor to dislodge it. Also note that while Eq. (1) may suggest a higher eccentricity protects a particle from the impact, its increased velocity relative to an impactor instead makes it easier to dislodge, [Eq. (2)].

To illustrate this point, consider a hypothetical impact that may have formed the Pluto-Charon pair while at the 3 : 2 resonance with Neptune. Figure 1 shows the threshold impactor mass, Eq. (2), that knocks Pluto out of libration at various phase f . The curves reveal that an eccentric target is most susceptible to being dislodged at perihelion and aphelion, where impacts occur with a largely tangential impulse, whereas impacts at $f = \pi/2$ and $3\pi/2$ result in an ineffective radial impulse. The figure also indicates that Pluto, having an eccentricity ~ 0.25 , is dislodged from libration over a large fraction of its orbit by an impactor having a mass \leq Charon's mass. This suggests that if Pluto was swept into resonance due to radial migration of Neptune [3], any Charon-forming impact more likely occurred before Pluto's eccentricity was pumped up to its present value.

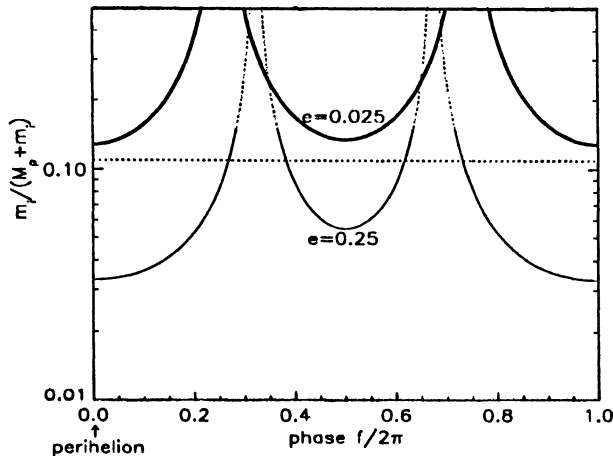


Figure 1. The minimum mass ratio $m_i/(M_p + m_i)$ that ejects the target from resonance as a function of its true anomaly f , for selected values of e . The dotted line indicates the Charon-Pluto value.

References

- [1] Peale S. (1986) *Satellites* (J. Burns and M. Mathews, Eds.)
- [2] Malhotra R. (1993) *Icarus*, **106**, 264.
- [3] Malhotra R. (1993) *Nature*, **365**, 819.