THE ASTROPHYSICAL JOURNAL, 440:L25–L28, 1995 February 10


# DISK TIDES AND ACCRETION RUNAWAY

WILLIAM R. WARD[1]
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109
AND
JOSEPH M. HAHN
Department of Physics, University of Notre Dame, Notre Dame, IN 46556
*Received 1994 September 12; accepted 1994 November 22*

## ABSTRACT

It is suggested that tidal interaction of an accreting planetary embryo with the gaseous preplanetary disk may provide a mechanism to breach the so-called runaway limit during the formation of the giant planet cores. The disk tidal torque converts a would-be shepherding object into a "predator," which can continue to cannibalize the planetesimal disk. This is more likely to occur in the giant planet region than in the terrestrial zone, providing a natural cause for Jupiter to predate the inner planets and form within the $O(10^7$ yr) lifetime of the nebula.

*Subject headings:* accretion, accretion disks — celestial mechanics, stellar dynamics — planets and satellites: general — solar system: formation

## 1. INTRODUCTION

One of the tightest constraints on models of solar system formation is the suspected $10^7$ yr lifetime of the solar nebula inferred from observations of T Tauri stars (Adams & Shu 1986; Walter 1986). The existence of gas giants like Jupiter and Saturn establish that the planet-building process for these objects was essentially completed before nebula dispersal. These planets are believed to have acquired their H/He component by gas accretion onto preexisting solid cores with estimated masses of 10–20 $M_\oplus$, where $M_\oplus = 6 \times 10^{27}$ g denotes the mass of the Earth (Mizumo, Nagazawa, & Hayashi 1978; Bodenheimer & Pollack 1986; Podolak, Hubbard, & Pollack 1993). If this model is correct, we must account for the accretion of $O(10^{29}$ g) cores within the lifetime of the gas disk.

## 2. RUNAWAY GROWTH

Current models of solid-body accretion indicate that large embryos can form in a relatively short timescale, owing to the onset of accretion runaway (Greenberg et al, 1978; Wetherill & Stewart 1989, 1993; Ida & Makino 1993). This runaway is due to a strong feedback loop in the growth rate, $M \sim \sigma\Omega\pi R^2 F_g$, through the gravitational enhancement factor $F_g$, where $\sigma$ is the surface density of solid material in the disk, $\Omega$ is the embryo's mean motion, and $M$ and $R$ are its mass and radius, respectively (e.g., Greenzweig & Lissauer 1990; Lissauer & Stewart 1993). The enhancement factor is the ratio of the effective collision cross section to the geometrical cross section. If the relative velocities $v$ are dominated by velocity dispersion instead of disk shear, the enhancement factor reads $F_g = 1 + (v_e/v)^2$, where $v_e \equiv (2GM/R)^{1/2}$ is the embryo's escape velocity. Eventually, the embryo will grow large enough to stir the local planetesimal disk (Lissauer 1987).[2] This limits the enhancement factor to $F_g \sim (v_e/\xi L\Omega)^2 \approx 10^3 \xi^{-2}(r/\mathrm{AU})$, and the characteristic growth time, $\tau_R \equiv R/\dot{R}$, becomes

$$\tau_R \approx \frac{\rho_p R}{\sigma\Omega F_g} \approx 16\xi^2\left(\frac{R}{\mathrm{km}}\right)\left(\frac{\rho_p}{\sigma}\right)\left(\frac{r}{\mathrm{AU}}\right)^{1/2} \mathrm{yr}\,. \qquad (1)$$

For values $\sigma = 4$ g cm$^{-2}$, $\rho_p = 2$ g cm$^{-3}$, $r = 5$ AU, $\xi \sim 4$, considered appropriate for the Jovian zone, equation (1) implies that a 15 $M_\oplus$ giant planet core could accrete in $\lesssim O(10^7)$ yr.

However, there is an apparent obstacle to the formation of such full-sized planetary cores via runaway growth: that of local mass exhaustion. Dynamical friction tends to cause embryo orbits to become very circular during their growth (Greenberg et al. 1978; Stewart & Wetherill 1988; Wetherill & Stewart 1989). There is a critical value of the Jacobi constant below which a test particle cannot enter the Hill sphere of a object in a circular orbit (e.g., Hayashi, Nakazawa, & Adachi 1977). This corresponds to a circular test-body orbit with differential semimajor axis $\Delta a_c = 2(3^{1/2})L$, where $L \equiv r(M/3\,M_\odot)^{1/3}$ is the Hill sphere radius. This has often been interpreted as an "effective accretion range" for a growing embryo (Lissauer 1987; Artymowicz 1987; Wetherill & Stewart 1989; Wetherill 1990; Lissauer & Stewart 1993). Although this range increases with the Hill radius, i.e., $\Delta a_c \propto M^{1/3}$, the width of the cannibalized zone should scale linearly with the mass. Thus, these widths become comparable when $M = \pi\sigma r(2\Delta a_c)$, which implies a limiting runaway mass of

$$M_R = 3^{1/4}(8\pi\sigma r^2/M_\odot)^{3/2}\,M_\odot\,. \qquad (2)$$

At this point the embryo is assumed to have largely isolated itself from the planetesimal disk, so that runaway growth stalls. Further growth would have to rely on diffusion of planetesimals into the gap (Hayashi et al. 1977; Artymowicz 1987) or on some other mechanism to supply new material such as gas drag (e.g., Weidenschilling 1977; Nakazawa & Nakagawa 1982). The timescales for these processes are not given by equation (1) and can be much longer.

For the Jovian zone, $M_R \approx 2\,M_\oplus$, which is about an order of magnitude too small for a giant planet core. It is the purpose

[1] Also at San Juan Capistrano Research Institute, San Juan Capistrani, CA.

[2] Objects experiencing a close encounter without impact will reencounter the embryo with a dispersion velocity on the order of the escape velocity from its Hill sphere, i.e., $v \sim O(\xi L\Omega)$, where $L$ is the Hill sphere radius and $\xi$ is a constant of order unity that depends on the damping mechanism(s) (e.g., Greenzweig & Lissauer 1990; Ida & Makino 1993).

of this *Letter* to point out that this so-called runaway limit may be breached by strong disk-planet tidal interactions between the forming embryo and the remnant nebula that have not previously been included in these calculations.

## 3. TORQUE BALANCE

It is well known from planetary ring studies that an object orbiting near a particle disk experiences a torque

$$T = \pm C_d \mu^2 (\sigma r^2)(r\Omega)^2 \left(\frac{r}{w}\right)^3 \tag{3}$$

that tends to repel the perturber from the disk, where $w$ is the distance to the disk edge, $r$ is the embryo's orbital radius, $\mu \equiv M/M_\odot$ is its mass normalized to the primary, and $C_d$ is a constant of order unity (e.g., Goldreich & Tremaine 1979b, 1980; Lin & Papaloizou 1979).[3] If the embryo occupies a gap in a disk, it experiences a positive (negative) torque from the inner (outer) portion of the disk. Acting alone, these torques keep the embryo roughly centered in the gap, i.e., $T(w_i) + T(w_o) = 0$ when $w_i \approx w_o$. However, in the presence of the nebula, the embryo experiences an additional tidal torque,

$$\Delta T_g = C_g \mu^2 (\sigma_g r^2)(r\Omega)^2 \left(\frac{r}{h}\right)^2 , \tag{4}$$

due to its density wave interactions with the gaseous disk (e.g., Goldreich & Tremaine 1980; Ward 1986, 1989; Korycansky & Pollack 1993; Artymowicz 1993). In equation (4), $\sigma_g$ represents the gas surface density, $h \sim c/\Omega$ is the scale height of the nebula, $c$ is the gas sound speed, and $C_g$ is a constant of order unity, the exact value of which depends on the structural details of the nebula (see § 5). Equation (4) arises from the gravitational attraction of the embryo for spiral density waves that are launched at various Lindblad resonances between the perturber and the gas disk (e.g., Goldreich & Tremaine 1979a; Shu 1984). Outer resonances exert negative torques on the perturber, while inner resonances exert positive torques. The origin of net culmulative torque, $\Delta T_g$, is a "mismatch" between the strengths of outer and inner Lindblad resonances due to global gradients of the disk (Goldreich & Tremaine 1980; Ward 1986). In addition, there is a torque contribution from corotation resonances that fall at the orbit of the perturber (e.g., Ward 1993; Korycansky & Pollack 1993). The sign of $C_g$ is negative for most model calculations, and we shall make that assumption here, although this is not crucial to our argument. This additional torque causes the embryo to be displaced inward from the center of the gap to occupy the torque balance position given by $T(w_i) + T(w_o) + \Delta T_g = 0$. This provides a constraint between the distance to the inner edge, $w_i$, and the gap's "aspect" ratio, $w_i/w_o \equiv \eta$,

$$C\left(\frac{w_i}{r}\right)^3 = \alpha \left(\frac{h}{r}\right)^2 (1 - \eta^3) , \tag{5}$$

where $\alpha \equiv \sigma/\sigma_g$ is the solid/gas ratio and $C \equiv |C_g|/C_d$. In the limit $C_g \to 0, \eta \to 1, w_i = w_o$; but for $|C_g| > 0, \eta < 1, w_i < w_o$.

If most of the disk material within the boundaries of the gap is accreted by the embryo, its mass is $M \approx 2\pi\sigma r w_i (1 + \eta^{-1})$. As before, the assumption is made that accretion runaway stalls when $w_i \geq \Delta a_c$. Substituting $\Delta a_c$ for $w_i$ yields

$$M = M_R \left(\frac{1 + \eta}{2\eta}\right)^{3/2} , \tag{6}$$

which can be combined with equation (5) to find

$$(1 - \eta^3)\left(\frac{2\eta}{1 + \eta}\right)^{3/2} = 8(3^{1/2}) C \left(\frac{\mu_R}{\alpha}\right)\left(\frac{r}{h}\right)^2 \equiv K . \tag{7}$$

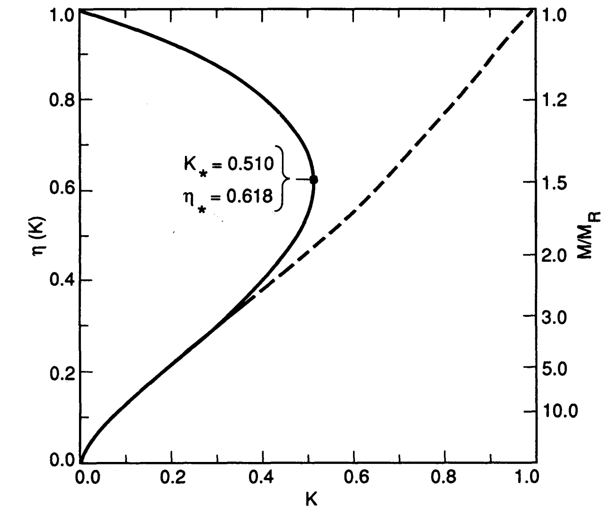


FIG. 1.—Gap aspect ratio, $\eta \equiv w_i/w_o$, as a function of nebula torque strength, parameterized by $K \equiv 8(3^{1/2})C(\mu_R/\alpha)(r/h)^2$. Upper branch ($\eta > \eta_*$) applies to runaway limit. The lower curve asymptotically approaches the dashed curve for an object orbiting $\Delta a_c$ outside a disk. Right-hand scale shows the corresponding embryo mass normalized to the value ($M_R$) when $K = 0$.

Figure 1 shows the behavior of $\eta(K)$; the right-hand scale shows the corresponding value of $M(K)$ normalized to $M_R$. A maximum allowed $K$ occurs at $dK/d\eta = 0$, for which $1 - 3\eta^3 - 2\eta^4 = 0$. This has the solution $\eta_* = 0.618$ and a mass from equation (6) of $M_* = 1.498 M_R$. The value of $K_*$ from equation (7) is $K(\eta_*) = 0.510$.

There are two branches to $\eta(K)$ for $K < K_*$; it is the upper branch that is relevant to the runaway process. The lower branch, at smaller $\eta$, corresponds to equilibrium states where the outer edge is increasingly remote and the torque balance is essentially between the nebula and inner disk tides only. For comparison, the curve $K = M_R/M = [2\eta/(1 + \eta)]^{3/2}$, for a perturber orbiting just outside a disk, i.e., $T(w_i = \Delta a_c) + \Delta T_g = 0$, is indicated by the dashed curve in Figure 1.

## 4. SHEPHERD OR PREDATOR

Torque balance with $w_i \geq \Delta a_c$ is not possible for $K > K_*$, which implies a critical value for the torque constant ratio of

$$C_{\text{crit}} = \alpha K_* \frac{(h/r)^2}{8\sqrt{3}\mu_R} , \tag{8}$$

above which the embryo cannot isolate itself from the planetesimal disk. If the torque constant ratio $C$ exceeds this threshold value, the nebula tidal torque will convert a would-be shepherding object into a "predator" that can continue to

[3] A value of $C_d \approx 0.83$ can be derived for a particle disk composed of nearly circular orbits (e.g., Goldreich & Tremaine 1982).

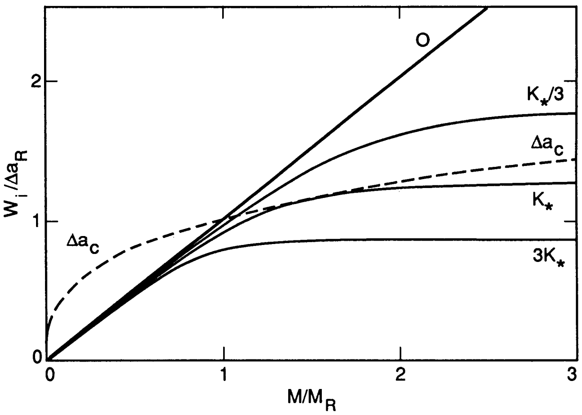


FIG. 2.—Distance from embryo to interior edge of gap, i.e., "stand-off distance," as a function of embryo mass for different values of $K$. Shown for comparison is the accretion range $\Delta a_c \equiv 2(3^{1/2})(M/3\,M_\odot)^{1/3}r$. Runaway growth stalls when $w_i \geq \Delta a_c$; this is possible for $K \leq K_*$.

consume the planetesimal disk. Relaxing the constraint $w_i = \Delta a_c$, equation (5) can be used to find the "stand-off" distance, $w_i$, as a function of $M/M_R$ for a given $K$,

$$\frac{M}{M_R} = \frac{1}{2}\left(\frac{w_i}{\Delta a_R}\right)\left\{1 + \left[1 - K\left(\frac{w_i}{\Delta a_R}\right)^3\right]^{-1/3}\right\}, \qquad (9)$$

where $\Delta a_R = \Delta a_c(M_R)$. Figure 2 compares $\Delta a_c$ with the stand-off distance given by equation (9) for several values of $K$. Note that for $M \gg M_R$, $w_i$ approaches a constant value, $w_i = \Delta a_R/K^{1/3} = 4(3^{1/4})(2\pi\sigma r^2/M_\odot)^{1/2}r/K^{1/3}$. This is because both $T$ and $\Delta T_g$ are proportional to $M^2$, so that once the outer edge is remote, $w_i$ must assume a constant value to balance the torques, independent of mass. It is clear that for $K > K_*$, $w_i$ never exceeds $\Delta a_c$ for any $M$, so that isolation cannot occur. As the edge of the planetesimal disk is stripped away, the nebula torque causes the embryo's orbit to decay, maintaining a small enough stand-off distance that the perturber can continue to accrete material.

So far, the effect of the reaction torque, $-T$, on the location of the disk particles has not been explicitly taken into account. For instance, it is well known that the shepherding action of a satellite on a planetary ring can nudge material away from the satellite (Goldreich & Tremaine 1979b). However, a key difference between shepherding in a planetary ring as opposed to a planetesimal disk is that in the latter the optical depth, $\tau \sim \sigma/\rho_p R \approx 10^{-5}(R/\mathrm{km})^{-1}$, is low and collisions are much less frequent.[4] In this case, any epicyclic motion generated at an enounter does not damp out, and particles repeatedly reencounter the embryo with a nonzero eccentricity. In both ring and disk environments, particles suffer shepherding, i.e., a recoil of their semimajor axes away from the embryo. The crucial difference is that the Jacobi constant is generally decreased by particle collisions in a planetary ring but is nearly conserved in the planetesimal disk when such collisions are rare. If we interpret the half-width, $w(J)$, appearing in equation (3) as referring to the $e = 0$ (reference) orbit of those particles with the greatest Jacobi constant, then $w(t)$ increases in a ring of high optical depth but remains nearly constant in the planetesimal disk. Hence, the shepherding of semimajor axes in the planetesimal disk does not remove particles from the accretion zone. In this interpretation of equation (3), any effect of heating on the torque strength has been absorbed into the coefficient, $C_d(e, w)$, which is now to be considered a function of the particles' epicycle motions. Its behavior could be found by phase averaging numerical integrations of Hill's equations.[5] We have not included such a procedure in this brief communication, but will report such results in a subsequent publication. Here we have been content to determine the critical torque coefficient in terms of the ratio $C \equiv |C_g|/C_d$.

## 5. TORQUE STRENGTH

In terms of the constants adopted for the Jovian zone, $\alpha \equiv \sigma/\sigma_g \sim 10^{-2}$, $h/r \sim 0.07$, $r = 5$ AU, $\mu_R \sim 6.3 \times 10^{-6}$, the critical torque ratio is

$$C_{\mathrm{crit}} \sim 0.29\left(\frac{\alpha}{10^{-2}}\right)\left(\frac{T}{160\ \mathrm{K}}\right)\left(\frac{r}{5\ \mathrm{AU}}\right)^{-2}\left(\frac{\sigma}{4\ \mathrm{g\ cm^{-2}}}\right)^{-3/2}. \qquad (10)$$

By comparison, setting $r = 1$ AU, $\sigma = 7$ g cm$^{-2}$, $T = 10^3$ K, $\alpha = 0.0036$, as representative of the terrestrial region, equation (10) gives $C_{\mathrm{crit}} \sim 7.0$. The increased threshold value, $C_{\mathrm{crit}}(1\ \mathrm{AU})/C_{\mathrm{crit}}(5\ \mathrm{AU}) \sim 24$, in the terrestrial zone, implies that embryo isolation is much easier to achieve there.

A recent attempt to determine $C_g$ in terms of the structural gradients of the nebula has been made by Korycansky & Pollack (1993). Numerical integration of the two-dimensional fluid equations for power-law models of the form $\sigma_g \propto r^{-k}$, $T \propto r^{-l}$, yields $C_g^L = -3.2(1 + 0.28k + 0.81l)$ for Lindblad torques and $C_g^c = 2.0(1 - 0.63k)$ for corotation torques, where only linear terms have been retained. The Lindblad value is consistent with an earlier semianalytical estimate by Ward (1986). Both authors found that the Keplerian rotation of the gas disk was an important source of differential torque. For disks with $k, l \sim O(1)$, $|C_g| = |C_g^L + C_g^C| \approx 6$. Ward noted that these values, which pertain to a two-dimensional disk, would likely be decreased by a finite thickness of the disk. This has been confirmed by Artymowicz (1993), who has used a vertically averaged potential together with an improved model for disk response at a Lindblad resonance to estimate the torque. Artymowicz's calculations indicate that vertical averaging causes nearly a 50% reduction in the differential torque.

Interestingly, if $C_d \lesssim O(1)$, $C$ could lie between the threshold values for the terrestrial and Jovian regions for reasonable disk models. Although current calculations of the torques are not yet reliable enough to make such precise predictions, this problem makes it clear that further refinement of these calculations is a high-priority issue.

## 6. CONCLUSION

A failure of an embryo to achieve isolation may permit its continued growth toward the critical core size for gas accre-

[4] Note that the collisional timescale, $\sim(\tau\Omega)^{-1}$, is longer than the synodic period at the Hill radius if $\tau \ll O(\mu^{1/3})$, which is easily satisfied by kilometer-sized planetesimals in the vicinity of an Earth mass embryo.

[5] Depending on phase, successive encounters can either increase or decrease $e$. From the Jacobi constant, changes in the semimajor axis obey $\delta a/a = -\delta(e^2)\Omega/(\Omega - \Omega_s)$, and the angular momentum, $H$, lost by the particle per synodic period is $\Delta H/P_{\mathrm{syn}} = \delta(e^2)(a\Omega)^2/4\pi = \dot{H}$.

tion. The eventual onset of gas accretion will alter the local configuration of the gaseous nebula. Development of a low-density zone in the gas disk surrounding the core should abort the nebula torque and stabilize the orbit. In addition, the protoplanet becomes a sink for any later decaying embryos which attempt to follow the same evolutionary track—unless they are well enough spaced at the outset, e.g., a potential Saturnian core. A distinct advantage of this scheme is that it does not require a large excess of material to account for rapid core growth, most of which must later be removed from the system (Lissauer 1987; Lissauer & Stewart 1993). An intriguing additional possibility is that embryo isolation could still have occurred in the terrestrial zone, stalling the runaway process there. Growth to planetary size in the inner solar system could have proceeded along the slower, stochastic accretion route as described by Wetherill (1990). This implies that Jupiter may predate the terrestrial planets, a situation consistent with ideas concerning its role in the development of the asteroid belt (Wetherill 1992). This also suggests that not all planetary systems will necessarily have giant planets. From equation (8), it is clear that small values of $\mu_R$ favor embryo isolation. Thus, small disks (low-mass and/or low-radius) may not be conducive to rapid core formation and may tend to develop systems of smaller, more numerous planets.

This work was supported by NASA under contract NBAS7-100 with the Jet Propulsion Laboratory, California Institute of Technology, and by the Department of Physics, University of Notre Dame.

REFERENCES

Adams, F. C., & Shu, F. H. 1986, ApJ, 308, 836
Artymowicz, P. 1987, Icarus, 70, 303
———. 1993, ApJ, 419, 155
Bodenheimer, P., & Pollack, J. B. 1986, Icarus, 67, 391
Goldreich, P., & Tremaine, S. 1979a, ApJ, 233, 857
———. 1979b, Nature, 277, 97
———. 1980, ApJ, 241, 425
———. 1982, ARA&A, 20, 249
Greenberg, R., Wacker, J. F., Hartman, W. K., & Chapman, C. R. 1978, Icarus, 35, 1
Greenzweig, Y., & Lissauer, J. J. 1990, Icarus, 87, 40
Hayashi, C., Nakazawa, K., & Adachi, I. 1977, PASJ, 29, 163
Ida, S., & Makino, J. 1993, Icarus, 106, 210
Korycansky, D. G., & Pollack, J. B. 1993, Icarus, 102, 150
Lin, D. N. C., & Papaloizou, J. 1979, MNRAS, 186, 799
Lissauer, J. J. 1987, Icarus, 69, 249
Lissauer, J. J., & Stewart, S. R. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 1061
Mizuno, H., Nagazawa, K., & Hayashi, C. 1978, Prog. Theor. Phys., 60, 699
Nakazawa, K., & Nakagawa, Y. 1982, Prog. Theor. Phys. Suppl., 70, 11
Podolak, M., Hubbard, W. B., & Pollack, J. B. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 1109
Shu, F. H. 1984, in Planetary Rings, ed. R. Greenberg & A. Brahic (Tucson: Univ. Arizona Press), 513
Stewart, G. R., & Wetherill, G. W. 1988, Icarus, 74, 542
Walter, F. M. 1986, ApJ, 306, 573
Ward, W. R. 1986, Icarus, 67, 164
———. 1989, ApJ, 345, L99
———. 1993, Ann. NY Acad. Sci., 675, 314
Weidenschilling, S. J. 1977, MNRAS, 180, 57
Wetherill, G. W. 1990, Ann. Rev. Earth Planet. Sci., 18, 205
———. 1992, Icarus, 100, 307
Wetherill, G. W., & Stewart, S. R. 1989, Icarus, 77, 330
———. 1993, Icarus, 106, 190