N-Body Simulations of Narrow Eccentric Ringlets

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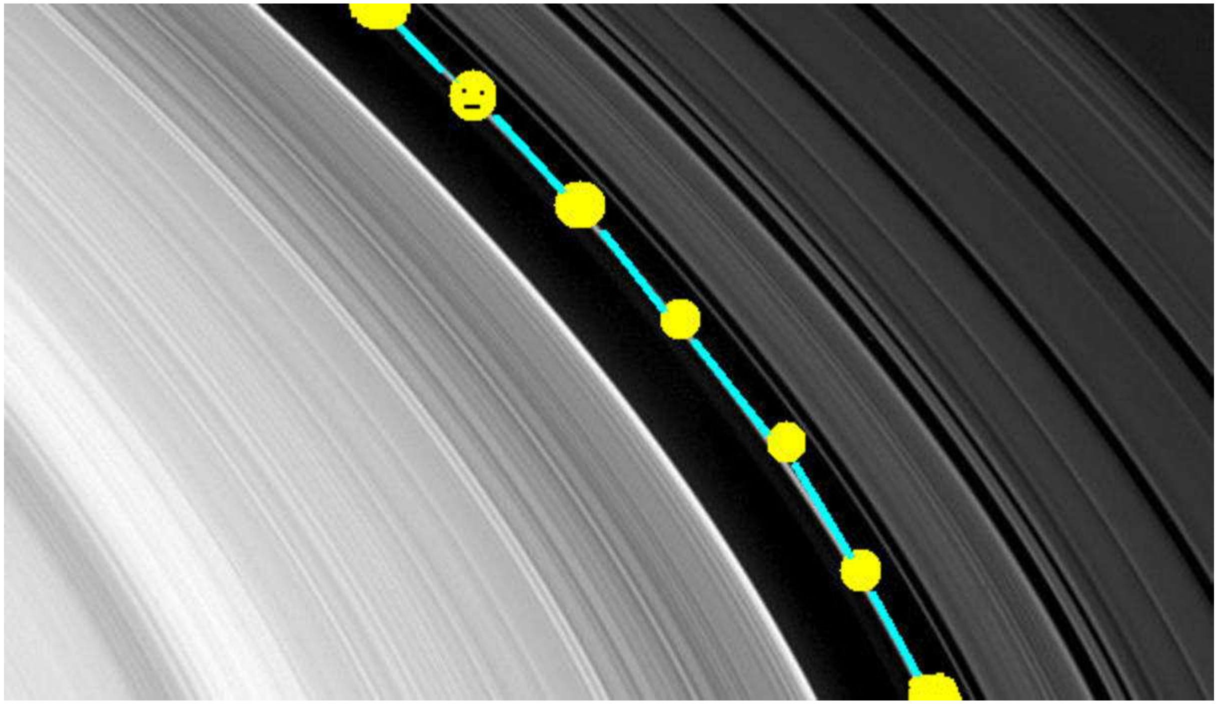


Cassini image of narrow 25km-wide Huygens ringlet, 200km beyond outer edge of B ring

The following describes an N-body algorithm that can simulate disturbances in planetary rings that are of low azimuthal wavenumber m, such as:

- scalloping at outer edge of A and B rings, $m = 2$ and 7,
- spiral density waves with $m =$ a few,
- libration of narrow eccentric ringlets. $m = 1$

One must model all $360°$ in ring to simulate these phenomena, cannot simulate a small patch in the ring.

Fortunately the radial width of the region of interest is small, $\Delta r \sim 100$km while orbit radius is $r \sim 10^3 \Delta r$.

- all particles are close in the radial sense,
- gravity that one streamline in the ring exerts on a particle is $A_g = -2G\lambda/\Delta r$, where $\lambda =$linear density (eg. GT79, BGT83, BGT85)

This N-body integrator uses trace particles to map streamlines within ring:

- no gravitational scattering when a particle nears a streamline,
- runs requires only $N \sim 1000$ particles,
- runtimes are minutes to hours on PC.

Each trace particle also represents a patch within the ring,

- ring surface density is $\sigma = \lambda/\Delta$,
- radial acceleration due to ring pressure is $A_p = -(\partial p/\partial r)/\sigma$
- viscous acceleration $A_\nu = -(\partial F_\nu/\partial r)/r\sigma$ where the ring's viscous angular momentum flux is $F_\nu = -\nu\sigma r^2(\partial\Omega/\partial r)$
- model also accounts for pressure drop and the large viscous torque at a ring's sharp edge

The code is a symplectic integrator, uses the kick-drift scheme of SYMBA and MERCURY (Duncan at al 1998, Chambers 1999).

- kick $\Delta v = A\Delta t$ is due to perturbing acceleration A during timestep Δt
- oblateness effects occur during the drift step, which is *epicyclic*, not keplerian:
  - mean anomaly drifts (advances) as $\Delta M = \kappa\Delta t$
  - longitude of peri advances as $\Delta\tilde{\omega} = (\Omega - \kappa)\Delta t$ where Ω, κ are the angular, epicyclic frequencies for orbits around oblate planet

Coordinates & velocities are related to epicyclic orbit elements via

$$r = a - ea\cos(M) + ... \qquad \theta = \tilde{\omega} + M + 2e(\Omega/\kappa)\sin(M) + ...$$

$$v_r = ea\kappa\sin(M) + ... \qquad v_\theta = a\Omega + ea\Omega\cos(M) + ...$$

(from B-R & Longaretti 1994).

Narrow eccentric ringlets

Note that planetary oblateness drives differential precession,
which would cause a narrow ringlet to lose its eccentric shape in only ~ 100 yrs,

- but GT79 showed that ring gravity can oppose differential precession, provided the ringlet has a positive eccentricity gradient $e' = a(de/da)$

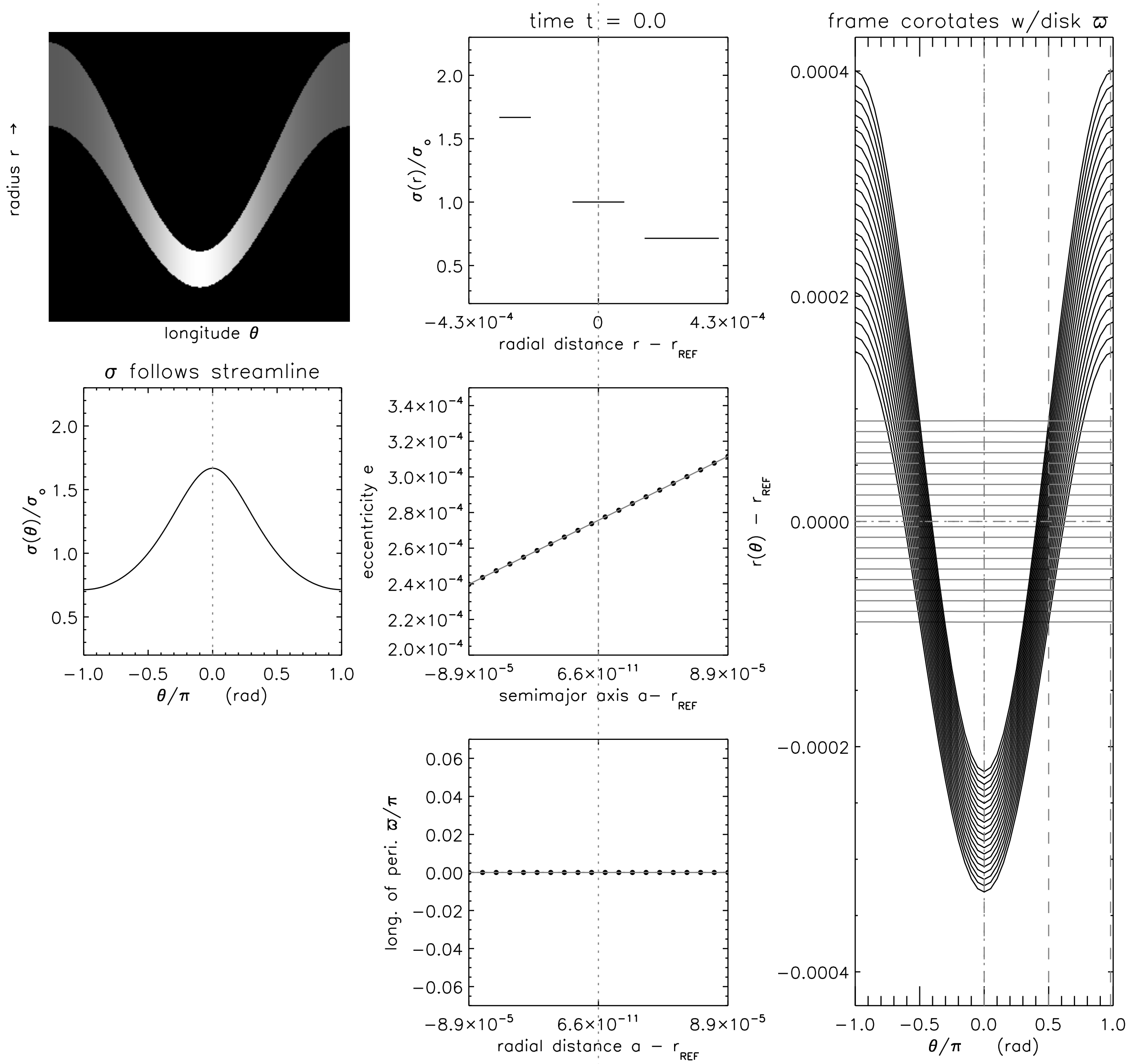
- BGT83 show that the ringlet is in static equilibrium (streamlines in ringlet show no relative motion) when $e' \simeq 16eJ_2(M_p/\pi\sigma a^2)(R_p/a)^2(\Delta a/a) \equiv e'_{eq}$,
  - and that a ringlet that is displaced from equilibrium will librate with period $T_{\text{lib}} = (M_p/\pi\sigma a^2)(\Delta a/a)T_{\text{orb}}$

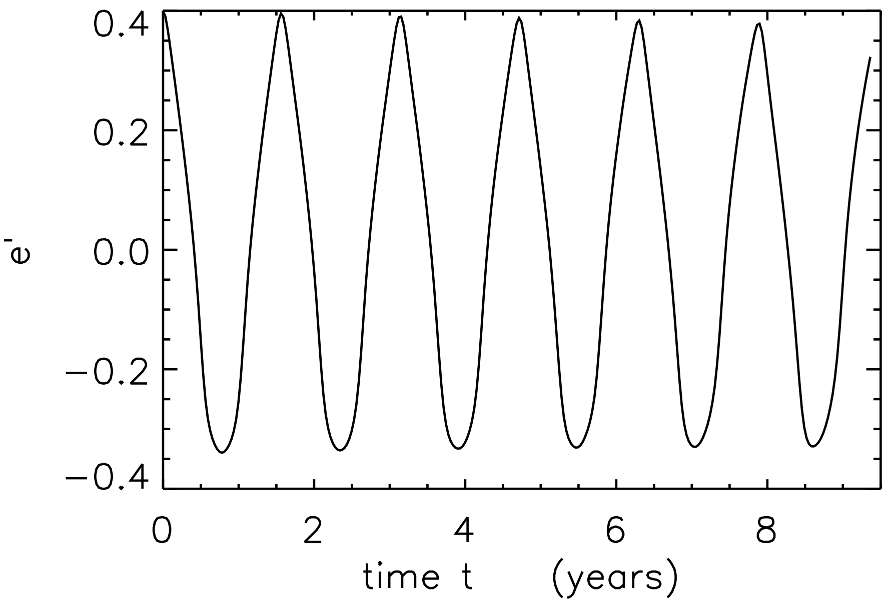
Meanwhile, JS has been using Cassini to monitor the Huygens ringlet
for several years, and it is clearly not in static equilibrium...

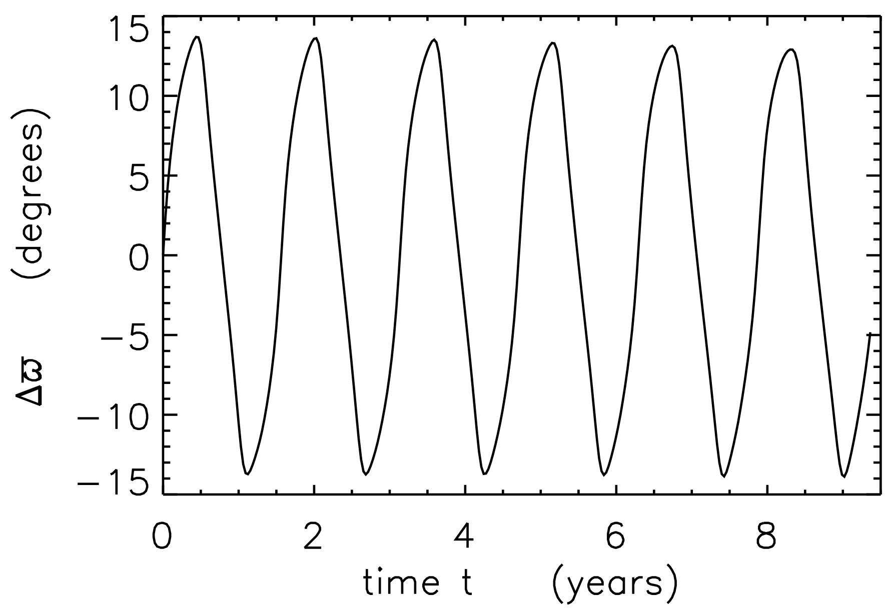
- so I am using N-body simulations as a guide to determine what Huygens is doing:
  - librating,
  - circulating
  - might have freely precessing modes,
  - experiencing radial oscillations,
  - or doing something else...

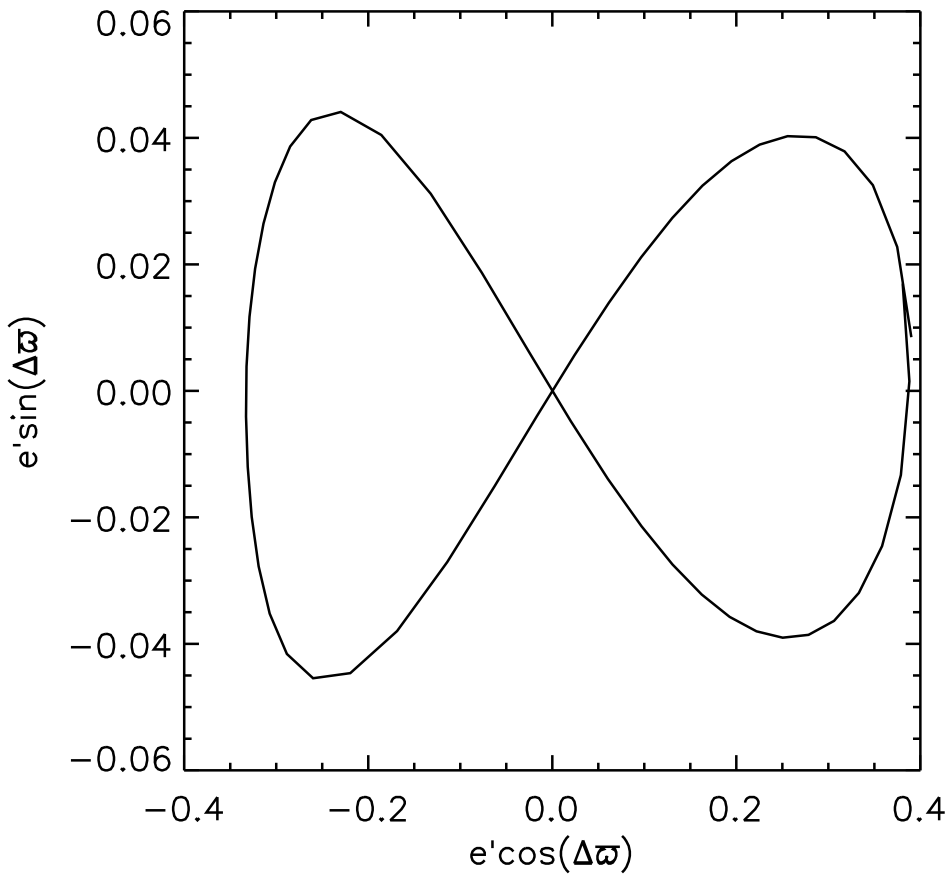
N-body simulation of a librating Huygens ringlet

- model ringlet is composed of $40 \times 20 = 800$ trace particles
- narrow, $\Delta a = 20$ km
- modest eccentricity, $e = 2.4 \times 10^{-4}$, $er \simeq 30$ km
- $\nu = 0$, so no radial spreading
- heavy, $\sigma = 100$ gm/cm^2, for fast libration
- initial $e' = 0.4$, for dramatic effect









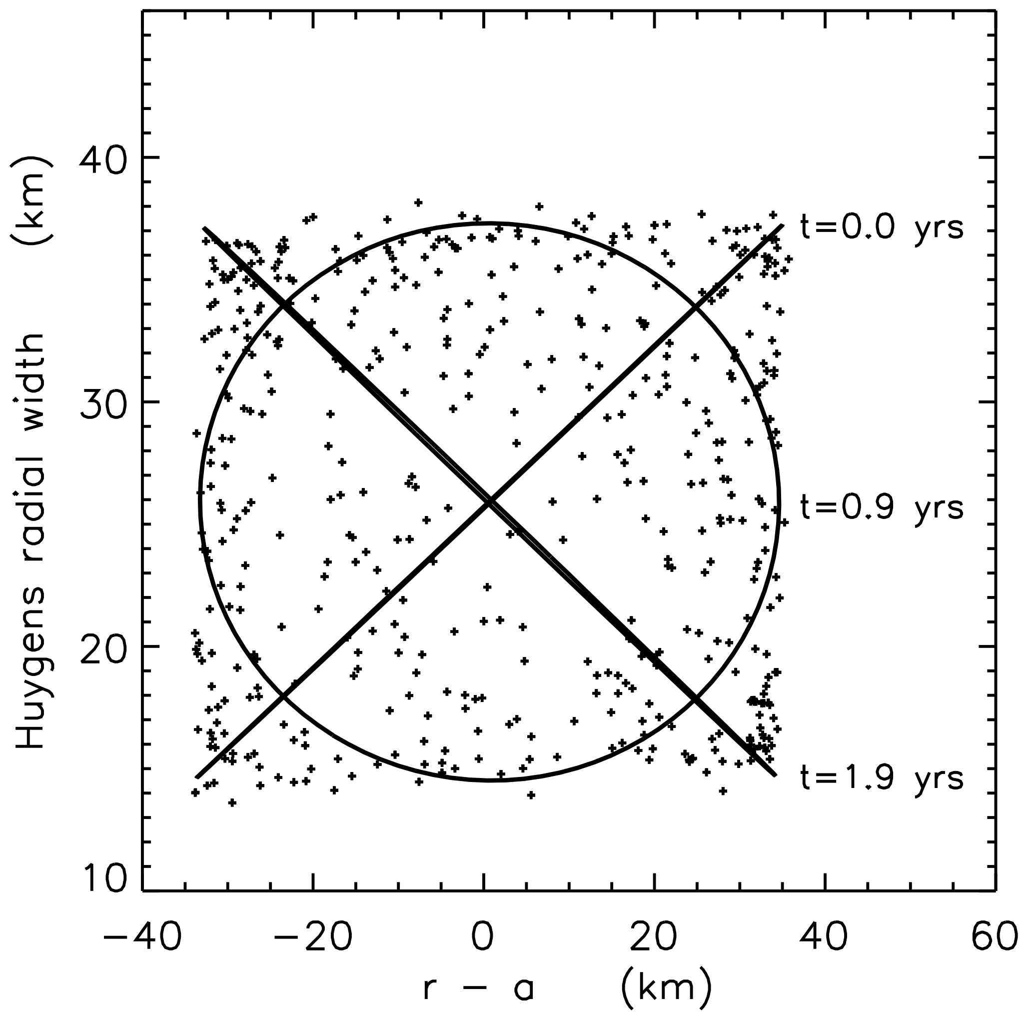
Trajectories through this phase space can resemble a figure-8, due to e' changing sign.

This occurs when the libration amplitude is large, $e'_{max} \gtrsim 2e'_{eq}$.

Width-radius plots

convenient way to compare models to ringlet observations

- periapse is on left, apoapse on right
- when $e' = 0 = \Delta\tilde{\omega}$ (ie ringlet isn't librating) then ringlet width is constant
- when $e' > 0$ and $\Delta\tilde{\omega} = 0$, ring is widest at apoapse
- when $e' < 0$ & $\Delta\tilde{\omega} = 0$, ring is widest at periapse
- when $e' = 0$ & $\Delta\tilde{\omega} \neq 0$, wider near quadrature



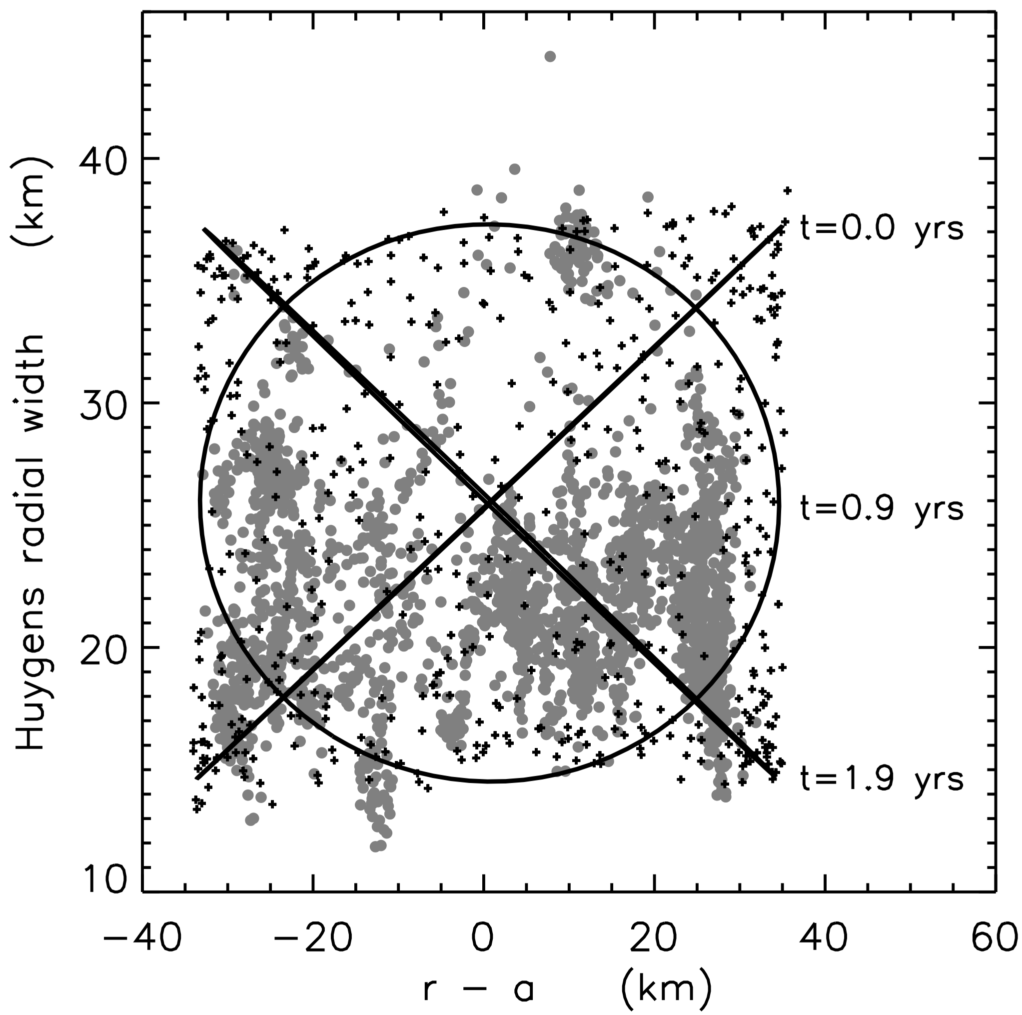
- cycle repeats every half-libration
- points = simulated ringlet's $w(r)$, sampled at random times and longitudes

a librating ringlet fills a box in width-radius plot

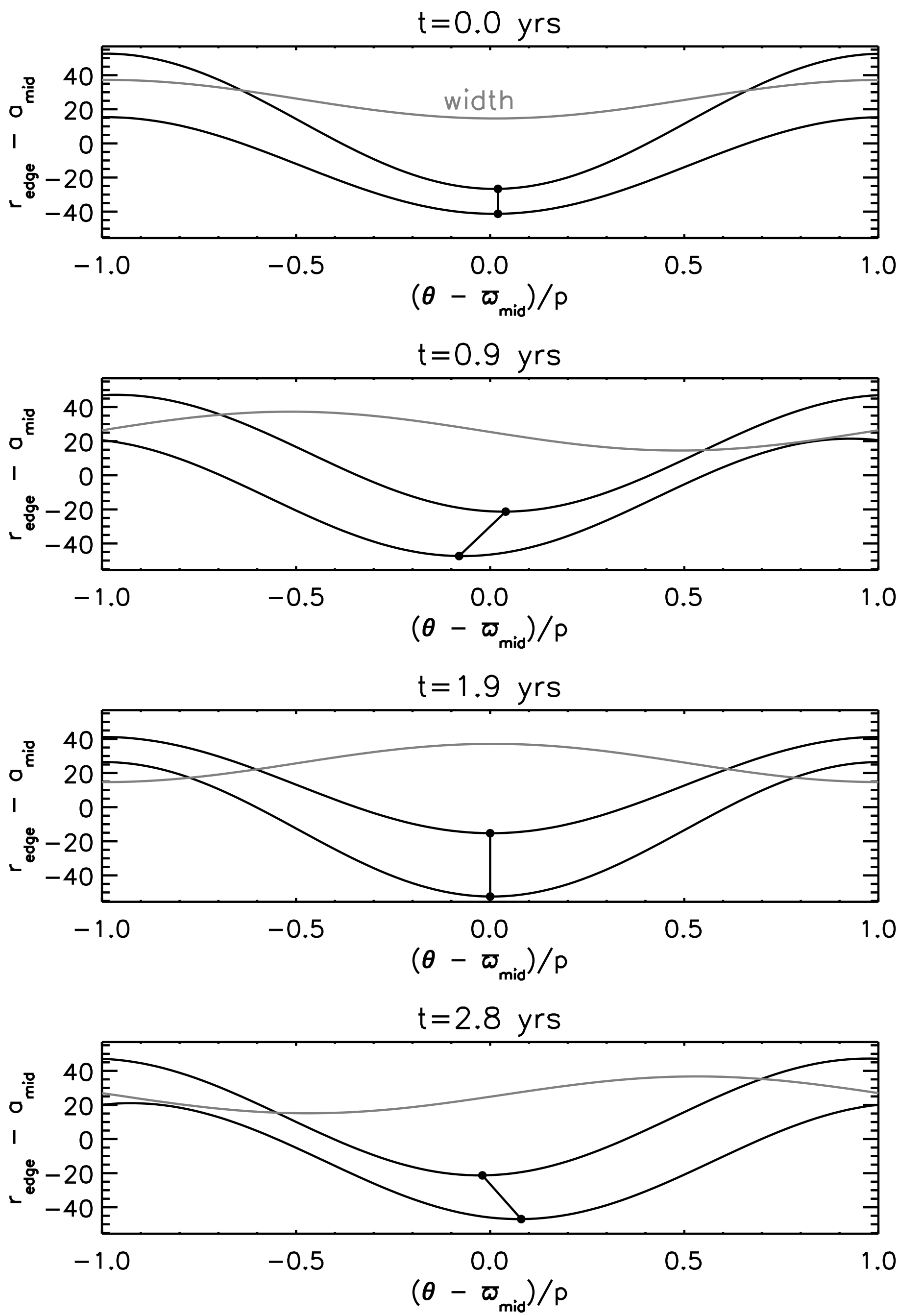
Huygens’ observed width-radius

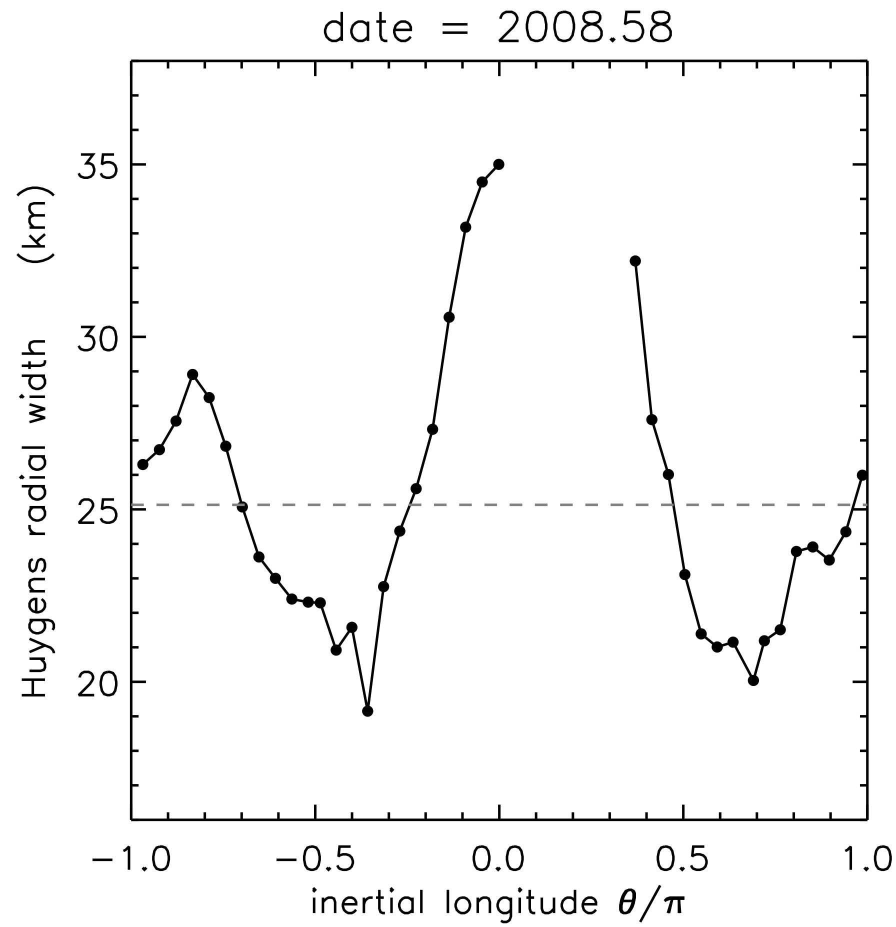
Cassini has monitored Huygens for several years
JS has extracted ringlet edge-radii and longitudes from hundreds of ISS images, and produced this $w(r)$ plot seen in grey

- a librating ringlet model *can* fill the observed box of grey width-radii data
- libration amplitude $e'_{\text{max}} = \mathbf{0.5}$
- this sim’s $\sigma = \mathbf{50}$ gm/cm^2



Simulated and observed width-longitude plots

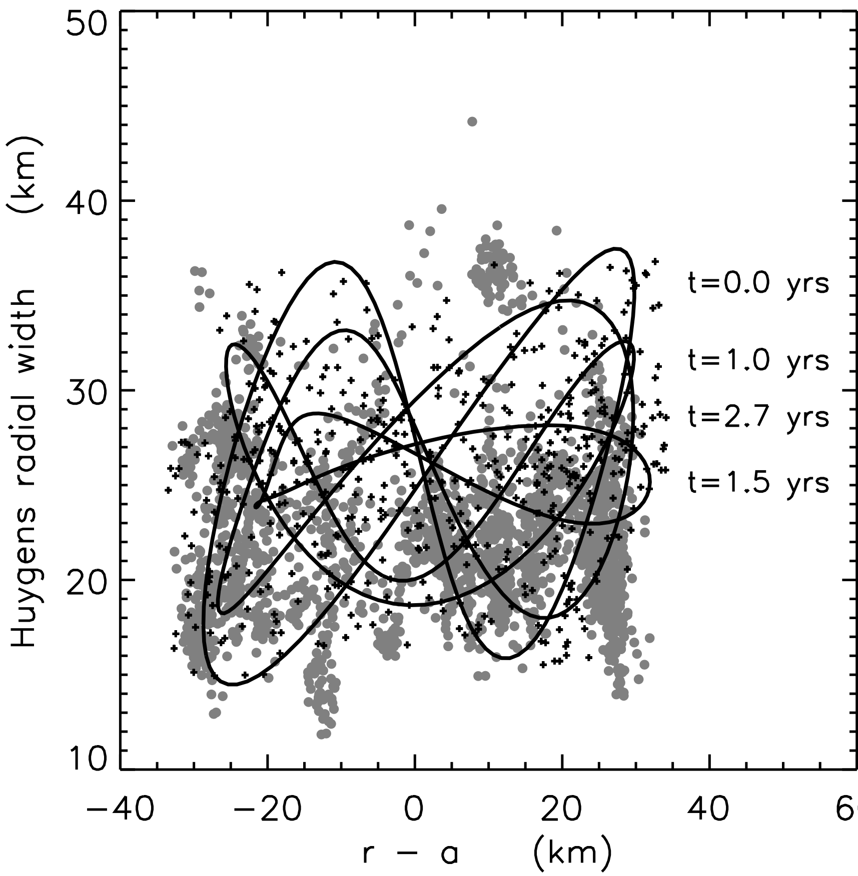


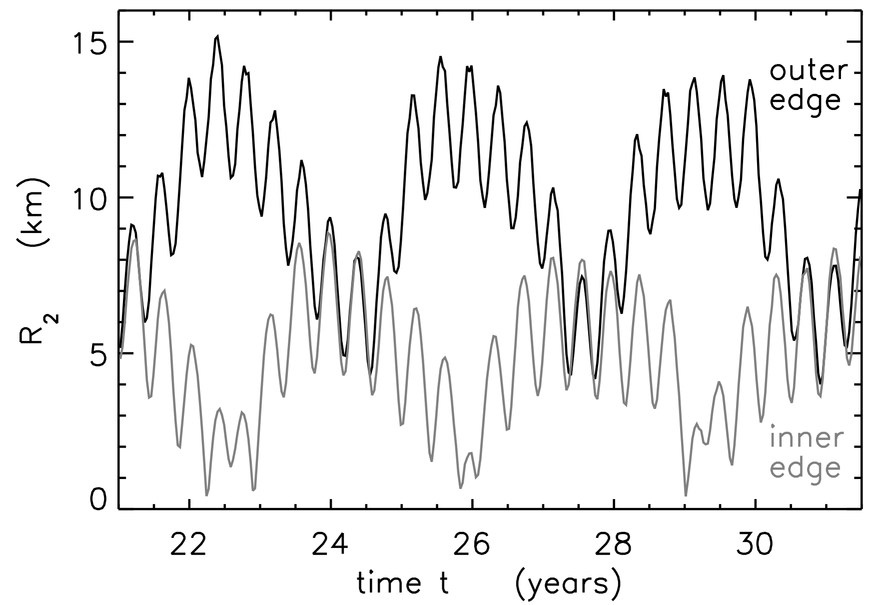


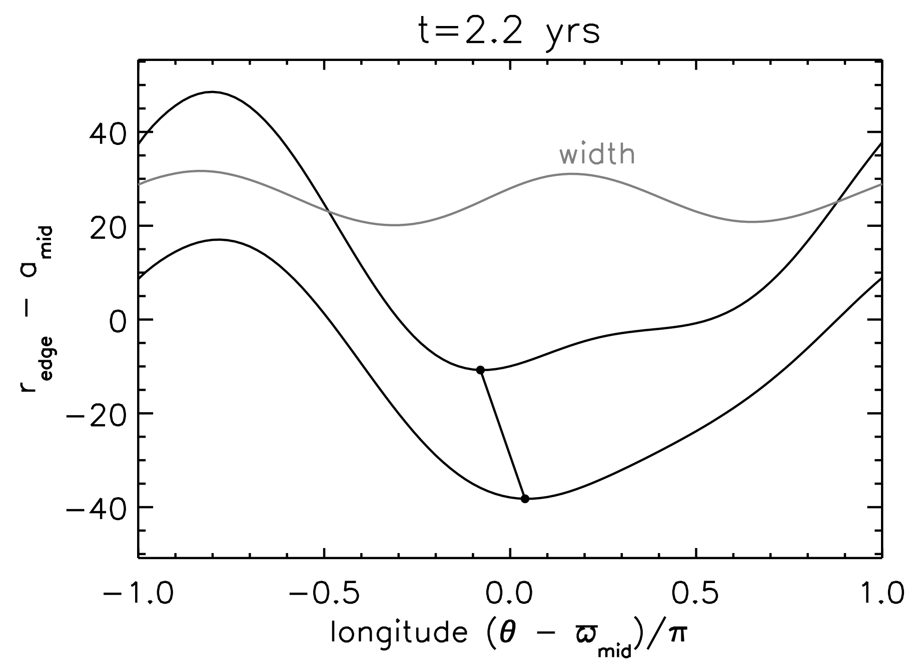
- plots of width-longitude $w(\theta)$ for a librating ringlet always show an $m = 1$ shape where $w(\theta) \sim \cos\theta$

- but Huygen's $w(\theta)$ exhibits a prominent $m = 2$ pattern

simulation of free $m = 2$ modes in Huygens







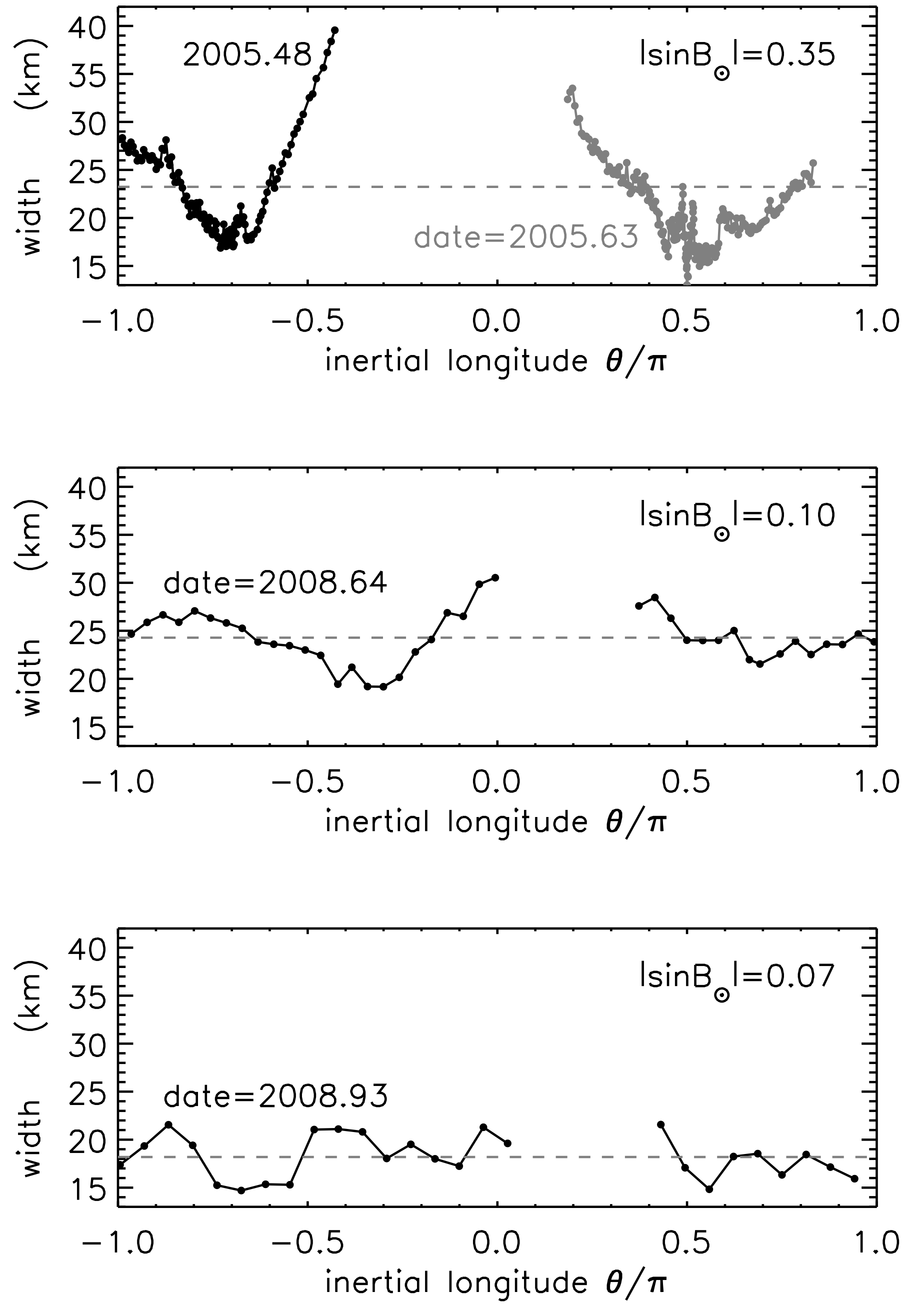
- an unforced $m = 2$ disturbance is planted in Huygens at time $t = 0$
- $m = 2$ mode sloshes back & forth across ringlet, possibly as spiral wave
- model does fill the observed $w(r)$ box, and produces $m = 2$ shape in $w(\theta)$
- R_2 at outer edge is ∼twice inner edge

Huygens' observed width-longitude relationship

- Huygens' $w(\theta)$ does show a prominent $m = 2$ pattern

- but this pattern is oriented towards the *Sun*:
  - Huygens is widest towards Saturn's shadow
  - and also wider towards the solar longitude
  - $m = 2$ amplitude shrank with solar elevation $|B_\odot|$
  - mean width shrank by $\sim$third

- evidently Huygens is *heliotropic* (Hedman et al 2010), but with some interesting distinctions



Keep in mind that JS is measuring edges & widths of the bright, *optically thick* part of Huygens.

If Huygens is heliotropic, then β for an optically thick patch in ring is

$$\beta \;=\; \frac{\text{RP}}{\text{gravity}} \propto \frac{\text{projected area of patch}}{\text{patch mass}} \propto \frac{\sin|B_\odot|}{\sigma_d}$$

(rather than the usual $\beta \propto 1/\rho R$ in optically thin ring)

The patch's forced eccentricity (from Hedman et al) is

$$e_f \;\simeq\; \beta\Omega/\dot{\tilde{\omega}}_{obl} \sim 3\times10^{-7}/\sigma_d$$

where σ_d is dust surface density in cgs units
and $\beta = Q_{pr}L_\odot(r/r_\odot)^2\sin|B_\odot|/4\pi c\sigma_d GM \sim 3\times10^{-9}/\sigma_d$

The epicyclic amplitude due to radiation pressure is
$R_{rp} = ae_f = 0.031\text{km}/\sigma_d \sim 10\text{km}$
$\Rightarrow \sigma_d \sim 0.003$ gm/cm^2
$m_d = \sigma_d A_d \sim 6\times10^{14}$ gm
equivalent solid body radius $R_e \sim 0.7$ km

Reality check:

dust optical depth is $\tau \simeq \sigma_d/\rho R_1 \sim 60/R_{\mu m}$
where $R_{\mu m}$ is typical dust size in microns

$\Rightarrow$ Huygens' dust is optically thick when $R_{\mu m} \lesssim 60\ \mu m$

Q: why does Huygens' $w(\theta)$ show $m = 2$ shape,
when plots of $r(\theta)$ for the heliotropic Charming ringlet (Hedman et al)
shows $m = 1$?

A: dust produce by Huygens' parent bodies have their parents' initial r, v at the moment of formation, but this fresh dust also suddenly feels RP,
resulting in orbits whose free e is comparable to the forced e_f that is due to RP

- this causes dust orbits to *circulate* about their forced orbit, rather than librate
- these circulating orbits are still preferentially oriented to & away from the Sun
- this circulation will make the dust ring widest at the solar longitude and 180° away, resulting in an $m = 2$ shape in $w(\theta)$
- or so I think. Need to confirm this by putting RP into the model.

Findings for Huygens

- plots of Huygens observed $w(\theta)$ shows that
  Huygens is disturbed, with $\Delta w(\theta) \sim \pm 10$km

- this disturbance is not due to libration about equilibrium,
  because the observed $w(\theta)$ shows an $m = 2$ shape

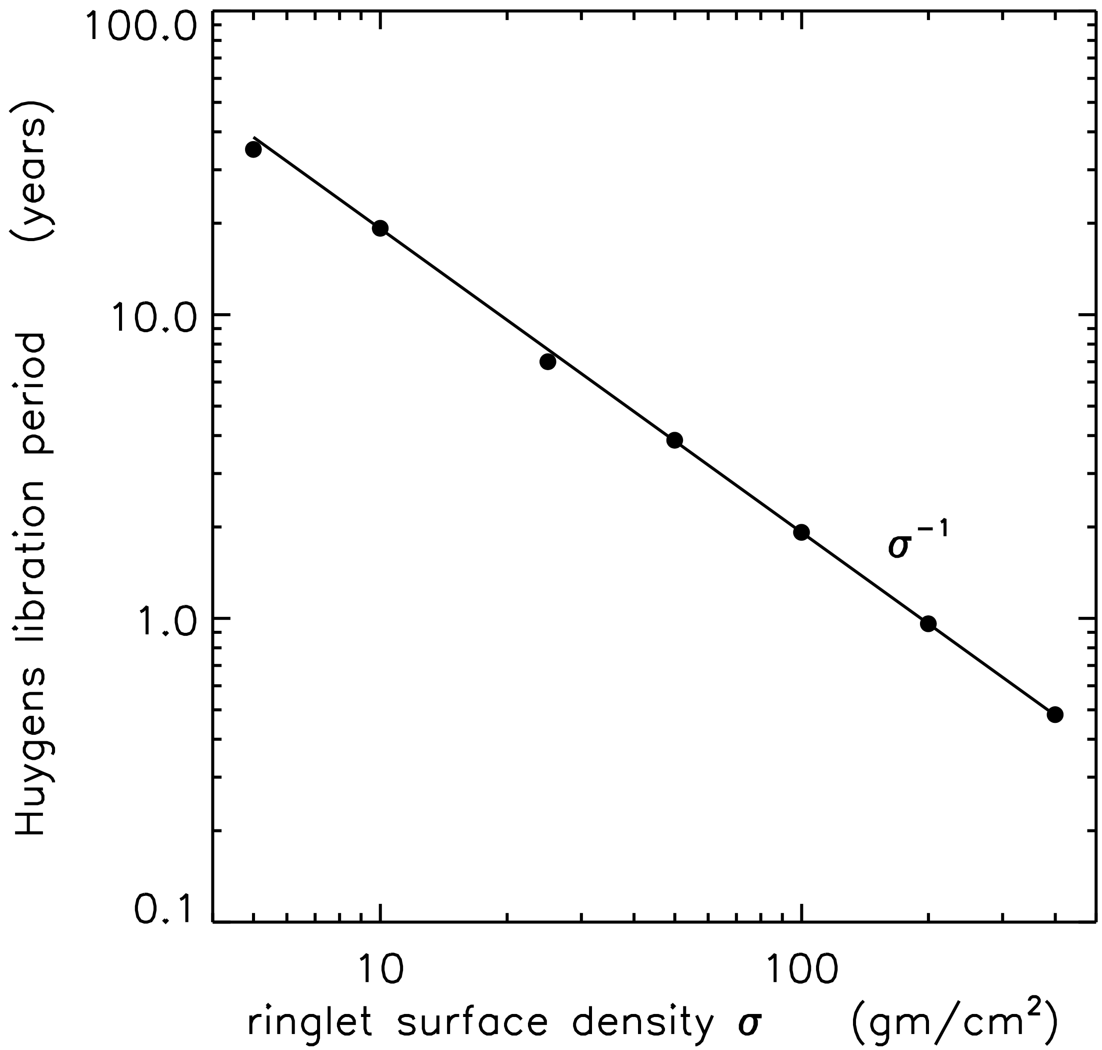
- the disturbance in Huygens is also not due to free $m = 2$ modes
  sloshing across the ringlet via self gravity
  - because the observed $m = 2$ pattern is oriented with the Sun,
    and its amplitude varies with solar elevation $|B_\odot|$

- this indicates that Huygens is an optically-thick ringlet-shaped cloud of dust
  that is generated by unseen parent bodies
  - this dust is heliotropic,
    and its epicyclic response to sunlight
    varies as $R \propto |\sin B_\odot|/\sigma_d$
  - $R \sim 10$ km implies $\sigma_d \sim 0.003$ gm/cm^2 and $m_d \sim 6 \times 10^{14}$ gm

- other ringlets and sharp-edged rings might also be dusty,
  and so might also exhibit these heliotropic behaviors at their edges, too
  - detecting these heliotropic disturbances in other ring-edges
    will provide a means of measuring the ring's dust content

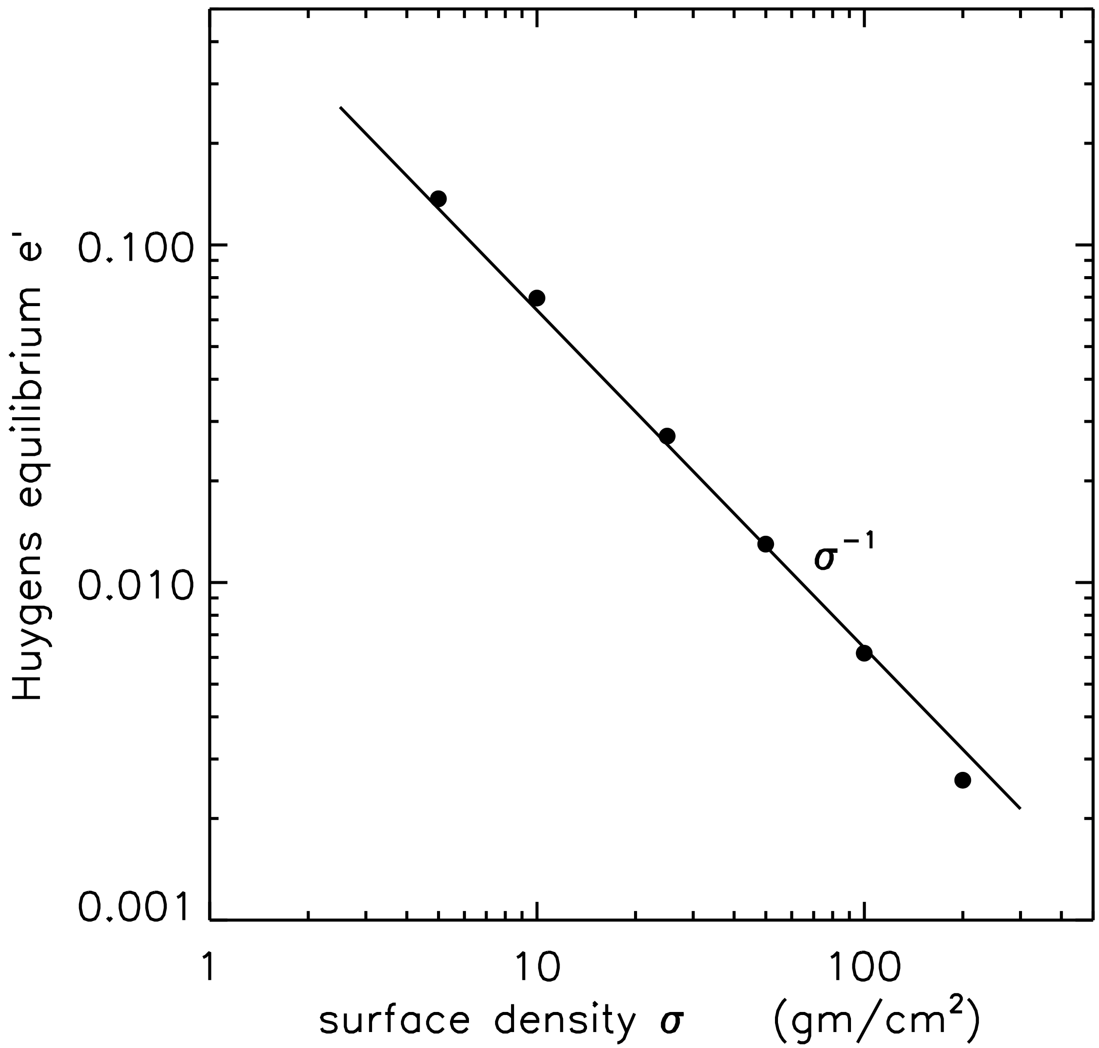
Libration period

- libration period $T_{lib} \propto \sigma^{-1}$, predicted by BGT83
- if you can observe libration, you can infer ringlet σ
  - will likely need $\sigma \gtrsim 50$ gm/cm^2

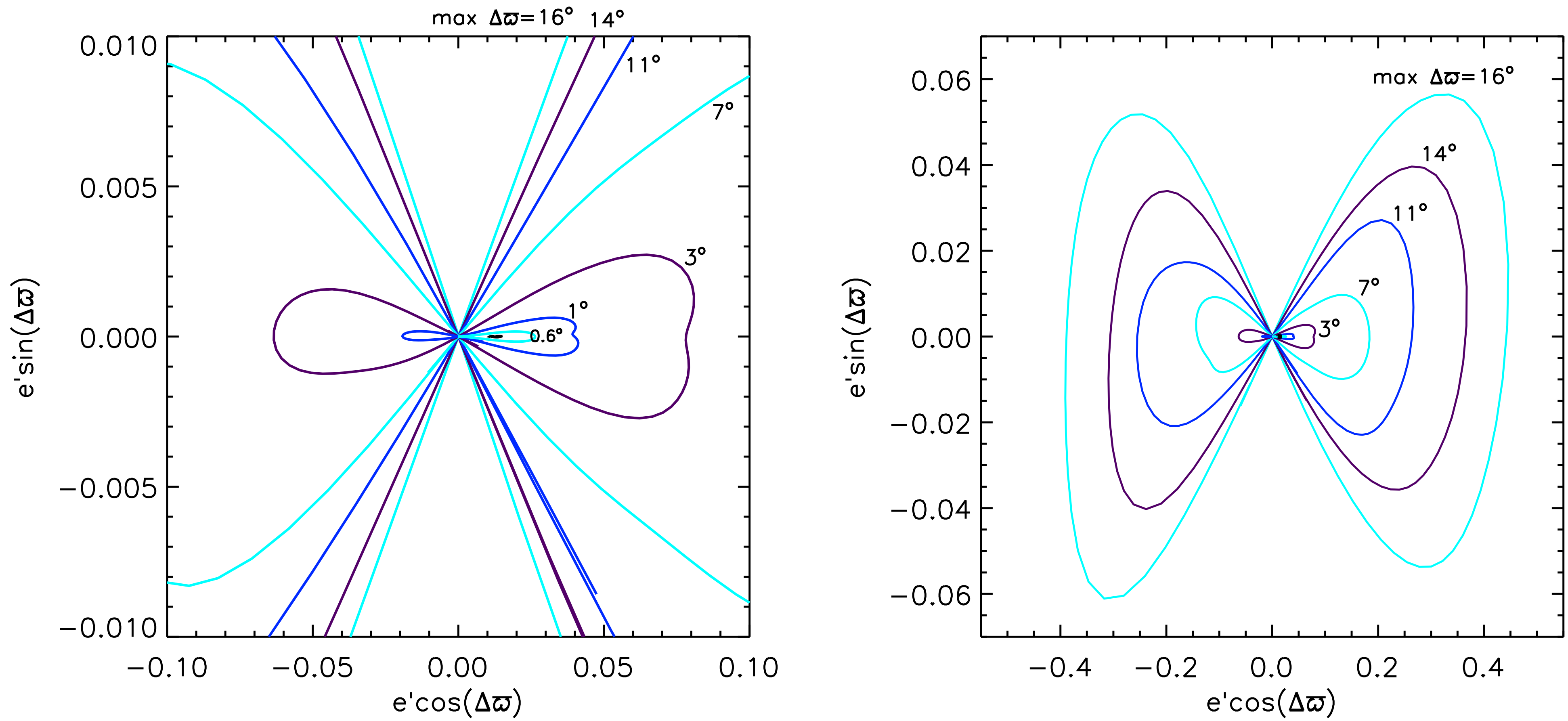


The equilibrium point

- if ringlet is not librating, then equilibrium $e'_{eq} = f(a, e, \Delta a, \sigma, J_2)$ (BGT83)
- observing e'_{eq} will also determine σ
  - likely requires $e' \gtrsim 0.1$ and $\sigma \lesssim 10$ gm/cm^2



Level curves for the Huygens ringlet



- level curve = polar plot of $e'(\Delta\tilde{\omega})$ over time, from N-body sim's of Huygens at various libration amplitudes
- left plot zooms in on center of right figure
- black dot just right of origin is the GT79 equilibrium point
  - low amplitude libration $0 < e' \lesssim 2e'_{eq}$ have teardrop-shaped level curves
  - high amplitude librations are figure 8s
  - ringlet evolves along a level curve with speed $\propto \sigma$

B ring model

- $\boldsymbol{\sigma = 230}$ gm/cm^2