Sculpting the Kuiper Belt via Neptune’s Orbital Migration

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What is a Kuiper Belt Object (KBO)?

- KBOs are distant, ice–rich debris that were left over from when Solar System first formed

- likely heavily cratered due to impacts w/other KBOs, ⇔ perhaps like Phoebe

- Phoebe is in a very wide, retrograde orbit about Saturn—was probably captured from heliocentric orbit
  - some suggest that Phoebe originated in the Kuiper Belt (maybe...)

- nonetheless, this pic’ of Phoebe might be a representative of a typical KBO

from CICLOPS: Cassini Imaging page.
What is the Kuiper Belt?

- a vast swarm of giant comets orbiting just beyond Neptune

- observed KBOs have radii $10 \lesssim R \lesssim 1000$ km
  - $N(R > 50 \text{ km}) \sim 10^5$
  - mass($R > 50 \text{ km}$) $\sim 0.1 \, M_\oplus$
  - $\sim 100 \times$ asteroid belt

- several dynamical subclasses
  - resonant populations (e.g., 3:2, 2:1, 5:2)
  - Main Belt ($40 \lesssim a \lesssim 50$ AU, i.e., between 3:2 and 2:1)
  - Scattered Disk ($a > 50$ AU & $30 < q < 40$ AU)
  - Centaurs ($a < a_{\text{Neptune}}$)
• accretion models (Stern 1995, Kenyon & Luu 1999) show that KBOs can only form in a quiescent environment, i.e., $e_{\text{initial}} \lesssim 0.001$

– some process has disturbed the Kuiper Belt & pumped up resonant KBOs’ $e$’s (and $i$’s)

• these eccentric KBOs orbiting at Neptune’s MMRs are generally interpreted as evidence for Neptune’s orbit having migrating outwards by $\Delta a_{\text{Nep}} \approx 9$ AU
3:2 $\Rightarrow$ evidence for planet migration

- outward migration causes Neptune's mean motion resonances (MMR's) to sweep out across the Kuiper Belt
- ex: the 3:2 is where a KBO orbits 2 times for every 3 orbits of Neptune
- Malhotra (1993) showed that KBOs get trapped at sweeping MMR's, are dragged outwards, and have $e$ pumped up
  - this mechanism accounts for Pluto, with $e = 0.25$ at 3:2
  - the $e$-pumping depends only on Neptune's displacement, $e = f(\Delta a)$

- KBOs at Neptune's 3:2 have $e = 0.33$, so $e = f(\Delta a) = 0.33 \Rightarrow \Delta a = 12$ AU, so they were dragged outwards from $a = 28 \rightarrow 40$ AU

- since Neptune's 3:2 resonance expanded by 12 AU, its semimajor axis evidently expanded by $\Delta a_{\text{Nep}} = 9$ AU
Why would the giant planets migrate?

- cores of giant planets formed within a planetesimal disk
- planet–formation was likely not 100% efficient
  - residual planetesimal debris is left over
- recently–formed planets scatter the planetesimal debris, exchange \( L \) with planetesimal disk
- \( \text{Nbody simulations (Fernandez & Ip 1984, Hahn & Malhotra 1999, Gomes, Morby, Levison 2004)} \) show planets evolve away from each other, ie, Jupiter inwards, Neptune outwards
- driving Neptune \( \Delta a_{\text{Nep}} \simeq 9 \text{ AU} \) requires disk mass \( M_D \sim 50 \text{ M}_\oplus \) over \( 10 < r < 50 \text{ AU} \).
Migration into a dynamically cold Kuiper Belt

- **red dots** = observed KBO orbits

- Mercury Nbody integrator (Chambers 1999) is used to simulate Neptune’s migration into Kuiper Belt (black dots)
  - 4 planets + $10^4$ massless p’s evolved for 4.5 Gyrs
  - planet migration is driven by an external torque on planets, $\Delta a_{\text{Nep}} = 9$ AU
  - initial KB is dynamically cold (ie $e_{\text{initial}} = 0 = i_{\text{initial}}$)

- note: observed Main Belt has $e_{\text{obs}} \sim 0.1$ while $e_{\text{sim}} \sim 0.03$

$\Rightarrow$ something has stirred--up the Kuiper Belt, either prior to, or after the onset of planet--migration
Migration into a dynamically hot Kuiper Belt

- assume KB is stirred–up prior to migration, ie, $e_{\text{initial}} \sim 0.1$
- simulation in better agreement with observed Main Belt
- weaker, higher–order res’nces (eg, 7:4, 5:2) trap particles
  - first noted in migration sim’s by Chiang et al (2003)
  - a surprise—the theory of resonance capture theory shows trapping probability $P \propto e_{\text{initial}}^{-3/2}$ (B&G 1984)...
- migration into a previously stirred–up KB having $e_{\text{initial}} \sim 0.1$ can account for:
  - Main Belt $e \sim 0.1$
  - the 7 KBOs known to librate at the 5:2
• don’t directly simulated $i$’s to observed KBO $i$’s $\leftrightarrow$ these are biased

• instead, compare *ecliptic $i$–distribution* $\rightarrow$ $i$’s of bodies with latitudes $|\beta| < 1^\circ$
  – this model can account for bodies with $i \lesssim 15^\circ$
  – but it does not account for bodies with higher $i$’s

• this is problematic since $\sim 1/2$ of all KBOs have $i > 15^\circ$ (eg, Brown 2001)
Dealing with telescopic selection effects

- telescopes select for larger & brighter KBOs that live nearest the Sun & ecliptic
  - discovery of low a, high e, and low i KBOs are favored
- use Monte Carlo methods to account for selection effects
  - replicate each Nbody particle \( \times 10^4 \), & randomize their positions along their orbital ellipses
  - assume a power–law in the bodies’ cumulative size distribution
    \( N(R) \propto R^{-Q} \)
  - assign apparent magnitudes via
    \[ m = m_\odot - 2.5 \log(pR^2AU^2/r^4), \]
    where \( p = \text{albedo} \)
• the size distribution $Q$ is obtained from the KBO luminosity function:

$$\Sigma(m) = \text{sky-plane number density of KBOs brighter than magnitude } m$$

$$- \quad \Sigma(m) = \int_{m}^{\infty} \frac{dN(R(m))}{dR} dR$$

$$\sim 10^{Qm/5}$$

- the HST KBO survey by Bernstein et al. (2004) shows that the ‘bright end’ of $\Sigma(m < 24)$ has logarithmic slope $\alpha = d\log\Sigma/dm = Q/5 = 0.88$

- observing the Belt 1 magnitude fainter yields $8 \times$ more KBOs

$$- \quad \Rightarrow Q = 5\alpha = 4.4$$
Nbody/Monte Carlo model of the Kuiper Belt

- use Monte Carlo method to assign sizes & magnitudes to Nbody sim’

- \( \sim 500 \) KBOs with known orbits; all have \( m < 24 \)

- also shown are 500 random Nbody/MC particles having \( m < 24 \)

- two notable discrepancies
  - model 2:1 is overdense
  - the model’s ‘Outer Belt’ of \( e \sim 0.1 \) particles beyond \( a > 50 \) AU is extremely overdense

* edge of Solar System at \( a \sim 50 \) AU (eg, Trujillo & Brown 2001)?
the apparent 2:1/Main Belt ratio

- plot the ratio of 2:1/Main Belt (MB) KBOs as a function of magnitude $m$
  - Note: although the number of known KBOs is sensitive to the sky–area surveyed $A(m)$ surveyed by various astronomers, their ratios are *not* sensitive to survey details

- the model’s 2:1/MB ratio $\approx 0.8$, while observed ratio $\approx 0.04$
  - the observed 2:1 population is *underabundant* by a factor of $0.8/0.04 \approx 20$, relative to model predictions

- this discrepancy has been known for some time—see previous figure
The 3:2 population

- but we didn’t know that the 3:2 is also depleted (relative to the MB) by a factor $\sim 6–60$

- note also that the 3:2/MB ratio decreases with $m$
• why?
  – a dearth of fainter objects in 3:2, *not an overabundance of faint MB objects!*
  – can be accounted for if the 3:2 population has *shallower* $Q = 2.7$ size distribution
  – why might the 3:2 population be so different?

* Note: asteroid families exhibit $2 \lesssim Q \lesssim 6$ (Tanga et al 1999)

- asteroid families result when a parent asteroid collides & breaks up; the physics of collisional breakup determines the fragments’ $Q$
- might the 3:2 KBO population be debris from the breakup of a large KBO?
Why are the observed resonant populations depleted (relative to model expectations)?

- blame it on other unmodeled effects:
  - planet migration is driven by scattering of planetesimals by planets
  - particularly large or close scatterings at Neptune will cause its orbit (and its resonances) to shudder some
  - likewise for particles at resonances
    * I expect this shaking of the resonance location & particles’ orbits reduces the trapping efficiency & depletes the resonant populations
Upper limits on an Outer Belt

- No KBOs have been detected in the Outer Belt (OB) beyond $a > 50$ AU
  - outer edge of the Solar System?
- can infer several distinct upper limits:
  - density of KBOs in OB is smaller than MB density by factor $f > 100$,
  - OR all OB bodies are fainter than the faintest KBO in the MB, $m = 24.5$
    * radii $R_{OB} \lesssim 80$ km
      (eg, Allen et al 2002)
  - OR large bodies in OB are rare
    * the OB size distribution is steep, ie, $Q > 6.0$
The Scattered Disk of KBOs

- Nbody integrations show that gravitational scattering by Neptune produces a swarm of bodies in wide, eccentric orbits at $a \gtrsim 50$ AU having perihelia $30 \lesssim q \lesssim 40$ AU (Duncan & Levison 1997)

- but in this simulation, very few scattered bodies persist over a Solar age

- rather, 90% of survivors in gray zone are trapped at various exotic resonances, eg, 9:4, 11:4, 7:2, etc

- only 10% are truly scattered, indicated by crosses

- KBOs in so-called Scattered Disk might not have had close approach to Neptune

  -- rather, they were placed there via resonance trapping
Neptune’s Trojans

- 5 Trojans survived at Neptune’s triangular Lagrange points for $4.5 \times 10^9$ years
- the simulation’s Trojan/MB ratio is $r_{T/MB} \sim 0.01$
Centaurs

- Centaurs have \( a < a_{\text{Neptune}} \)
- only 7 spotted during simulation’s final 2 Gyrs
- simulated Centaurs are rare:
  - due to short dynamical lifetime \( \sim 10^7 \) yrs
  - and sparse time sampling, \( \Delta T = 100 \) Myrs
- observed Centaurs are prominent, due to proximity to Sun
- open circles show that all 7 simulated Centaurs emerged from MMRs
- simulation’s Centaur/MB ratio is \( r_{T/MB} \sim 6 \times 10^{-4} \)
The surface density of the Kuiper Belt

- curves show how Neptune has dynamically eroded the inner KB
  - Note: model does not include collisional erosion, another important and unmodeled effect
- however 2:1 & 3:2 are very depleted, and the Outer Belt (a > 50 AU) is absent or unseen
  - form a truncated Belt that ignores depleted populations
- surface density of simulated truncated Belt agrees quite well with the KBOs’ observed $\sigma(r)$ from Trujillo & Brown (2001)
Calibrate the Kuiper Belt model

- to estimate the total KBO population \( N \), note the Belt’s luminosity function \( \Sigma(m) \propto N \)

- estimate \( N \) by fitting the simulation’s \( \Sigma_{\text{sim}} \) to the observed \( \Sigma_{\text{obs}} \) of Bernstein et al (2004):
  - recall that the simulation’s \( i \)'s are too low, ie, my Belt is too thin
    - median \( i_{\text{sim}} \gtrsim 3^\circ \),
      while median \( i_{\text{obs}} \gtrsim 15^\circ \)
      (from Brown 2001)
    - simulated \( \Sigma_{\text{sim}} \) is overdense by factor \( f_i \sim i_{\text{obs}}/i_{\text{sim}} \sim 5 \)

- to compensate, first divide \( \Sigma_{\text{sim}} \) by \( f_i \) and then fit \( \Sigma_{\text{sim}} \) to \( \Sigma_{\text{obs}} \)

- the final tally: there are \( N(R > 50 \text{ km}) \sim 2 \times 10^5 \) KBOs larger than 50 km
Census of the Kuiper Belt

- assumptions:
  - albedo $p = 0.04$ (eg, comet Halley’s albedo)
  - body density $\rho = 1$ gm/cm$^3$
  - $Q = 4.4$ size distribution, except 3:2 population has $Q = 2.7$

<table>
<thead>
<tr>
<th>Subclass</th>
<th>$r_x/MB$</th>
<th>$N(R &gt; 50 \text{ km})$</th>
<th>mass ($M_\oplus$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centaurs</td>
<td>0.001</td>
<td>100</td>
<td>$7 \times 10^{-5}$</td>
</tr>
<tr>
<td>Trojans</td>
<td>0.008</td>
<td>1,000</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>3:2</td>
<td>0.02</td>
<td>3,000</td>
<td>0.003</td>
</tr>
<tr>
<td>2:1</td>
<td>0.04</td>
<td>5,000</td>
<td>0.002</td>
</tr>
<tr>
<td>Scattered Disk</td>
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<td>25,000</td>
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</tr>
<tr>
<td>Main Belt</td>
<td>1.0</td>
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</tr>
<tr>
<td>Total</td>
<td></td>
<td>160,000</td>
<td>0.08</td>
</tr>
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</table>
• these results are all within factors of \( \sim 2 \) of other estimates that generally adopt rather simple models of the KB:

  – TJL (2001): \( N(R > 50 \text{ km}) \sim 70,000 \) and mass \( \sim 0.06 \text{ M}_\oplus \)

  – extrapolate Bernstein et al (2004) over *entire* Belt:
    \( N(R > 50 \text{ km}) \sim 170,000 \) and mass \( \sim 0.08 \text{ M}_\oplus \)

  – Sheppard et al (2000): \( N_{\text{Centaurs}}(R > 50 \text{ km}) \sim 100 \)

• but recent HST observations of KBO binaries reveal albedos of \( p \sim 0.1 \) (ie, \( 2.5 \times \) larger than previously assumed)

  – so KBO sizes are probably overestimated by \( \sqrt{2.5} \) or 60%

  – and masses overestimated by \( 2.5^{3/2} \sim 4 \Rightarrow m_{\text{KB}} \sim 0.02 \text{ M}_\oplus \)
• Neptune’s migration into a dynamically cold Kuiper Belt (KB) cannot account for the $e \sim 0.1$ that are observed in the Main Belt
  – some other unknown mechanism was also responsible for stirring up the KB

• migration into a hot KB does account for the Main Belt $e$’s, as well as the KBOs trapped at Neptune’s 5:2 (first noted by Chiang et al 2003)
  – trapping also occurs at many other exotic resonances: 11:6, 13:7, 13:6, 9:4, 12:5, 8:3, 11:4
  – this mechanism also parks particles in eccentric orbits in the Scattered Disk
    * most of the simulation’s particles inhabiting the so–called Scattered Disk at $a \lesssim 80$ AU were never scattered...
• a comparison of the model to observations of the KB reveals:

  – the model Belt is ‘too thin’ by a factor of \( f_i \sim \frac{i_{\text{obs}}}{i_{\text{sim}}} \sim 5 \); this is the main deficiency of the model

  – also reveals that the observed resonant populations are depleted relative to model predictions (for example, 2:1 & 3:2 are depleted by \( \times 20 \))

    * could be due to (unmodeled) scatterings at Neptune, or among particles

  – if a hypothetical Outer Belt beyond \( a > 50 \) AU exists, it must

    * be underdense by a factor \( f \gtrsim 100 \) relative to Main Belt

    * or be composed of small bodies, \( R \lesssim 80 \) km

    * or be composed of bodies having a steep size distribution, \( Q > 6.0 \)

• a census of the Kuiper Belt reveals

\[
N(R > 50 \text{ km}) \sim 160,000 \text{ having a mass } \sim 0.02–0.08 \text{ M}_\oplus
\]
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