The Secular Evolution of the Primordial Kuiper Belt

Joseph M. Hahn (LPI)

March 20, 2003
What Happened to the Kuiper Belt?

- KBO orbits indicate that some process stirred up the KB
- Neptune’s outward migration does explain the 3:2.
  - but does not account for high $i$’s
  - nor the low-$q$ KBOs in the Scattered Disk.

* Gomez (2003) showed that high $e, i$ Scattered KBOs can ‘invade’ the Main Belt, but $\varepsilon \sim 0.001$.
- will investigate more efficient processes
The Curious KBO Inclinations

- Main Belt has bimodal $i$’s (Brown 2001)
  - $i \sim 2^\circ$ (dynamically cold, flat disk)
  - and $i \sim 20^\circ$
    (a hotter halo of KBOs?)

- again, Gomez’ ‘invasion’ of the Main Belt can explain the bimodal $i$’s, but $\varepsilon \sim 0.001$

- I’ll explore a more efficient mechanism for stirring up the KB—possibly too efficient?
Secular Evolution of the Kuiper Belt

- secular perturbations are the constant or low–frequency gravitational forces exerted by a perturber

- of particular interest are secular resonances, which are sites where a perturber’s precession rate matches a small body’s:
  - large $e$’s are excited where $\dot{\omega}_{\text{particle}} = \dot{\omega}_{\text{perturber}}$
  - large $i$’s are excited where $\dot{\Omega}_{\text{particle}} = \dot{\Omega}_{\text{perturber}}$

- in a gravitating disk, this $e$–disturbance can propagate away from resonance as a spiral density wave [aka, apsidal wave (Ward and Hahn 1998)].

- the $i$–disturbance can propagate away from resonance as a spiral bending (or nodal) wave (Ward and Hahn 2003).
The Rings Model

- note that the secular evolution of a system of point-masses is identical to that of gravitating rings (e.g., Murray and Dermott 1999).
- treat a disk of numerous small bodies as a nested set of interacting rings of mass $m_j$, orbits $(a_j, e_j, i_j, \tilde{\omega}_j, \Omega_j)$ and thickness $h_j$ due to their particles dispersion velocities $c_j$.
- the planets are thin $h_j = 0$ rings.
- evolve the system as per the Lagrange planetary equations
  - apply the well-known Laplace–Lagrange solution to obtain the system’s secular evolution
  - note, however, that the rings’ finite thickness $h$ softens their gravity, which in turn requires softening the solution’s Laplace coefficients over the scale $h/a$. 
WKB Analysis

- A WKB solution (e.g., an approximate solution) to the planetary equations yields the properties of these waves:
  - Two types of apsidal density waves:
    * Long waves with wavelength $\lambda_L \propto M_{KB}$
    * Short waves with wavelength $\lambda_S \lesssim 10h$
  - There are only long nodal bending waves with wavelength $\lambda_L \propto M_{KB}$

- Apsidal density waves propagate between a resonance and the $Q$–barrier, which lies where $h$ exceeds the threshold

$$h_Q \approx 0.3 \frac{M_{KB}}{M_{\text{Sun}}} \left| \frac{n}{\Omega_{\text{pattern}}} \right| a$$

- If long density waves encounter a disk edge or a $Q$–barrier, they reflect as short density waves.

- Nodal bending waves propagate between resonance and the disk edge, or else they stall where $h \approx 3h_Q \iff \text{New!}$
Simulation of Apsidal Density Waves in a $M_{KB} = 10 \, M_\oplus$ Kuiper Belt with $h = 0.01a$
Summary of Apsidal Density Waves in the KB

- simulated Belt’s have masses $M_{KB} = 30$ to $0.2 \, M_\oplus$ (e.g., the Belt’s primordial mass to its current, eroded mass) and $h = 0.002a$

- density waves reflect at the disk edge at 70 AU or at a $Q$–barrier.
  - reflected short waves are nonlinear, ie., $\Delta \sigma/\sigma \sim 1$

- the giant planets deposit $\sim 0.5\%$ of their $e$–AMD into the disk in the form of spiral density waves.
  - consequently, larger $e$’s get excited in lower–mass disks
  - waves excite large $e$’s in low–mass disks, $e \sim 0.3$ for $M_{KB} \sim 0.2 \, M_\oplus$
  - but this requires a very thin disk, $h \sim 0.002a$
• the giant planets deposit $\sim 10\%$ of their $i$–AMD into the disk in the form of spiral bending waves.
  
  – again, larger $i$’s get excited in lower–mass disks

• bending waves also reflect at the disk edge at 70 AU or else they stall where $h \gtrsim 3h_Q$
  
  – note the low $i$’s interior to the stall–zone
Implications for the Primordial Kuiper Belt

- when the KB was still young and quite massive, $M_{KB} \sim 30 \ M_\oplus$, then low-amplitude apsidal density waves ($e_{\text{max}} \sim 0.02$) and nodal bending waves ($i_{\text{max}} \sim 0.5^\circ$) were sloshing about the KB.

  - wave propagation times were short,

  $$T_{\text{prop}} \sim 10^6 \left( \frac{\Delta a}{30 \ \text{AU}} \right) \left( \frac{M_{KB}}{30 \ M_\oplus} \right)^{-1} \ \text{years} \quad (2)$$

  - the density waves eventually reflect and return as nonlinear short waves having $\Delta \sigma / \sigma \sim 1$ which dominate the Belt’s surface density structure

  - there was no localized excitation of $e$’s and $i$’s since there are no resonances in this massive Belt.
Implications for the Current Kuiper Belt

- over time, gravitational stirring by large KBOs increased the disk thickness \( h \) while collisional erosion decreased \( M_{KB} \rightarrow 0.2 \, M_\oplus \)

- stirring/erosion draws the \( Q \)-barrier and the stall–zone inwards to the secular resonances at \( \sim 40 \, \text{AU} \) which ultimately shuts off wave action

- if stirring/erosion happened quickly, in less than \( T_{\text{prop}} \sim 50 \times 10^6 \) years (the time for waves to propagate out to the Main Belt) then wave–action did not excite the KB \( \Leftarrow \text{least exotic outcome} \)

- but if the stirring/erosion timescale \( \geq T_{\text{prop}} \), then bending waves with \( i_{\text{wave}} \sim 10^\circ \) would have destroyed the Main Belt’s low \( i \sim 2^\circ \) component \( \Leftarrow \text{not allowed} \)

- so if bending waves did get into the Main Belt, they must have stalled far downstream

- if this distant stall–zone slowly migrated inwards to \( \sim 50 \, \text{AU} \) due to KB erosion, this would have terminated the Main Belt at \( \sim 50 \, \text{AU} \) by lofting the more distant KBOs into high \( i \) orbits \( \Leftarrow \text{a bit exotic, but does agree with observations} \)
alternatively, waves could have avoided destroying the Main Belt by propagating into a more distant reservoir of as–yet–unseen KBOs orbiting beyond 50 AU
– however many KB astronomers object to this utterly speculative scenario...