The Secular Evolution of the Primordial Kuiper Belt

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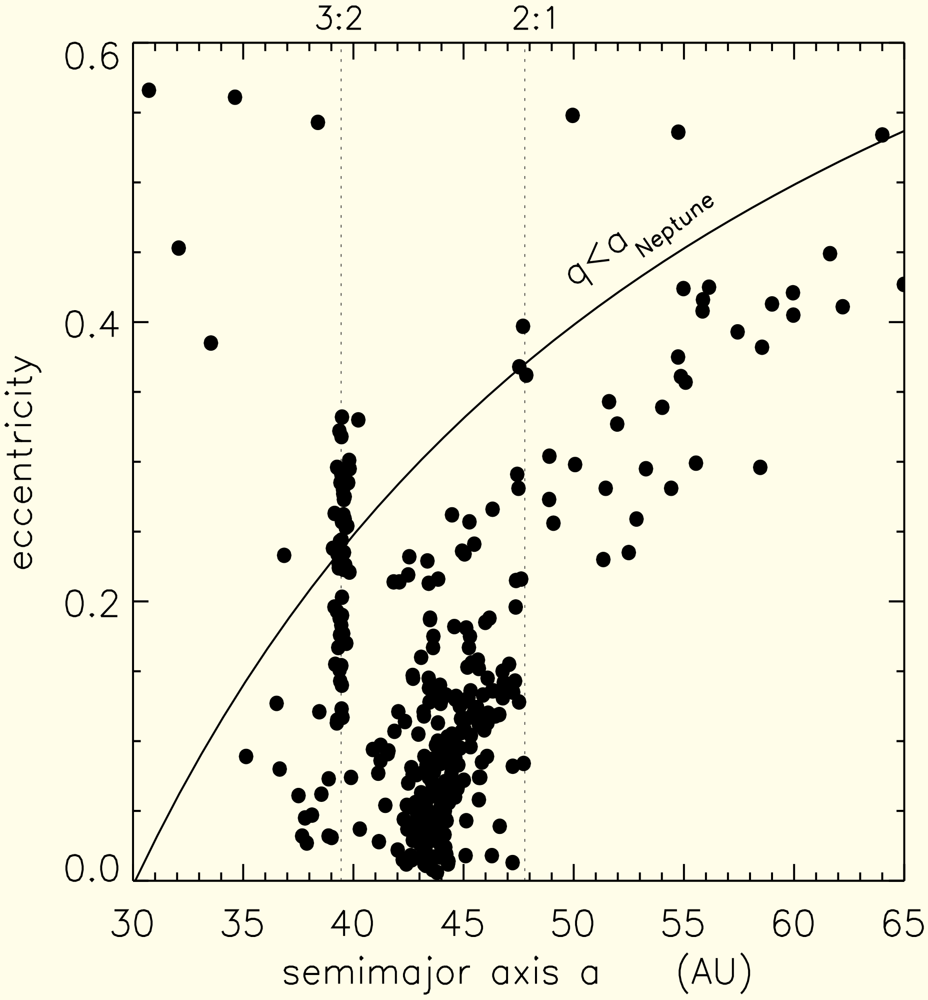
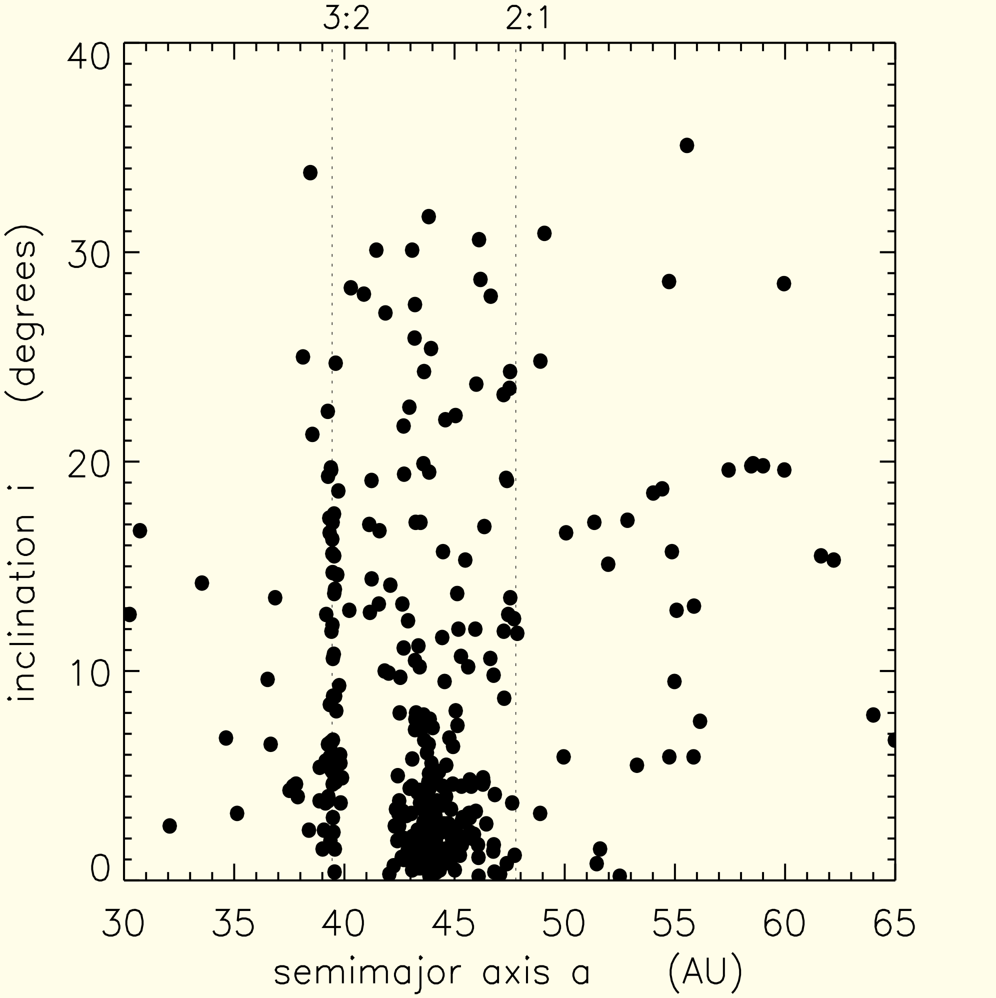
What Happened to the Kuiper Belt?

- KBO orbits indicate that some process stirred up the KB
- Neptune's outward migration does explain the 3:2.
  - but does not account for high i's
  - nor the low-q KBOs in the Scattered Disk.
    - $\ast$ Gomez (2003) showed that high e, i Scattered KBOs can 'invade' the Main Belt, but $\varepsilon \sim 0.001$.
- will investigate more efficient processes



The Curious KBO Inclinations

- Main Belt has bimodal i's (Brown 2001)
  - $i \sim 2^\circ$ (dynamically cold, flat disk)
  - and $i \sim 20^\circ$ (a hotter halo of KBOs?)
- again, Gomez' 'invasion' of the Main Belt can explain the bimodal i's, but $\varepsilon \sim 0.001$
- I'll explore a more efficient mechanism for stirring up the KB—possibly too efficient?



Secular Evolution of the Kuiper Belt

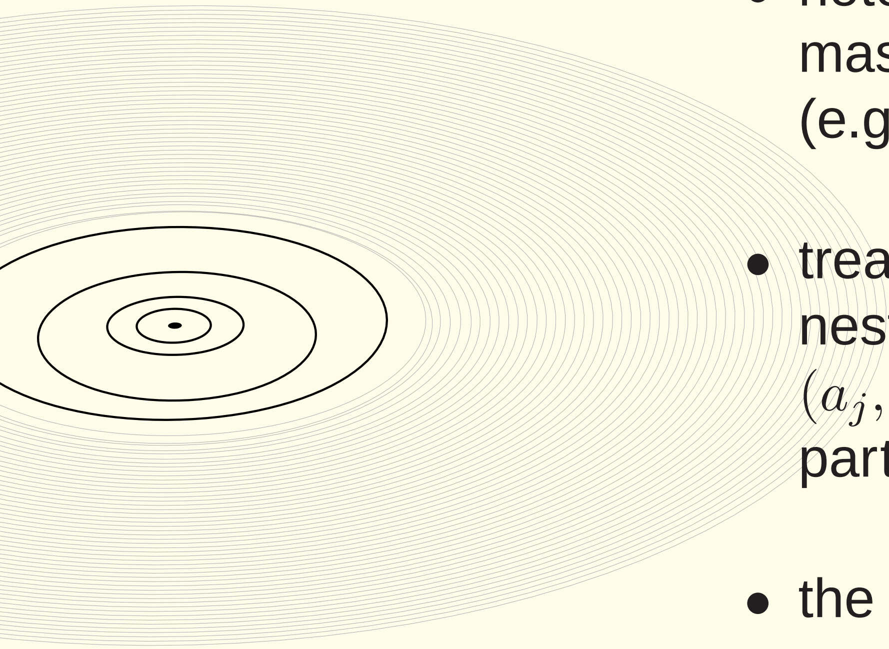
- secular perturbations are the constant or low–frequency gravitational forces exerted by a perturber

- of particular interest are secular resonances, which are sites where a perturber's precession rate matches a small body's:
  - large e's are excited where $\dot{\tilde{\omega}}_{\text{particle}} = \dot{\tilde{\omega}}_{\text{perturber}}$
  - large i's are excited where $\dot{\Omega}_{\text{particle}} = \dot{\Omega}_{\text{perturber}}$

- in a gravitating disk, this e–disturbance can propagate away from resonance as a spiral density wave [aka, apsidal wave (Ward and Hahn 1998)].

- the i–disturbance can propagate away from resonance as a spiral bending (or nodal) wave (Ward and Hahn 2003).

The Rings Model



- note that the secular evolution of a system of point-masses is identical to that of gravitating rings (e.g., Murray and Dermott 1999).

- treat a disk of numerous small bodies as a nested set of interacting rings of mass m_j, orbits $(a_j, e_j, i_j, \tilde{\omega}_j, \Omega_j)$ and thickness h_j due to their particles dispersion velocities c_j.

- the planets are thin $h_j = 0$ rings.

- evolve the system as per the Lagrange planetary equations
  - apply the well-known Laplace–Lagrange solution to obtain the system's secular evolution
  - note, however, that the rings' finite thickness h softens their gravity, which in turn requires softening the solution's Laplace coefficients over the scale h/a.

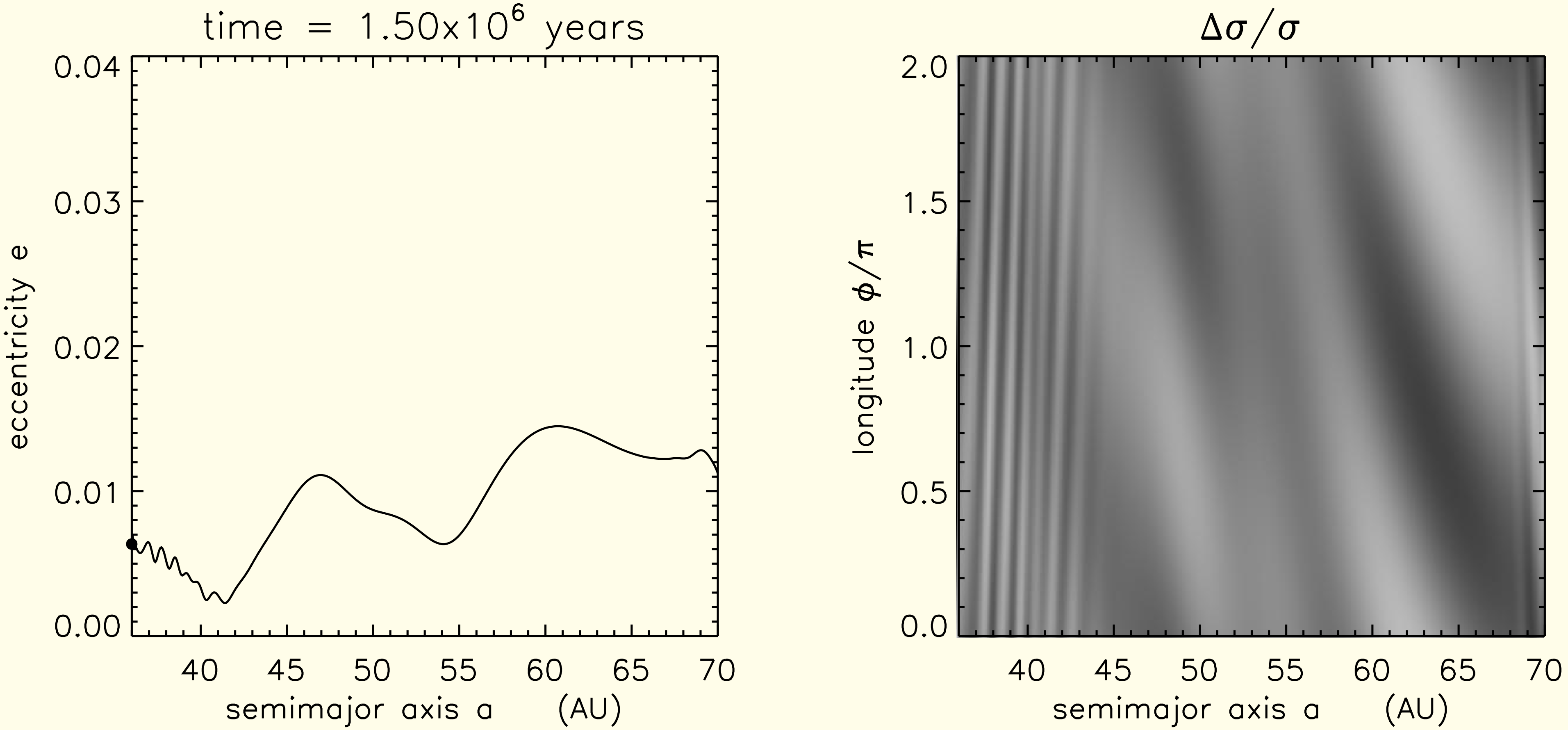
WKB Analysis

- a WKB solution (e.g., an approximate solution) to the planetary equations yields the properties of these waves
  - two types of apsidal density waves:
    - long waves with wavelength $\lambda_L \propto M_{KB}$
    - short waves with wavelength $\lambda_S \lesssim 10h$
  - there are only long nodal bending waves with wavelength $\lambda_L \propto M_{KB}$
- apsidal density waves propagate propagate between a resonance and the Q–barrier, which lies where h exceeds the threshold

$$h_Q \simeq 0.3 \frac{M_{KB}}{M_{\text{Sun}}} \left| \frac{n}{\Omega_{\text{pattern}}} \right| a \tag{1}$$

- if long density waves encounter a disk edge or a Q–barrier, they reflect as short density waves
- nodal bending waves propagate between resonance and the disk edge, or else they *stall* where $h \simeq 3h_Q$ ⇐ New!

Simulation of Apsidal Density Waves
in a $M_{KB} = 10$ M$_\oplus$ Kuiper Belt with $h = 0.01a$

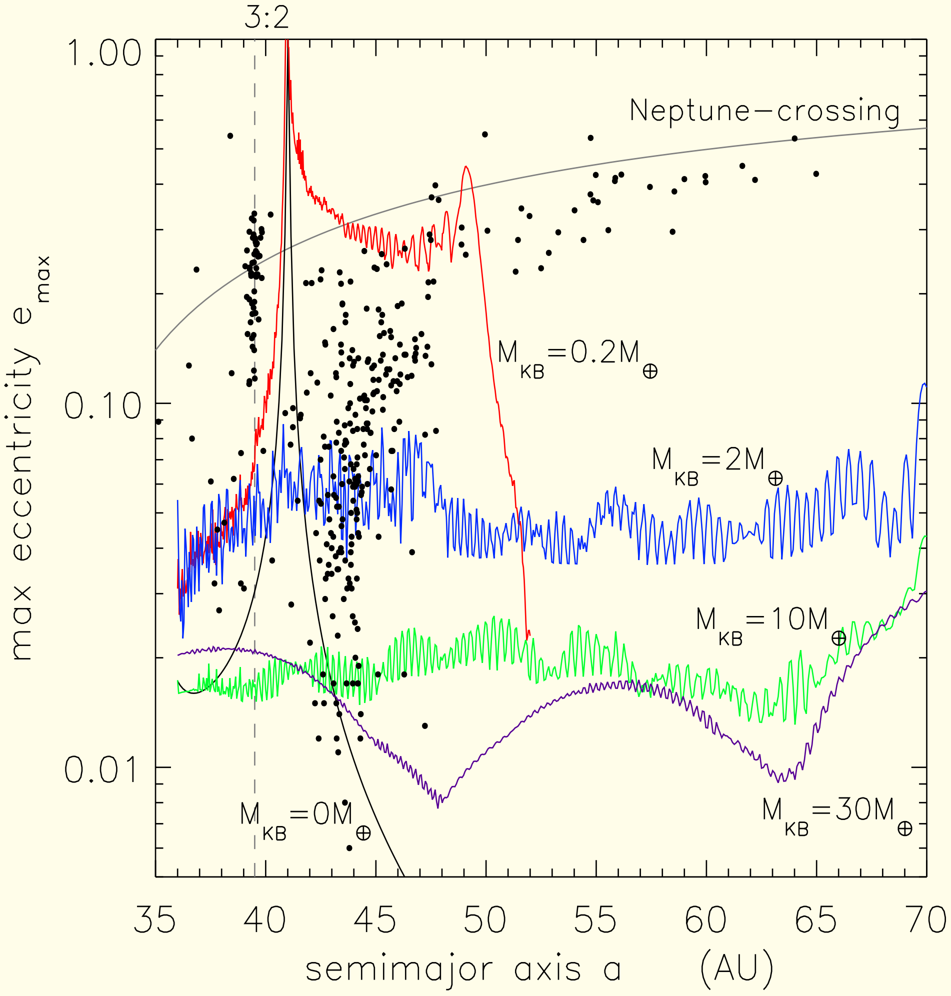


Summary of Apsidal Density Waves in the KB

- simulated Belt's have masses $M_{KB} = 30$ to 0.2 M$_\oplus$ (e.g., the Belt's primordial mass to its current, eroded mass) and $h = 0.002a$

- density waves reflect at the disk edge at 70 AU or at a Q–barrier.
  - reflected short waves are nonlinear, ie., $\Delta\sigma/\sigma \sim 1$

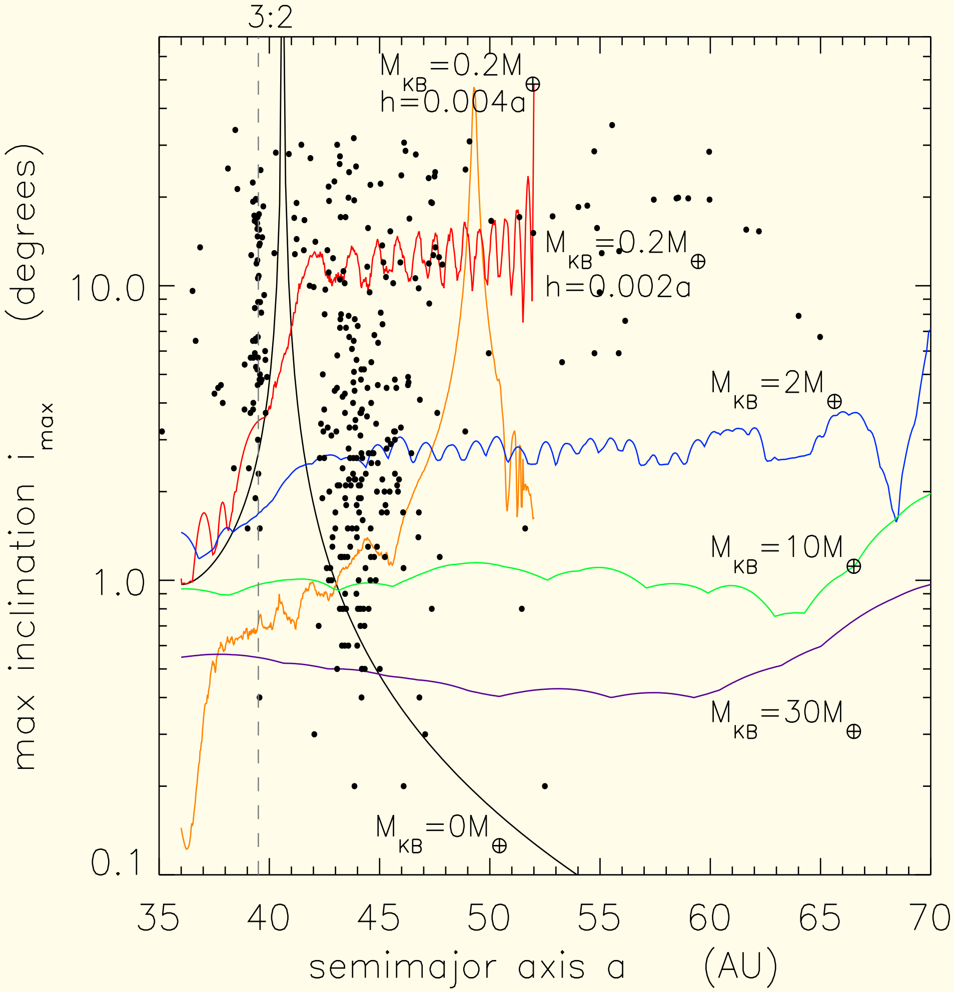
- the giant planets deposit $\sim 0.5\%$ of their e–AMD into the disk in the form of spiral density waves.
  - consequently, larger e's get excited in lower–mass disks
  - waves excite large e's in low–mass disks, $e \sim 0.3$ for $M_{KB} \sim 0.2$ M$_\oplus$
  - but this requires a very thin disk, $h \sim 0.002a$



Summary of Nodal Bending Waves in the KB

- the giant planets deposit $\sim 10\%$ of their i–AMD into the disk in the form of spiral bending waves.
  - again, larger i's get excited in lower–mass disks

- bending waves also reflect at the disk edge at 70 AU or else they stall where $h \gtrsim 3h_Q$
  - note the low i's interior to the stall–zone



Implications for the Primordial Kuiper Belt

- when the KB was still young and quite massive, $M_{KB} \sim 30$ M$_\oplus$, then low–amplitude apsidal density waves ($e_{\rm max} \sim 0.02$) and nodal bending waves ($i_{\rm max} \sim 0.5^\circ$) were sloshing about the KB.
  - wave propagation times were short,

$$T_{\rm prop} \sim 10^6 \left(\frac{\Delta a}{30\ \rm AU}\right)\left(\frac{M_{KB}}{30\ \rm M_\oplus}\right)^{-1}\ \text{years} \tag{2}$$

  - the density waves eventually reflect and return as nonlinear short waves having $\Delta\sigma/\sigma \sim 1$ which dominate the Belt's surface density structure
  - there was no localized excitation of e's and i's since there are no resonances in this massive Belt.

Implications for the Current Kuiper Belt

- over time, gravitational stirring by large KBOs increased the disk thickness h while collisional erosion decreased $M_{KB} \rightarrow 0.2$ M$_\oplus$
  - stirring/erosion draws the Q-barrier and the stall–zone inwards to the secular resonances at ~ 40 AU which ultimately shuts off wave action

- if stirring/erosion happened quickly, in less than $T_{\text{prop}} \sim 50 \times 10^6$ years (the time for waves to propagate out to the Main Belt) then wave–action did not excite the KB $\Leftarrow$ least exotic outcome

- but if the stirring/erosion timescale $> T_{\text{prop}}$, then bending waves with $i_{\text{wave}} \sim 10^\circ$ would have destroyed the Main Belt's low $i \sim 2^\circ$ component $\Leftarrow$ not allowed

- so if bending waves did get into the Main Belt, they must have stalled far downstream
  - if this distant stall–zone slowly migrated inwards to ~ 50 AU due to KB erosion, this would have terminated the Main Belt at ~ 50 AU by lofting the more distant KBOs into high i orbits $\Leftarrow$ a bit exotic, but does agree with observations...

- alternatively, waves could have avoided destroying the Main Belt by propagating into a more distant reservoir of as–yet–unseen KBOs orbiting beyond 50 AU
  - however many KB astronomers object to this utterly speculative scenario...