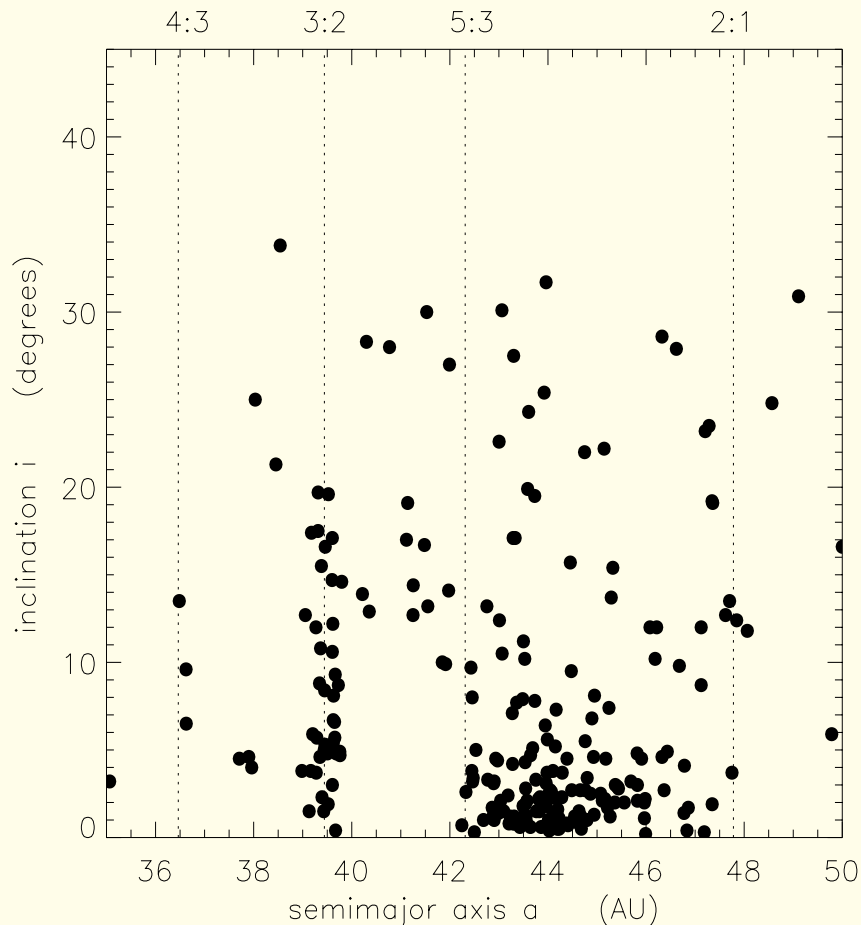# Secular Resonance Sweeping in a Self–Gravitating Kuiper Belt

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What Stirred Up the Kuiper Belt?

4:3   3:2   5:3   2:1

inclination i   (degrees)

40

30

20

10

0

36   38   40   42   44   46   48   50

semimajor axis a   (AU)

- Pluto and $R \sim 100$ km KBOs only form when initially in nearly circular orbits, *i.e.*, $e_0 \sim \sin i < 0.01$ (Kenyon and Luu 1999).

- However Main Belt KBOs $(40 \lesssim a \lesssim 50$ AU$)$ are bimodal, *i.e.*, $i \sim 2°$ and $i \sim 17°$ (Brown 2001).

- Possible explanations:

  – scattering by giant planets (Levison and Stern 2001),

  – scattering by long–gone protoplanets (Petit *et al* 1999),

  – perturbations from a passing star (Ida *et al* 2000),

  – i–pumping by secular resonances that sweep across the solar system as the solar nebula gas dissipates (Nagasawa and Ida 2000).
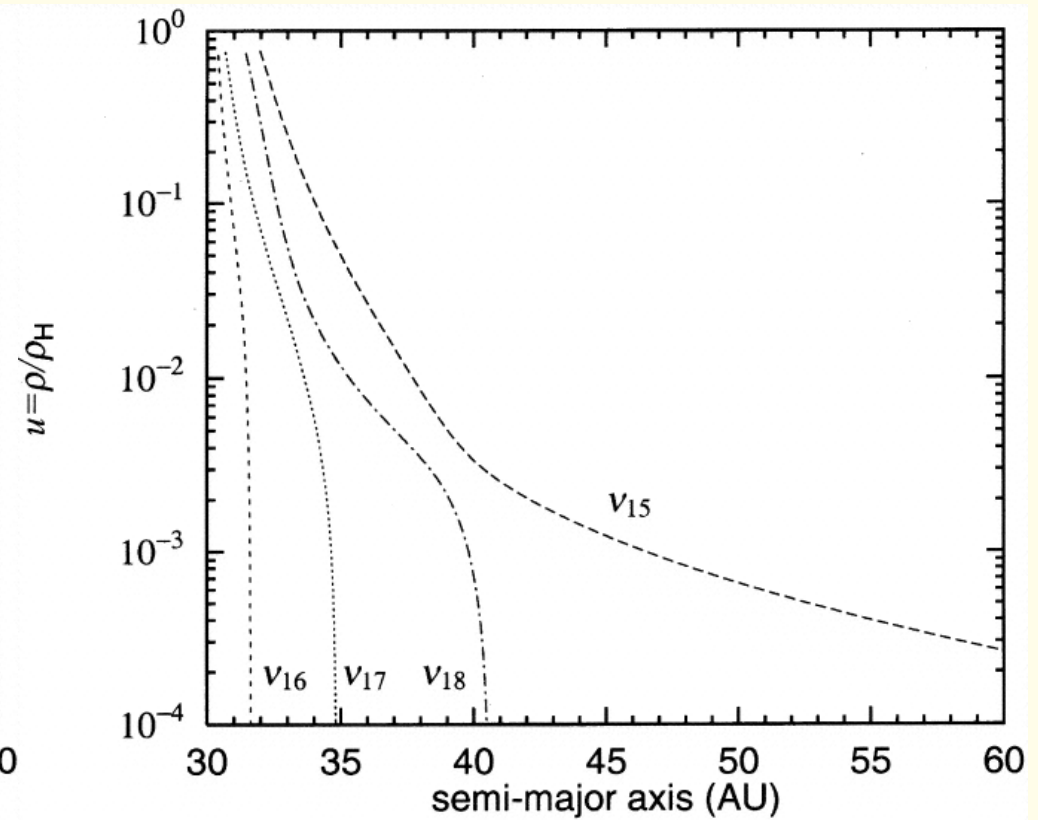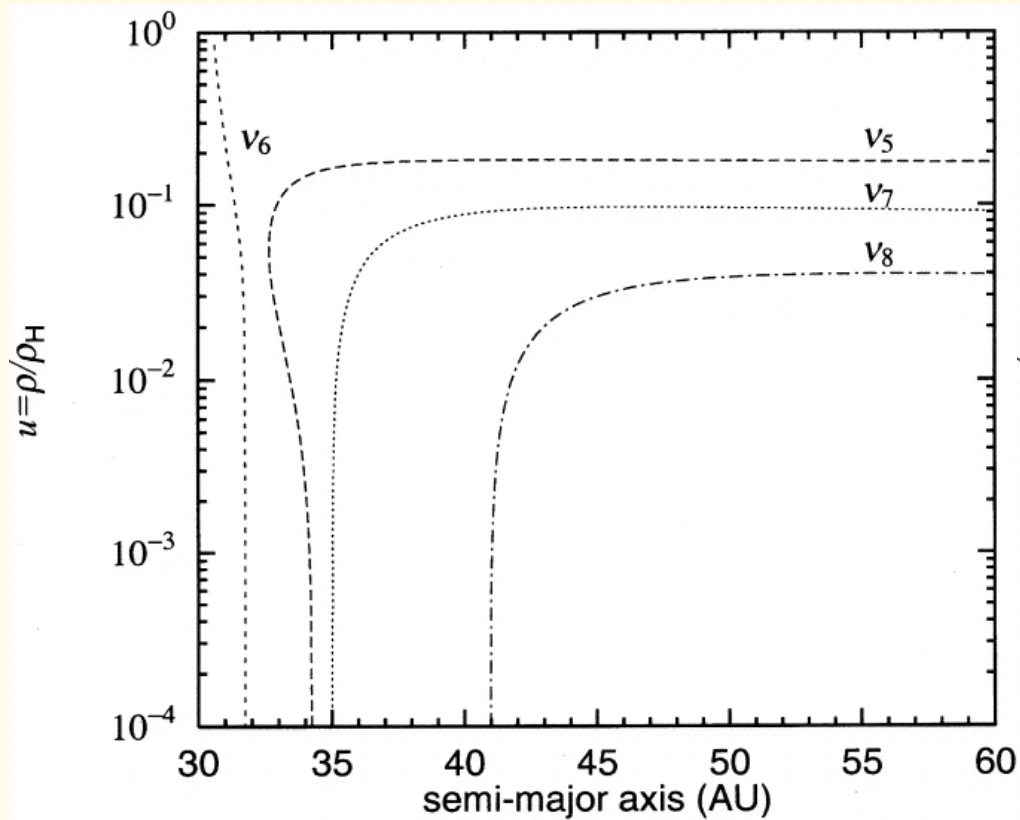
Secular resonances are sites where a small body's orbital precession rates matches one of the solar system's natural eigenfrequencies. Large e's are excited at the ν_j resonances (left figure) and large i's are excited at the ν_{1j} (right). Fig's from Nagasawa and Ida (2000).

As the solar nebula gas is depleted, the ν_5, ν_7, and the ν_8 resonances sweep inwards across the KB and excite eccentricities.

The ν_{15} and ν_{18} also sweep across the KB and excite inclinations.

2

Revisit Sweeping Secular Resonances
Using an 'N–Ring' Integrator

- solve the linearized Laplace–Lagrange eqn's of motion for the secular evolution of N bodies that are perturbed by a minimum–mass solar nebula whose gas content decays exponentially of a timescale τ.

- e.g., treat the 4 giant planets + numerous small bodies at a set of nested gravitating rings whose mutual perturbations cause the rings to flex and tilt, causing their e's and i's evolve over time.

$M_{disk} = 0M_{\oplus}$

$t = 0.0\tau$

$M_{disk} = 10M_{\oplus}$

$\rho(t)/\rho_o = 1.0E+00$

4
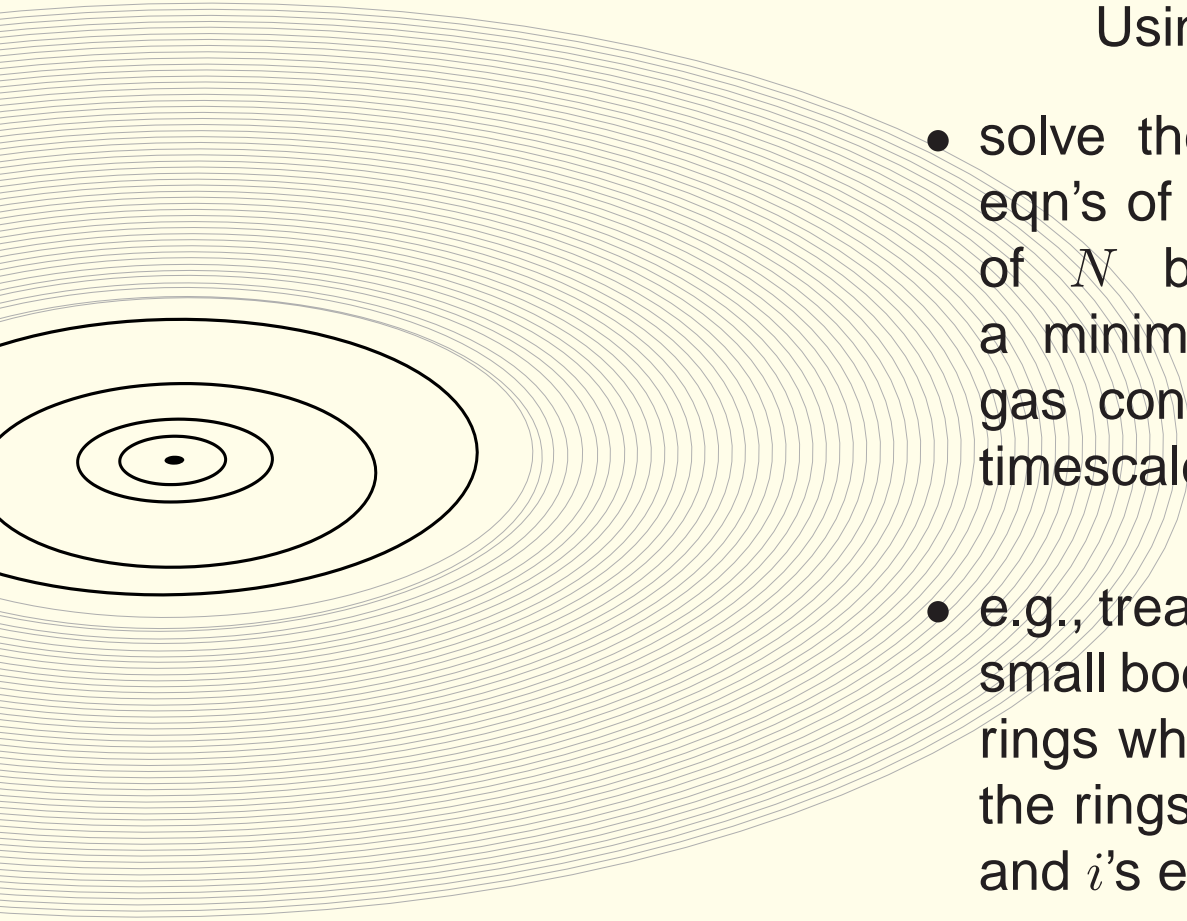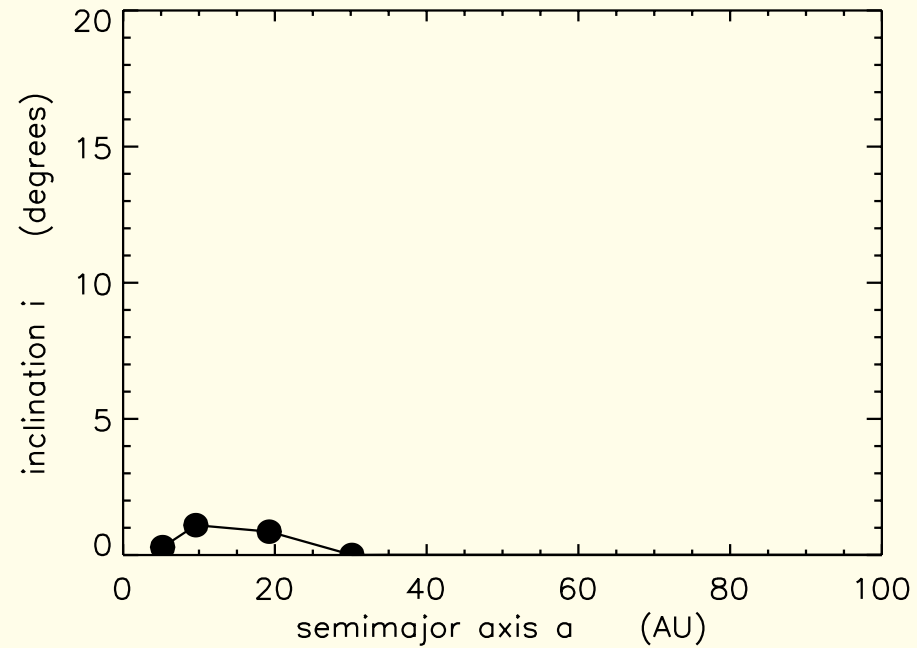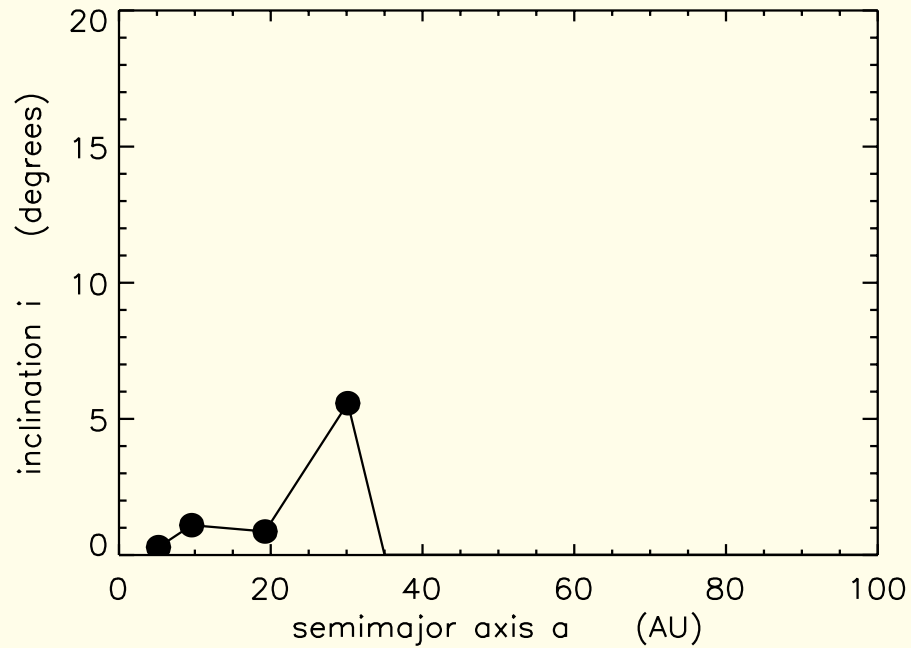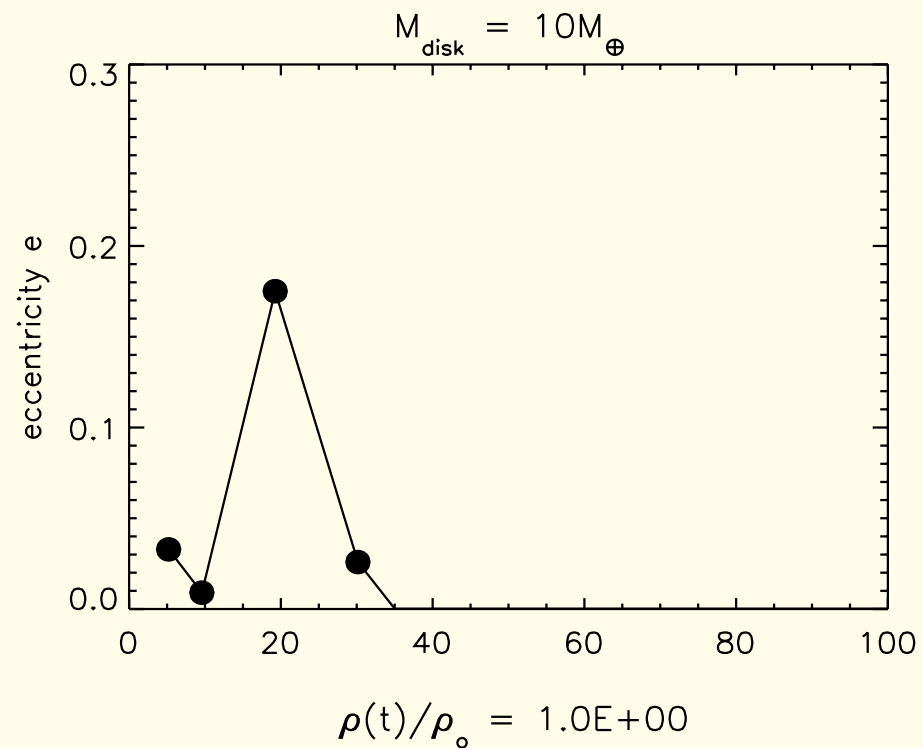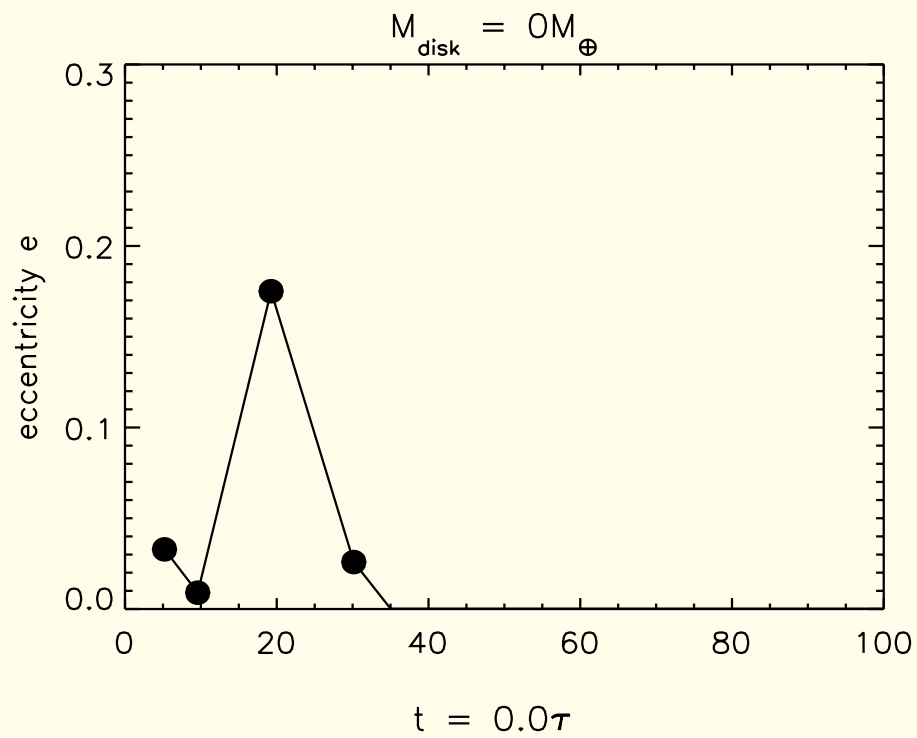
Results for a Massless Kuiper Belt $M_{disk} = 0$

- Repeats Nagasawa and Ida (2000) result for $\tau = 10^7$ years.

- sizable eccentricities are excited in the KB over $30 \lesssim a \lesssim 50$ AU.

- very large inclinations of $> 10°$ are excited by ν_{18} at $a = 41$ AU and by $\nu_{15} \rightarrow \infty$.

- NOTE: Nagasawa and Ida put their nebula in the *ecliptic* which is tilted $\Delta i = 1.6°$.

  - repeating this experiment with the nebula midplane in the giant–planets' invariable plane reduces the i excitation by *orders of magnitude*.

Results for a Self–Gravitating Kuiper Belt of mass $M_{disk} = 10\ M_\oplus$

- This particular simulation has $\sigma_{solids} \simeq \frac{1}{5}\times$ solids in a minimum–mass nebula. Other runs with $10\times$ more/less mass have also been performed.

- When $t \simeq 2.5\tau$ and $\rho(t) \simeq 0.1\rho_0$, the giant planets launch a spiral density wave that propagates from the inner to the outer edge of the KB where it reflects multiple times.

- When $t \simeq 5\tau$ and $\rho(t) \simeq 0.01\rho_0$, a spiral bending wave is launched.

- The effect of the giant planets secular perturbation is analogous to tugging on a sheet:

  - When there is no tension in this Kuiper Belt sheet (*i.e.*, no mass or self–gravity), the giant planets perturbations' generate large–amplitude 'wrinkles' in the inner part of this sheet nearest the planets.

  - But when the sheet has tension (e.g., $M_{disk} > 0$), the planets' pushes and pulls gets transmitted across the length of this sheet in the form of spiral waves. This results in very low–amplitude excitation that spans the entire KB.

The Fine Print

- My equations of motion are formally valid only when adjacent rings do not intersect. I still need to generalize these results for cases were the rings get eccentric enough for streamlines to cross.

- I used only $N = 50$ rings in my KB, which is only marginally sufficient to resolve the spiral waves. These results will be confirmed by higher resolution simulations using more rings.

- This model treats the nebula gas as a 'rigid' disk, whereas a real disk might flex somewhat due to the planets' perturbations, which may lead to an additional higher order perturbations upon the Kuiper Belt. This effect can be studied by treating the gas disk itself as a set of nested, 'fat' rings that interact via their internal pressure.

Conclusions

- Secular resonance sweeping due to nebula dispersal can excite the KB only if:

  - $M_{disk}(30 < a < 100 \text{ AU}) \lesssim 1 \text{ M}_\oplus$, or
    if $\sigma_{solids} \simeq \frac{1}{50} \times$ solids in a minimum–mass nebula.

  - and if the midplane of the solids has a slight tilt relative to the gas midplane. But this may be a difficulty since the planets and KBOs accreted from dust that sedimented right in the gas midplane...

- But if $M_{disk} \gtrsim 1 \text{ M}_\oplus$, then disk self–gravity allows spiral waves to propagate. This smears the giant planets' perturbations clear across the KB. The excitation is no longer localized in the Main Belt, and low–level excitation instead spans the entire Belt.

- Speculation: if the asteroid belt was sufficiently massive during nebula dispersal, wave–action might also mute the excitation that is thought to occur there, too.