The Secular Evolution of a Close Ring–Satellite System

Joseph M. Hahn
Space Science Institute
Austin, TX
jhahn@spacescience.org

Introduction

The following considers the dynamical evolution of a small satellite orbiting in a dense planetary ring. Examples of such systems include the gap-embedded moonlets Pan (which inhabits the Encke gap), and Daphnis (which orbits in the Keeler gap), both of which reside in Saturn's main A ring.

Our interest here is the long-term orbital stability of a gap-embedded satellite, which is actually uncertain due to the satellite's resonant interactions with the ring. For instance, a satellite's back-reaction to its many vertical resonances in the ring tends to pump up the satellite's inclination i over time (Borderies et al 1984). Similarly, the satellite's many Lindblad and corotation resonances in the ring will also cause the satellite's eccentricity e to evolve (Goldreich & Tremaine 1981, 1982). Consequently, it is a curiosity that Pan and Daphnis inhabit nearly circular orbits coplanar with the ring, despite their resonant interactions with the ring.

The next Section summarizes the effects of the satellite's resonant interactions with the ring. Subsequent Sections then calculate the rate at which the satellite's *secular* perturbations of the ring also cause the satellite's e and i to evolve. These evolutionary rates are then compared in the final Section, to determine whether this secular ring-satellite interaction can in fact stabilize the orbit of a gap-embedded satellite.

A gap-embedded satellite's resonant interactions with a ring

Borderies et al (1984) calculate the rate at which an inclined satellite's vertical resonance in a planetary ring tends to pump up the satellite's inclination i. When that interaction is then summed over all of the satellite's resonances in the ring, its inclination is excited at the net rate

$$\frac{di}{dt} \simeq f_v \left(\frac{\mu_s \mu_d}{\Delta^4}\right) in, \tag{1}$$

where the factor $f_v \simeq 0.02$, the satellite's mass μ_s is in units of the central planet's mass M_p, the so-called normalized ring mass $\mu_d = \pi\sigma a^2/M_p$ where σ is the ring surface density, Δ is the gap's fractional half-width, and a and n are the satellite's semimajor axis and mean angular velocity (Borderies et al 1984, Ward & Hahn 2003). For Pan, which orbits in the Encke gap (whose full width is $2\Delta a \simeq 300$ km) in Saturn's A ring (where the ring surface density is $\sigma \simeq 50$ gm/cm^2), this phenomenon pumps up the satellite's inclination over a timescale $\tau_i = |i/(di/dt)| \sim 10^5$ years. Note that this i-pumping timescale is very short compared to the age of Saturn's ring system, so it remains a mystery as to why the orbits of the gap-embedded satellites Pan and Daphnis are in fact coplanar with the ring.

A satellite's eccentricity e will also evolve due to its many Lindblad and corotation resonances in the ring (Goldreich & Tremaine 1981, 1982). Summing contributions from all of a gap-embedded satellite's Lindblad resonances in the ring yields

$$\left.\frac{de}{dt}\right|_L \simeq f_L \left(\frac{\mu_s \mu_d}{\Delta^4}\right) en, \tag{2}$$

where the factor $f_L = 1.52$. However this e-excitation is in competition with the satellite's corotation resonances, whose e-damping rate is similar to Eqn. (2) but with a lead coefficient $f_C = -1.60$. This competition seemingly results in a net damping of the satellite's eccentricity at rate of $\dot{e} \propto f_{LC}$ where $f_{LC} = f_L + f_C = -0.08$, which means that e-damping due to corotation resonances wins, but only by a 5% margin. However, if the motions of the ring particles orbiting at these corotation resonances are *saturated*, then the torque on the satellite due to its corotation resonances is ineffective (Goldreich & Tremaine 1981), and the satellite's e grows due to its Lindblad resonances over a very short timescale, $\tau_e = |e/(de/dt)_L| \sim 10^3$ years.

Consequently, the orbit of a gap-embedded satellite is seemingly unstable, since the satellite's interaction with its vertical resonances in the ring tends to pump up the satellite's i. Also, the stability of the satellite's e is uncertain. For instance, if the satellite's corotation resonances in the ring are saturated, then Lindblad resonances will pump up the satellite's e until it crashes into the nearby gap edge.

Secular interactions with a planetary ring

Another type of gravitational ring-satellite interaction worth considering are secular perturbations, which is a slowly varying gravitational disturbance that can alter a body's e and i, as well as drive orbital precession. For instance, a perturber's secular perturbations can launch spiral waves at a secular resonance in a disk (Ward & Hahn 1998, 2003), as well as at a non-resonant site like a nearby gap edge (Goldreich & Sari 2003). This secular interaction might also provide some orbital stability to a gap-embedded satellite, since this phenomenon tends to damp a perturber's e and i.

simulating secular ring-satellite interactions

A body's secular perturbations are equivalent to that exerted by a gravitating ring (Murray & Dermott 1999). Consequently, it is convenient to simulate the secular evolution of a ring-satellite system by treating a broad planetary ring as if it where composed of numerous narrow rings whose shapes and orientations are described by their individual semimajor axes a_j, eccentricities e_j, inclinations i_j, longitude of periapse $\tilde{\omega}_j$, and longitude of ascending nodes Ω_j. The subsequent evolution of the rings' and satellite's orbital elements is then given by the secular solution to the Lagrange planetary equations (Murray & Dermott 1999), which is conveniently calculated using the so-called rings model of Hahn (2003). In these simulations, the rings are given masses m_j that correspond to a surface density of $\sigma = 50$ gm/cm^2. Each ring is also assumed to have a small thickness h due to the individual ring-particles' dispersion velocity, which has the effect of softening the gravity that is exerted by adjacent rings.

Figure 1 illustrates the evolution of a 10km satellite as it orbits in the center of a 100km-wide gap in Saturn's main A ring, which corresponds loosely to the gaps maintained by Pan or Daphnis. The satellite is given a small initial eccentricity e_s and inclination i_s, while all the other rings in this system have initial $e = 0 = i$. As that figure shows, the satellite's secular perturbations excites the eccentricities of the rings at the nearby gap edge, which in turn excites the e's of the more distant rings due to their self-gravity. The right-hand part Fig. 1 also plots the ring's longitudes $\tilde{\omega}(a)$, which steadily rotates with distance a in the ring, indicating that this disturbance is in fact a trailing $m = 1$-armed density wave. A similar disturbance is also seen in the ring's inclinations i and $\Omega(a)$, which reveals that the inclined satellite also launches a leading one-armed spiral bending wave at the gap's outer edge. As Fig. 1 shows, the wavelength of these waves is $\lambda \sim 0.002a_s \sim 300$km, which is much longer than the $\lambda \sim O(10)$ km waves routinely launched at mean-motion resonances throughout the ring (Tiscareno et al 2007). The simulated waves also propagate at the group speed $c_g = \mu_d an \sim 30$ km/year in Saturn's A ring.

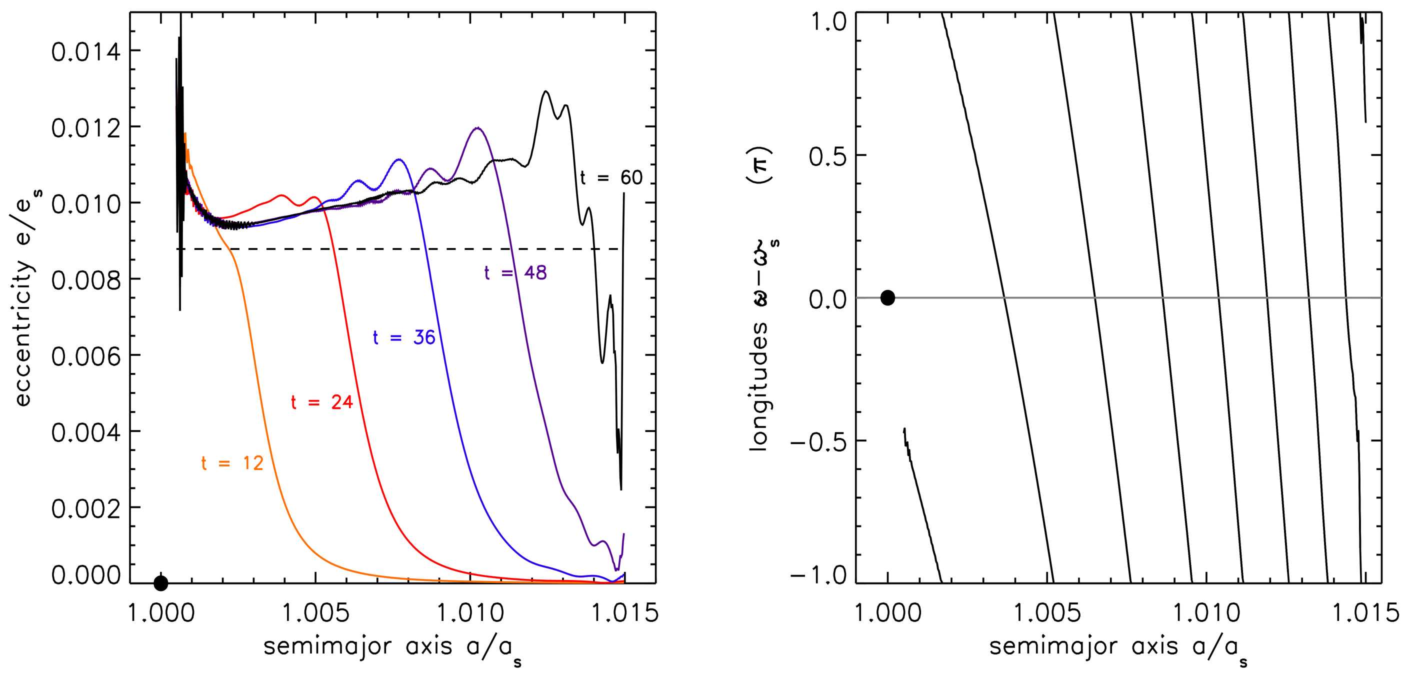

FIGURE 1: Left figure shows the time-evolution of a planetary ring that is disturbed by a small eccentric satellite orbiting in a narrow gap in the ring, indicated by the dot. Colored curves show the rings' eccentricities e plotted versus their semimajor axes a at selected times t in years. The dashed line is the expected wave amplitude, from Eqn. 3. Right figure plots the rings' longitudes of periapse $\tilde{\omega}(a)$ at time $t = 60$ years. Note that this curve wraps up with distance a, which indicates that this disturbance has the form of a trailing spiral density wave whose wavelength is $\lambda \sim 0.2\%$ of the satellite's orbital radius a_s.

An analytic description of these waves

The amplitude of these waves, as well as their dispersion relation, is derived from the Lagrange planetary equations. Those derivations are provided in Hahn (2007a,b), with the results summarized below.

wave amplitude and dispersion relation

The amplitude of these very long-wavelength spiral density and spiral bending waves is

$$e \simeq \frac{\mu_s e_s}{3\mu_d \Delta} \quad \text{and} \quad i \simeq \frac{\mu_s i_s}{3\mu_d \Delta}, \tag{3}$$

which compares well with the amplitudes seen in the simulations (dashed line in Fig. 1).

The waves' dispersion relation also yields their wavenumber $k = -\partial\tilde{\omega}/\partial a$, where

$$|k(x)| \simeq \frac{2 + (\mu_c/\mu_d)(1 + x/\Delta)}{3a\Delta} \quad \text{with} \quad \mu_c = \frac{21\pi}{4}\left(\frac{R_p \Delta}{a}\right)^2 J_2, \tag{4}$$

where R_p is the central planet's radius and J_2 its second zonal harmonic. Note that the planet's oblateness (described by J_2) causes the wavenumber $|k|$ to increase with the distance x from the ring edge, which causes the wavelength to shorten with distance, as is evident in the simulation (right Fig. 1).

surface density variations

The surface density variations due to this density wave is $|\Delta\sigma/\sigma| \simeq |eak|$ (Borderies et al 1985, Hahn 2003) where the wavenumber $|k| \simeq 2\pi/\lambda \sim 0.02$ km^{-1}. The rings model is used to determine the amplitude of the waves launched by Pan, which yields $e \sim 3\times10^{-7}$. Inserting these values into the above then yields $|\Delta\sigma/\sigma| \simeq 0.001$ due to the density waves that Pan can launch at the outer edge of the Encke gap. This of course is likely far to small to be observed by the Cassini spacecraft. Similarly, the height of the bending waves that an inclined Pan might launch are also likely too small for direct detection.

Instead, the main import of this ring-satellite interaction will be in the rates at which this secular phenomenon damps the satellite's e and i, provided below.

e and i damping due to the secular ring-satellite interaction

The Lagrange planetary equations also provide the rates at which this secular phenomena alters the satellite's e and i. Since the excitation of these waves transports angular momentum from the satellite to the ring, this interaction damps the satellite's e and i at the rates (from Hahn 2007a,b)

$$\left.\frac{de}{dt}\right|_S \simeq -\left(\frac{\mu_s}{15\Delta^2}\right) en \quad \text{and} \quad \left.\frac{di}{dt}\right|_S \simeq -\left(\frac{\mu_s}{15\Delta^2}\right) in. \tag{5}$$

A comparison of a satellite's i-damping rate due to its secular interaction with the ring (Eqn. 5) to its i-excitation rate due to its vertical resonances in the ring (Eqn. 1) shows that the satellite's inclination is stable ($di/dt < 0$) when the satellite's gap is sufficiently wide,

$$\Delta \gtrsim 0.5\sqrt{\mu_d}. \tag{6}$$

Both Pan and Daphnis satisfy this requirement, so their inclinations are stable.

When the satellite's e-damping rates (due to this secular interaction plus the corotation resonances) are compared to e-excitation due to the Lindblad resonances, we find that the satellites' $de/dt < 0$, so Pan and Daphnis' eccentricities are seemingly stable. But this of course is due to the near cancellation of the Lindblad and corotation torques.

If, however, the corotation resonances are saturated, then their e-damping effects are inoperative. But in this case, the secular e-damping overcomes the Lindblad e-excitation when

$$\Delta \gtrsim 3.4\sqrt{\mu_d}. \tag{7}$$

However, neither Pan's Encke gap nor Daphnis' Keeler gap is wide enough to satisfy this requirement. But since both satellites do in fact reside in nearly circular orbits, this then suggests that $de/dt < 0$, which implies that these satellites' corotation resonances are not saturated.

Main findings

- The secular perturbations exerted by a gap-embedded satellite results in the excitation of spiral density and bending waves at the gap's outer edge. In Saturn's A ring, waves launched by Pan and Daphnis will have very long wavelengths, with $\lambda \sim$ hundreds of km, and very low amplitude, $\Delta\sigma/\sigma \sim 0.001$, which is likely too low for direct detection.
- This secular ring-satellite interaction also damps the satellite's e and i on a short timescale that is or order $\tau \sim 10^3$ years. The pace of this e and i-damping dwarfs the excitation that is due to the satellite's Lindblad, corotation, and vertical resonances in the ring, thus stabilizing these satellite's e and i and confining them to the center of their gaps.
- However, e-damping due to this wave-action does not exceed e-excitation due to Lindblad resonances; corotation resonances are also required to assist in the e-damping. This implies that Pan and Daphnis' corotation resonances are not saturated.

Acknowledgements

This research was supported by NASA's Outer Planets Research Program.

References

Borderies, Goldreich, & Tremaine, 1984, ApJ, 284, 429.
Borderies, Goldreich, & Tremaine, 1985, Icarus, 63, 406.
Goldreich & Tremaine, 1981, ApJ, 243, 1062.
Goldreich & Tremaine, 1982, ARA&A, 20, 249.
Goldreich & Sari, 2003, ApJ, 585, 1024.
Hahn, 2003, ApJ, 595, 531.
Hahn, 2007a, ApJ, 665, 856.
Hahn, 2007b, in preparation.

Murray & Dermott, 1999, Solar System Dynamics, Cambridge U. Press.

Tiscareno, Burns, Nicholson, Hedman & Porco, 2007, Icarus, 189, 14.

Ward & Hahn, 1998, AJ, 116, 489.
Ward & Hahn, 2003, AJ, 125, 3389.

Preprint requests

If you wish to receive a reprint of the bending wave paper (Hahn 2007a) or a preprint on the density wave paper (Hahn 2007b), please write your email address here in the margin.