Spiral Bending Waves Launched at a Vertical Secular Resonance in a Planet–Forming Disk

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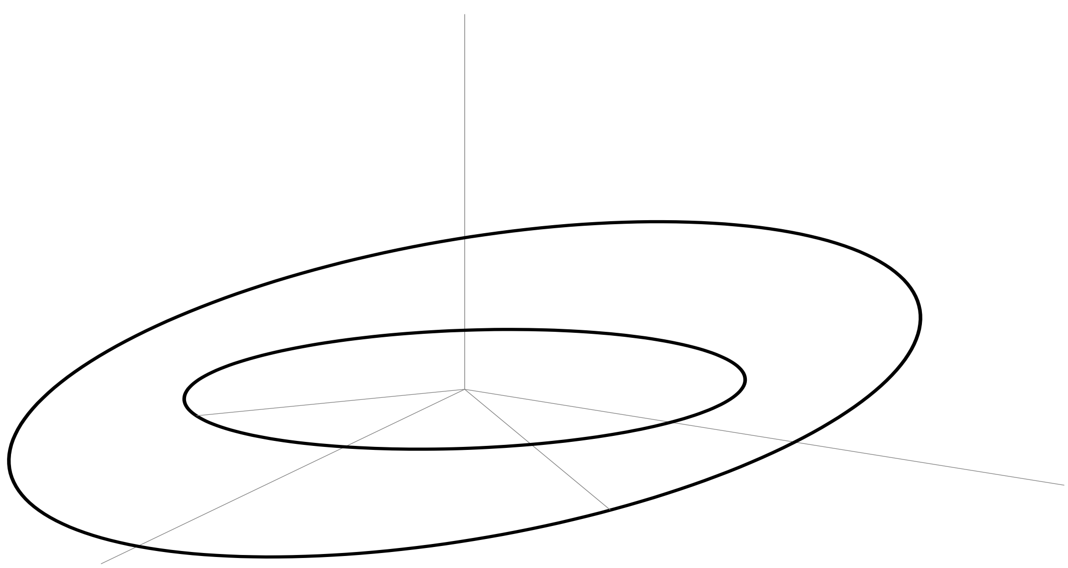
1. Consider a disk (planetesimal and/or nebula gas) perturbed by a protoplanet on an inclined orbit. The vertical secular resonance is the site where the disk's longitude of ascending node precesses at a rate

$$s = \frac{d\Omega}{dt}$$

that matches the protoplanet's

$$s_p = \frac{d\Omega_p}{dt}.$$



Large i's can get excited wherever $s(r) \simeq s_p$.

2. The disk's precession rate is determined by the disk's self–gravity + the protoplanet's gravity:

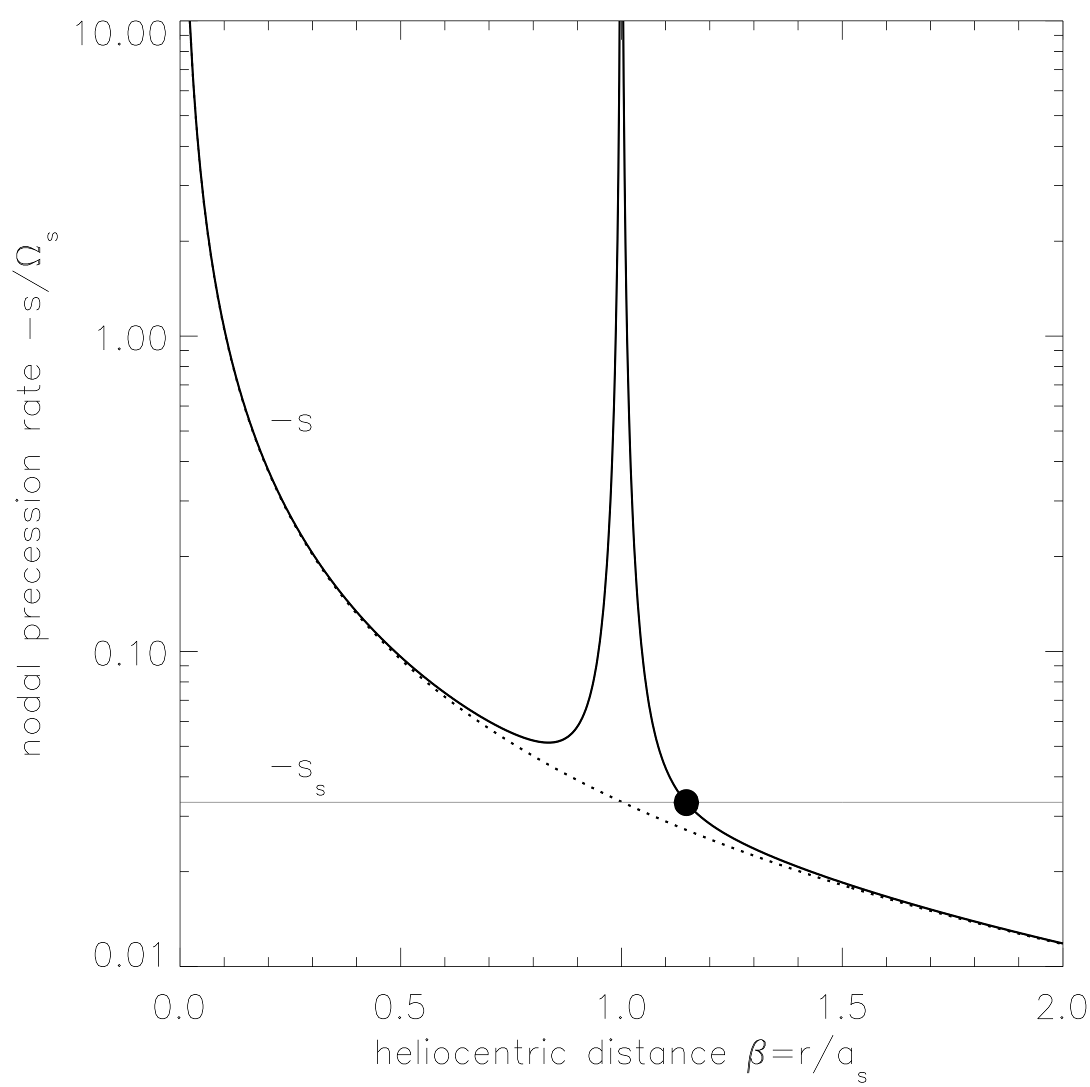
$$s(r) = \frac{d\Omega}{dt} = -\underbrace{\frac{2\pi G\rho_{\text{disk}}}{\Omega}}_{\Uparrow \; -\Omega/Q} - \frac{1}{4}\mu_p\beta^2 b_{3/2}^{(1)}(\beta)\Omega$$

The protoplanet's node precesses due to disk gravity:

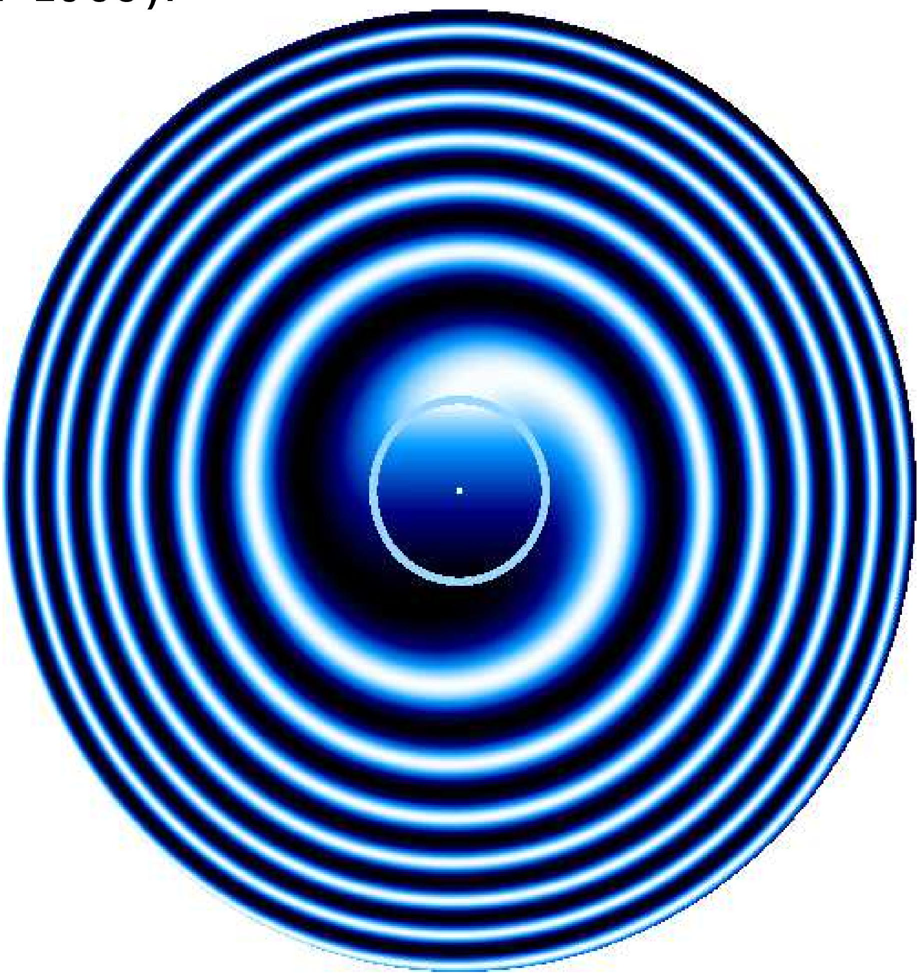
$$s_p = -\Omega(a_p)/Q$$

The secular resonance lies where $s(r) = s_s$, or

$$\Delta r \simeq \left(\frac{Q\mu_p}{3\pi}\right)^{1/3} a_p$$



- protoplanet excites i in the disk at resonance.
- the disk's self–gravity causes this vertical disturbance to propagate away as a spiral bending wave (e.g., Shu, Cuzzi, Lissauer 1983).



$$\lambda_0 \simeq 1.7\sqrt{h a_p}$$
$$\sim \begin{cases} 0.01 Q^{1/2} a_s & \text{in planetesimal disk} \\ 0.5 a_s & \text{in solar nebula} \end{cases}$$

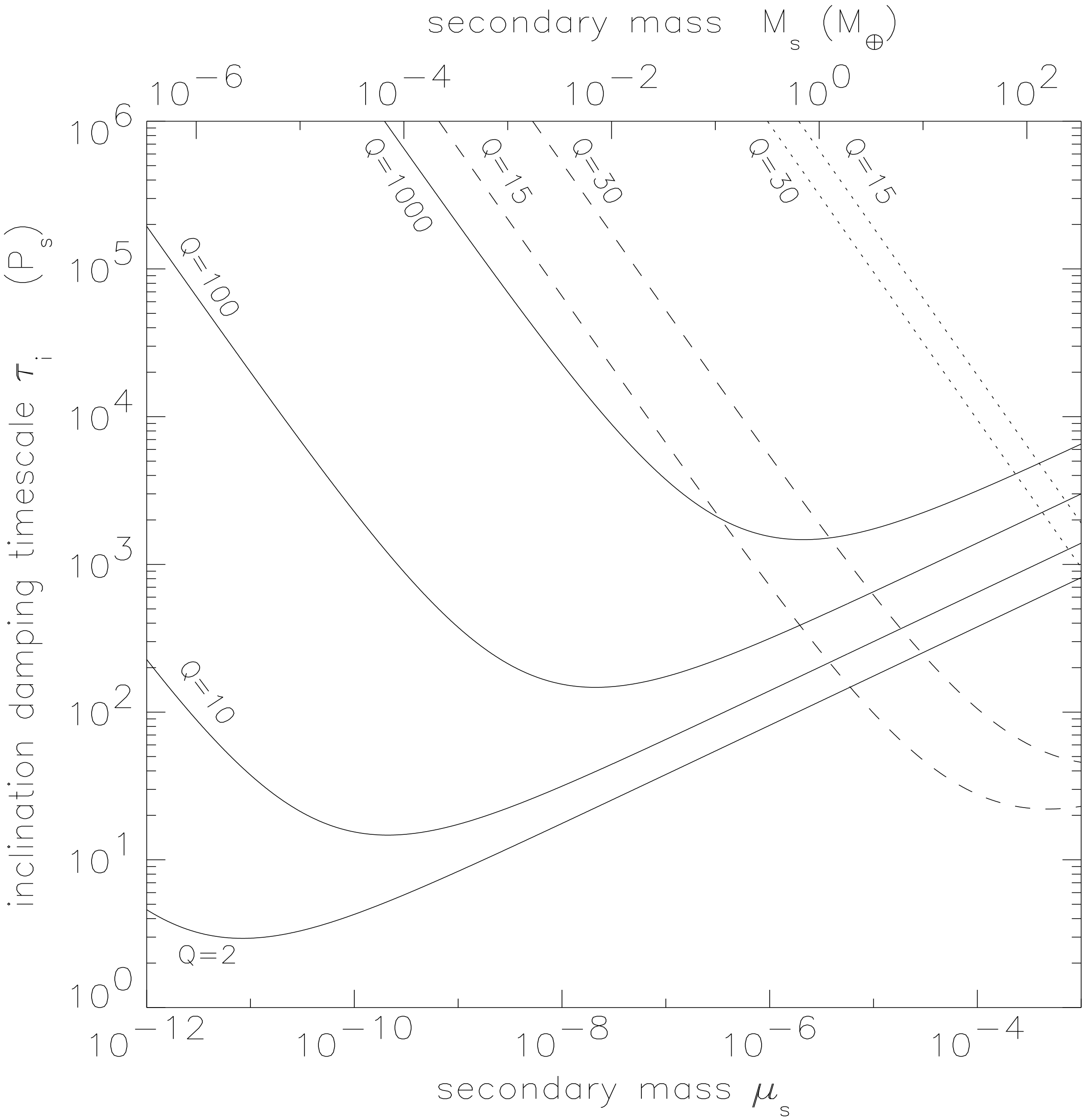
$$\sin i_{\text{disk}} \simeq \frac{0.2}{(h/r)^{3/2}} \frac{\mu_p}{\mu_d} \sin i_p$$

Damping Protoplanet's Inclinations

The protoplanet loses in–plane angular momentum as it excites disk inclinations. This damps protoplanet's i_p at the rate

$$\frac{1}{i_p}\frac{di_p}{dt} \simeq -\frac{\pi}{8}\mu_p\mu_d\beta^2[B_{3/2}^{(1)}(\beta,h')]^2\left|\frac{\Omega}{rds/dr}\right|\Omega_p \equiv \tau_i^{-1}$$

- recall that $\Delta r \propto (Q\mu_p)^{1/3}$
- τ_i is sensitive to disk–stirring & gap formation.



Implications for Planet–Formation

- the excitation of spiral bending waves at the vertical secular resonance damps i_p on timescales
  - $\tau_i(\mu_p, Q) \sim 10$ to 10^3 orbits in the planetesimal disk, τ_i grows with μ_p and Q.
  - i_p damping stalls when the protoplanet stirs–up/opens a gap in the disk.

- larger $M_p \gtrsim 1$ M$_\oplus$ bodies suffer i_p damping due to bending–wave excitation in the gas disk:
  - initially $\tau_i \sim 200 \left(\frac{M_p}{M_\oplus}\right)^{-1}$ orbits
  - $\tau_i \sim 10^4 \left(\frac{M_p}{20 M_\oplus}\right)^{-1}$ orbits
    when protoplanet opens a narrow gap of width $w \sim h$.

- spiral waves will keep the planetesimal disk cool since wave–propagation distributes the protoplanet's disturbance across the disk—there is no localized disk heating at resonances when waves propagate.

- inclination–damping might confine protoplanets to a narrow plane, allowing their mutual accretion to proceed faster.

- this might be especially important in the Uranus–Neptune zone, where many accretion models tend to produce a scattered mess of debris rather than core–sized protoplanets.