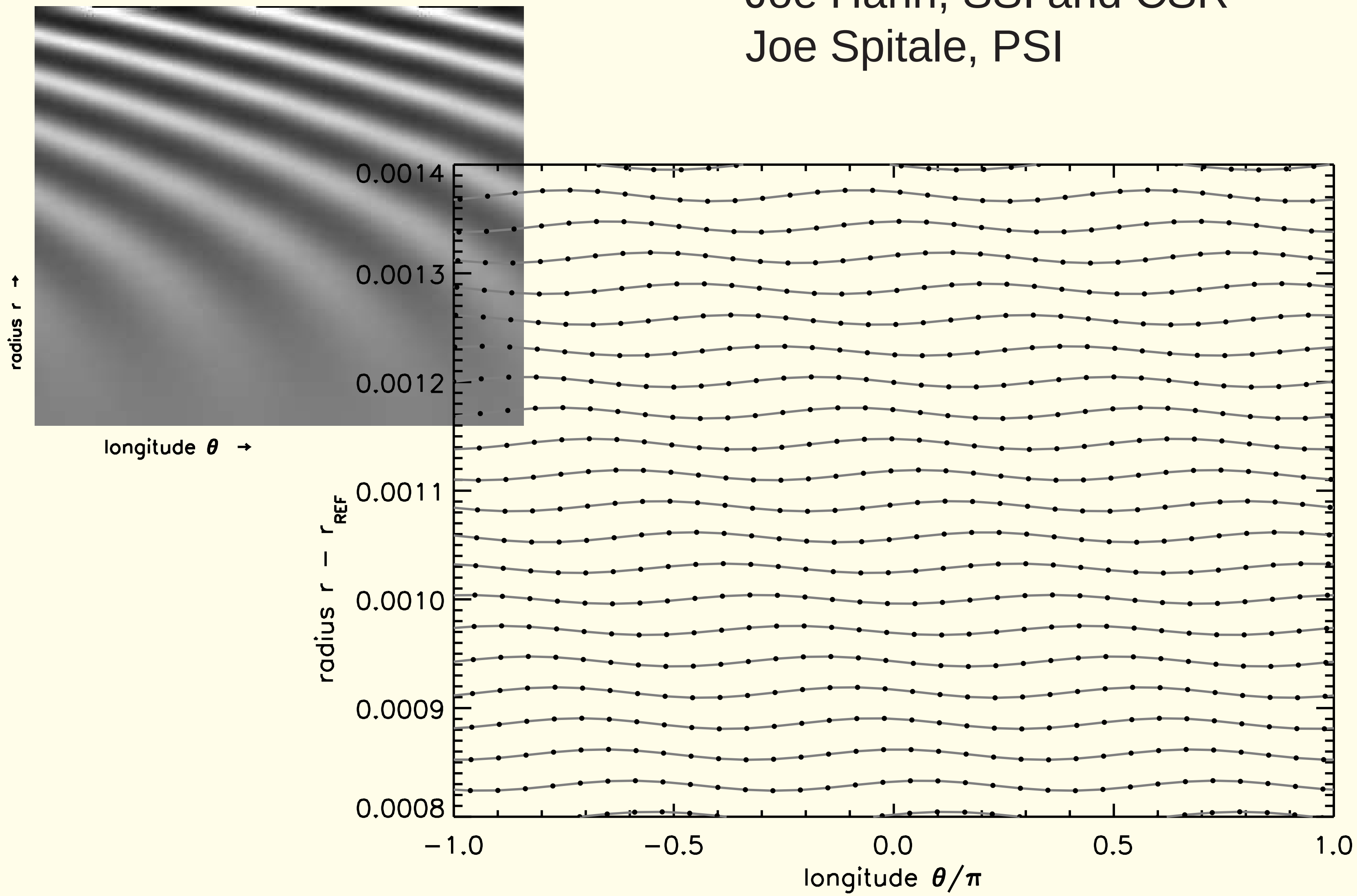
N–body simulations of the outer edge of Saturn’s B ring

Joe Hahn, SSI and CSR
Joe Spitale, PSI



N–body simulation of $m = 3$ spiral density wave

The N-body code `epi_int` *(epicyclic integrator)*

Designed to simulate the collective phenomena seen in rings:
spiral waves scalloped ring-edges

Code uses same step-kick-drift algorithm as SYMBA or MERCURY,
except for two differences:

1. *how forces on ring particles are computed*

- particles are assigned to streamlines having linear density $\lambda = \sum m_i / 2\pi r$

- particles only interact with other streamlines, not with other particles

- all streamlines are close in the radial sense, $\Delta r < 10^{-3} r$,
  so each particle perceives each nearby streamline as a straight line of matter

- gravity is simply $g = 2G\lambda/\Delta$, a smooth function of distance,
  almost no scattering, few particles, ~ 1K, needed to simulate a gravitating ring

- surface density $\sigma = \lambda/\Delta$
  gradients in $\sigma \Rightarrow$ pressure forces
  gradients in $v \Rightarrow$ viscous forces

$\mathbf{2}$. *the symplectic integrator’s ‘drift’ step occurs along an unperturbed orbit*

- the planet is oblate, so the unperturbed orbit is *epicyclic* rather than keplerian (B-RL 1994):

$$r = a(1 - e\cos M) \quad \theta = \tilde{\omega} + M + 2e(\Omega/\kappa)\sin M \quad v_r = a\kappa e \sin M \quad v_\theta = \ldots$$

where Ω, κ are the angular and epicyclic frequencies

So an orbital drift over time Δt advances the orbit elements

$$\Delta M = \kappa \Delta t \qquad \Delta\tilde{\omega} = (\Omega - \kappa)\Delta t$$

the effects of oblateness J_2 are accounted for in the integrator’s drift step

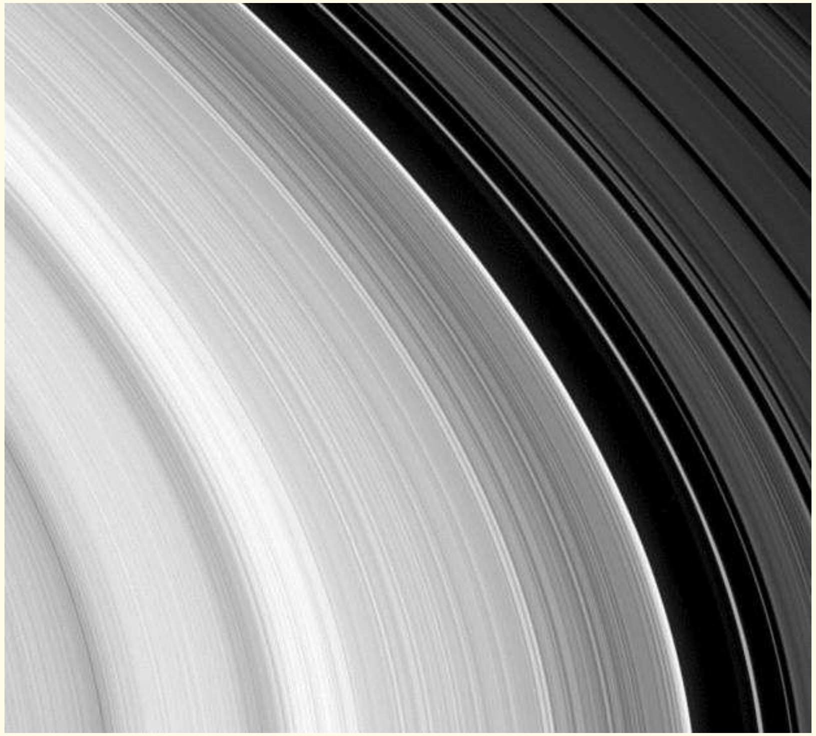
The outer edge of Saturn's B ring

- ring is 'confined' by Mimas' $m = 2$ ILR
  $a_{\text{edge}} = a_{\text{res}} + 12$km

- edge has expected $m = 2$ shape where

$$r(\theta) = a - R_2 \cos m(\theta - \tilde{\omega}_2)$$

SP 2010 measure forced pattern
with epicyclic amplitude $R_2 = 35$ km,
corotates with Mimas,
$\dot{\tilde{\omega}}_2 = \Omega_{\text{Mimas}} = 382.0$ deg/day



- SP 2010 also see other unforced modes at B ring edge, so

$$r(\theta) = a - \sum_m R_m \cos m(\theta - \tilde{\omega}_m)$$

- free $m = 1$ pattern with $R_1 = 21$ km
  that precessed due to oblateness,
  $\dot{\tilde{\omega}}_1 = 5$ deg/day

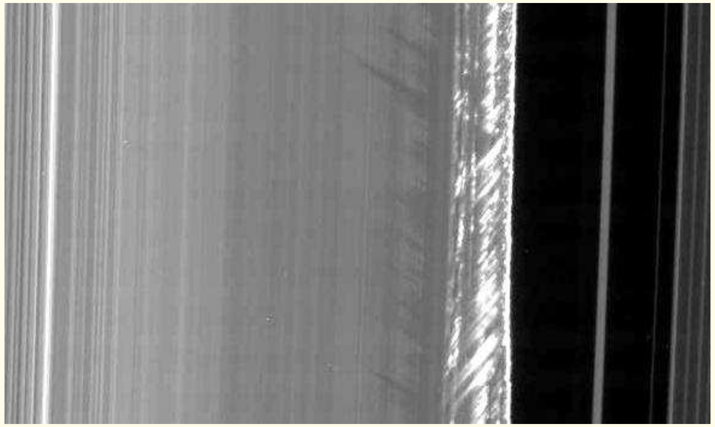
- free $m = 2$ pattern with $R_2 = 37$ km,
  rotates slightly faster than Mimas,
  $\dot{\tilde{\omega}}_2 = \Omega_{\text{Mimas}} + 0.1$ deg/day

- free $m = 3$ pattern with $R_3 = 12$ km
  whose speed $\dot{\tilde{\omega}}_3 = 1.33\Omega_{\text{Mimas}}$
  corresponds to ILR at $\Delta a = 24$ km interior to ring's edge

- NFH 2012 also see $m = 4, 5$ patterns with $R_m \sim 5$ km

Are unforced modes due to the instability of BGT 1985?

- BGT 1985 show that if ring particles are a close-packed incompressible particle fluid, then density waves are unstable. BGT argue that these waves can amplify when trapped between a ring edge and a nearby LR, which could account for the free modes



- if so, then the edge must be very thin, thickness $h = \sigma/2\rho \sim 1$ m assuming $\sigma \sim 100$ gm/cm^2 and $\rho \sim 0.5$ gm/cm^3

- and dynamically very cold, with $c \ll (h\Omega \sim 0.1)$ mm/sec

- but shadows cast by km-high structure suggests B ring edge is not thin or cold

- so this model employs a compressible EOS for a dilute particle gas, $p = c^2\sigma$

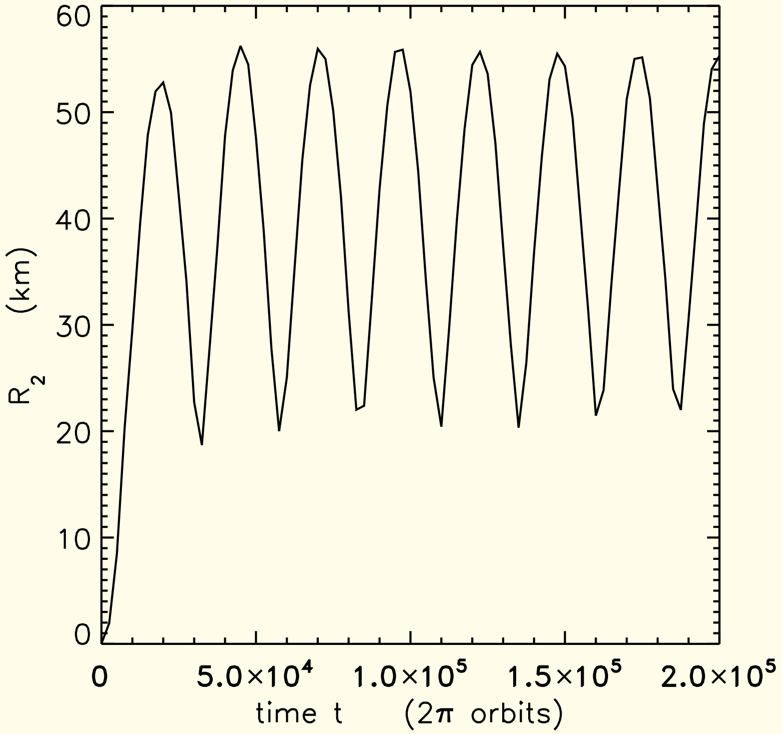
How to create the B ring's forced & free $m = 2$ patterns

- Start with circular ring + Mimas
  at time $t = 0$ and evolve N-body model
  coldstart initial condition, not physical

- this creates a free $m = 2$ pattern
  that nulls Mimas' forced pattern
  at time $t = 0$

- ring gravity causes the free pattern to rotates slightly faster than Mimas' motion

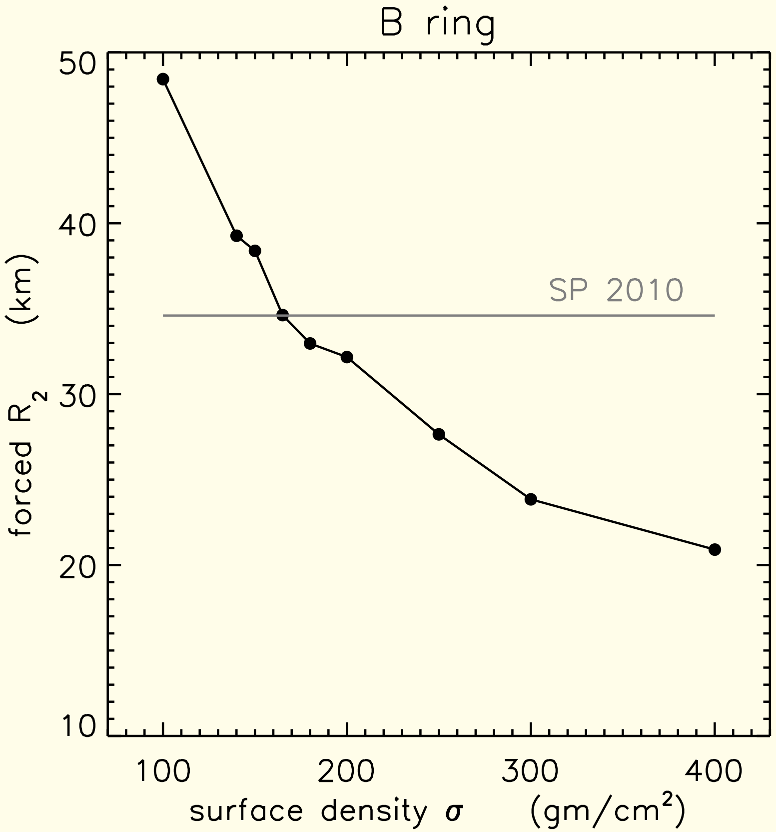
- superposition causes R_2 to oscillate, which is what Cassini sees

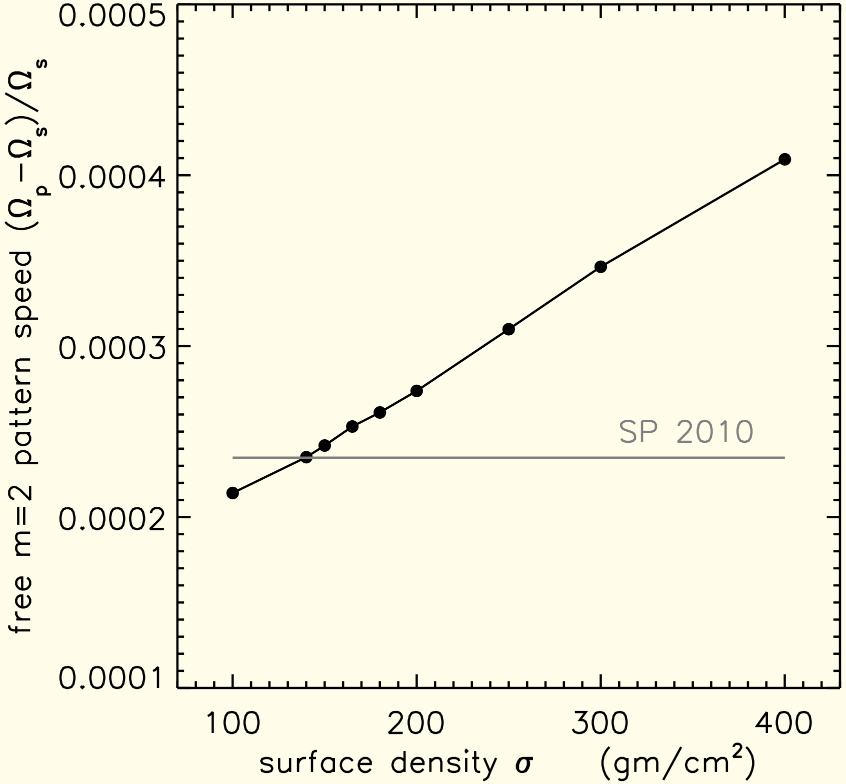


- this simulation has viscosity $\nu = 200$ cm^2/sec and evolved for $t = 300$ yrs

- simulation shows no sign of viscous damping of free mode during run
  ⇒free mode could have been created hundreds (thousands?) of years ago,
  and it would still persist despite the ring's viscosity

$m = 2$ patterns are sensitive to B ring σ



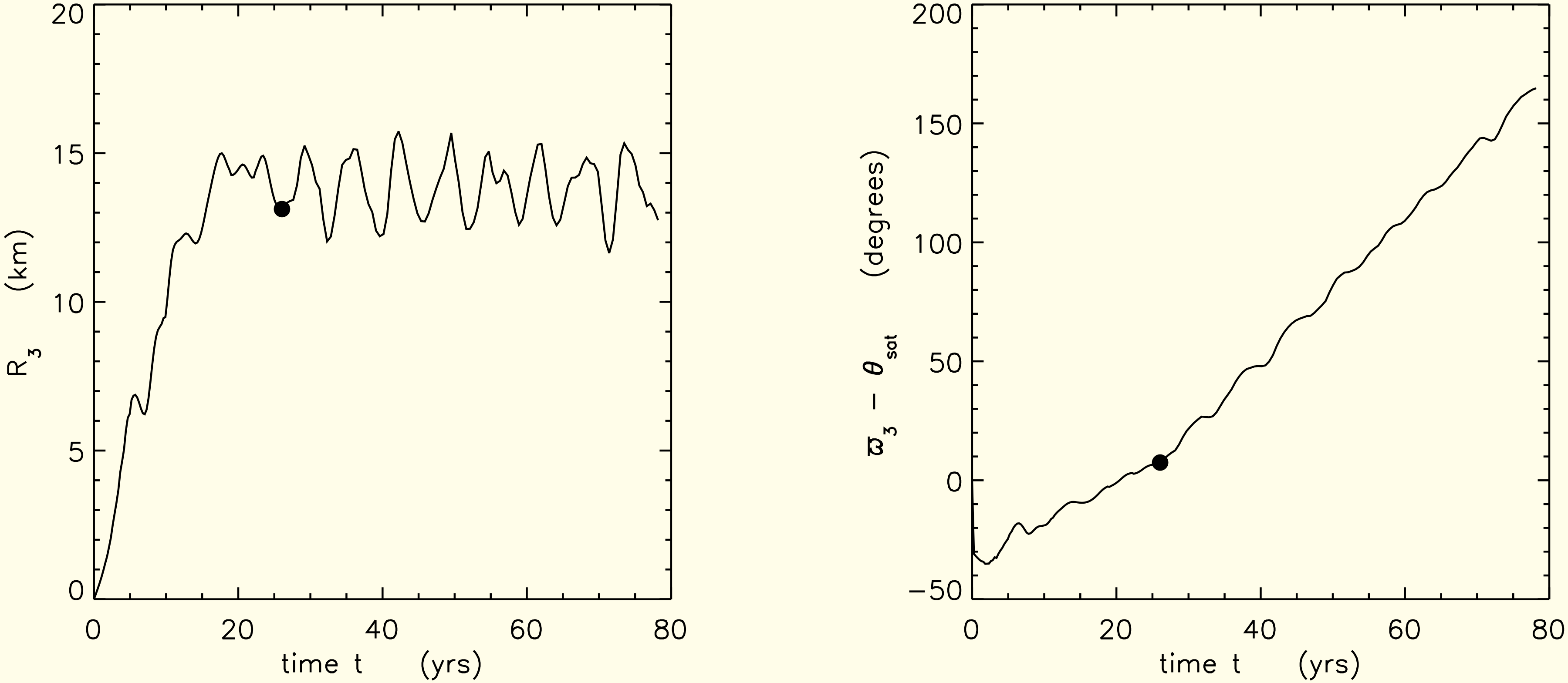


- comparison of B ring simulations to SP 2010 measurements of $m = 2$ modes indicates ring $\sigma \simeq 150$ gm/cm^2

- origin of free $m = 2$ is unknown, but might be due to recent impulsive event:
  impact by cometary debris cloud? (eg. HBETO 2011, SHB 2011)
  close flyby by large cometary Centaur?

the B ring’s free $m = 3$ pattern

To make a free $m = 3$ pattern at the B ring’s edge, slowly grow a fictitious Janus-sized satellite at an orbit 1.4% beyond its current, which puts an $m = 3$ ILR at the ring’s edge that excites a forced pattern that corotates with the satellite



Then turn off the satellite’s mass, converting the forced $m = 3$ pattern into a free pattern whose precession rate is sensitive to ring self gravity and σ.

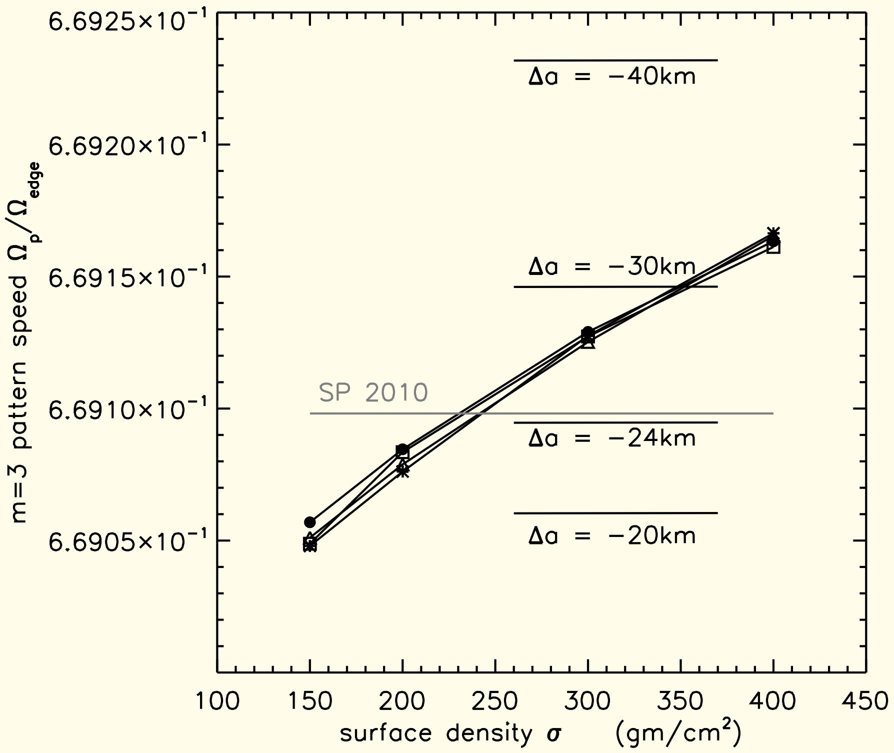
Again, no damping of ring’s free $m = 3$ pattern in $\Delta t = 55$ yrs despite $\nu = 100$ cm^2/sec.

free $m = 3$ pattern speed again depends on ring σ

- results are insensitive to where $m = 3$ ILR lies in the B ring, these simulations use $-40 < a_{\text{res}} - a_{\text{edge}} < -20$ km

- nonetheless, free pattern speed Ω_p depends only on ring σ

- not sure what this means...

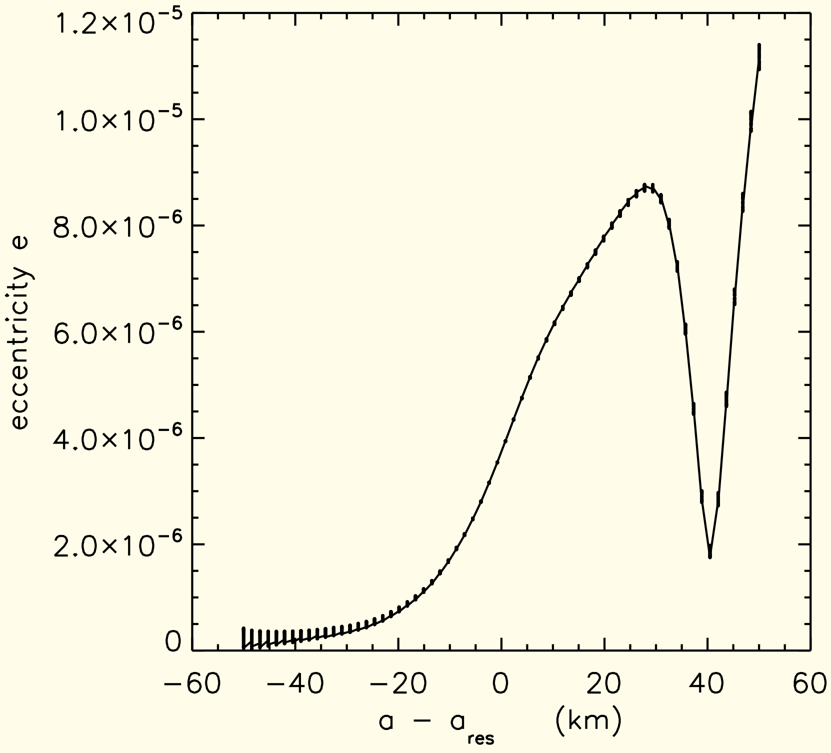


- simulations of σ pattern suggest B ring-edge surface density $\sigma \sim 230$ gm/cm^2 discrepancy with $m = 2$ result, $\sigma \sim 150$ gm/cm^2? need to do higher-res simulations...results might not have converged

free patterns at ring’s *outer* edge are due to *inner* LR

To make a free pattern’s at ring (or ringlet’s outer edge),
I place a fictitious satellite’s ILR inside ring’s outer edge

- ILR launches a spiral density wave, trapped between LR and edge
- wave amplitude grows until satellite is turned off
- disturbance stays at outer edge
- to make the ring’s inner edge eccentric, use an OLR instead

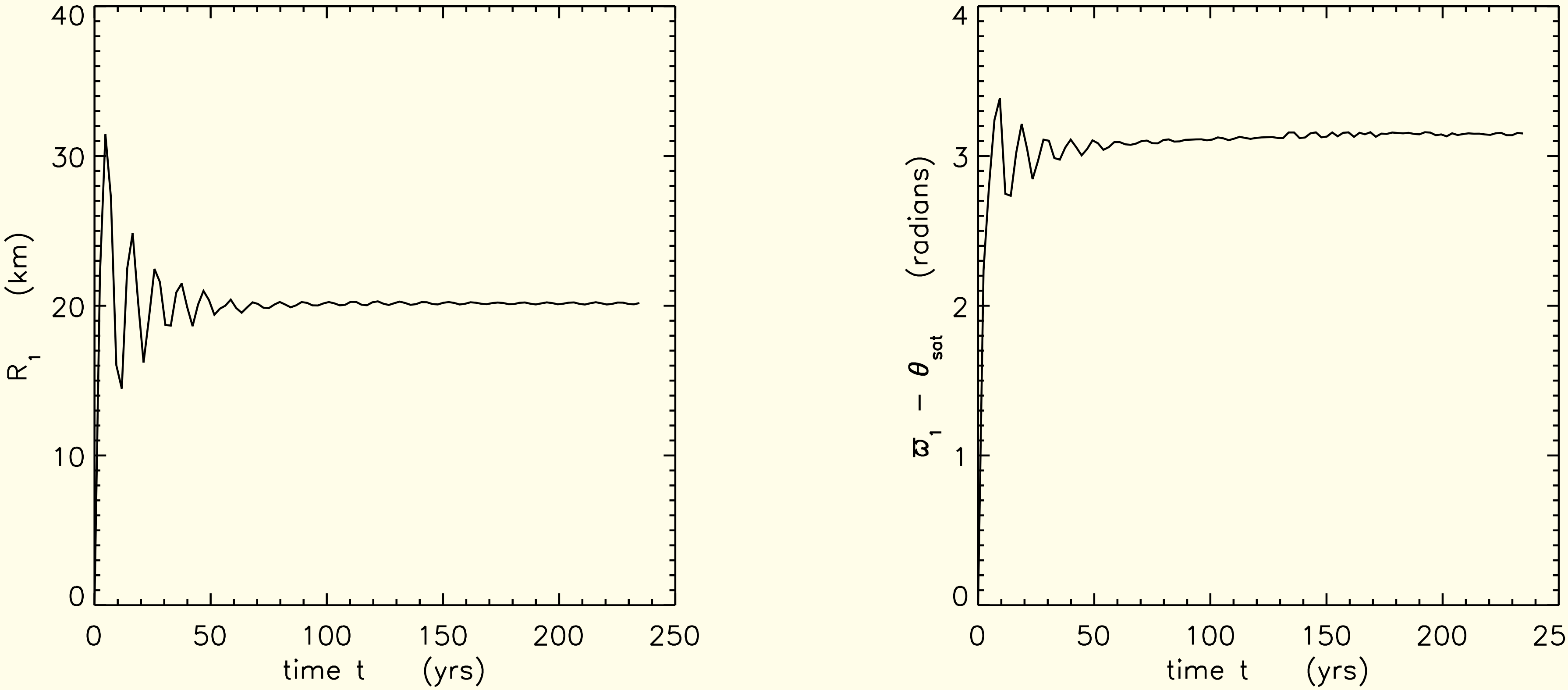


- Note: most ringlets in Cassini Division have circular inner edges and eccentric outer edges
  - could be due to transient ILRs in the past
  - due to impulsive event? impact by comet dust trail? Centaur flyby?

the B ring’s puzzling free $m = 1$ pattern

- the $m = 1$ ILR is special—is a *secular* resonance where (for ex.) $\dot{\varpi}_{\text{ring}} = \Omega_{\text{sat}}$

- Iapetus’ $m = 1$ ILR is in vicinity, in outer Cassini Division, is *powerful*
(this sim uses *half* Iapetus’ mass + temporary drag force to damp free $m = 1$)



- problem: B ring *and* Cassini Division’s $m = 1$ patterns should corotate with Iapetus’ longitude, but SP2010 find pattern speed $\dot{\varpi}_2$ due to J_2

Results thus far

- B ring's forced and free $m = 2$ patterns indicates $\sigma \simeq 150$ gm/cm^2
  - but free $m = 3$ pattern suggests $\sigma \simeq 230$ gm/cm^2
  - hopefully higher-resolutions simulations
    will cause σ to converge on a single value

- B ring's free patterns can persist for hundreds (thousands?) of years, despite viscosity $\nu = 100$ cm^2/sec
  - free patterns could be due to transient LRs that existed in recent past
  - due to impulsive event? impact by cometary dust trail? Centaur flyby?
  - density waves launched at *inner* LR
    can account for ringlets' noncircular outer edges,
    even after the LR has since 'turned off'

- N-body model says that Iapetus' $m = 1$ ILR at $\dot{\varpi}_{\text{ring}} = \Omega_{\text{sat}}$ should create a large forced $m = 1$ pattern at B ring edge *and* all across the Cassini Division. That pattern should *corotate* with Iapetus...not observed...puzzling