Nbody Simulations of the Huygens Ringlet

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introduction

Huygens is a prominent ringlet in the inner part of the Cassini Division. The ringlet's radial with is $w = 21$ km, and it orbits $\Delta = 260$ km beyond the outer edge of Saturn's B ring, which is confined by an $m = 2$ inner Lindblad resonance with the satellite Mimas. Cassini has revealed that this ringlet's dynamical behavior is reminiscent of the B ring-edge's behavior. So by studying this ringlet in greater detail, we hope to gain insight into the dynamics that also affects the massive and mysterious B ring.

the Huygens ringlet might be young

This ringlet's narrowness may be an indicator of its youth, since it is presumably viscous and is apt to spread radially. The timescale for a narrow ringlet to spread a radial width w is $t_\nu \simeq w^2/50\nu$ (Pringle 1981). If $\nu \sim 50$ cm^2/sec, then its viscous age is only $t_\nu \sim 60$ years, or 4×10^4 orbits about Saturn.

Of course the ringlet could instead be much older, but that would require confining the ringlet via unseen shepherd satellites. If so, then the mass of those shepherds must be well in excess of the ringlet's mass. Later it is shown that the ringlet's mass is at least $m_r \sim 10^{18}$gm, which corresponds to compact mass of ice whose diameter would be at least $d \sim 10$ km. Cassini is effective at detecting 10km satellites. Since none have been detected in the vicinity of the Huygens ringlet, we conclude that the ringlet is not confined by shepherd satellites, and thus is very young.

ringlet properties

Although Huygens orbits just exterior to Mimas' $m = 2$ inner Lindblad resonance, its shape in not just a simple $m = 2$ shape; rather, Cassini observations shows that the ringlet's shape is a more complicated superposition of modes

$$r(\theta,t) = a\left\{1 - \sum_j e_j \cos\left[m_j(\theta - \tilde{\omega}_j - \Omega_j t)\right]\right\}, \qquad (1)$$

where eccentricity e_j is the amplitude of mode j that rotates at the angular rate Ω_j (Spitale et all 2009).

Fitting equation (1) to five years of monitoring by Cassini reveals the following (Spitale & Porco 2011). (*i.*) Huygens has a strong $m = 1$ shape whose epicyclic amplitude is $R_1 = 28$ km. (*ii.*) The ringlet's inner edge has a forced $m = 2$ pattern that corotates with Mimas, as expected, with amplitude is $R_2 = 1.9$ km. But there is also some free motion there, such that the ratio of free/forced motion is 0.2. (*iii.*) This is reversed at the ringlet's outer edge where the free/forced amplitude ratio is 2.0. (*iv.*) At the ringlet's edges, the free $m = 2$ patterns rotate over time, but at rates slightly slower than Mimas' motion such that $\Delta\Omega_j = -1.3$ degrees/day, which is 0.3% of Mimas' angular rate.

a scenario for the Huygens ringlet

This ringlet's apparent youth, plus its strong $m = 1$ component, suggests that this ringlet is debris from a parent moonlet that was shattered by an impacting comet some $t_\nu \sim 60$ years ago. That debris would have quickly sheared out into an annulus whose eccentric $m = 1$ shape is a relic of that impact event. And because this hypothetical impact event occurred in the vicinity of an $m = 2$ Lindblad resonance, that ringlet should also have free and forced $m = 2$ components as well.

To test our thinking further, we use Nbody simulations to determine whether a viscous and self-gravitating annulus of debris can in fact settle into a ringlet whose dynamical state resembles the Cassini observations of the Huygens ringlet.

the model

The Nbody integrator uses the same kick-kick-drift scheme used in Symba and Mercury (Duncan et al 1998, Chambers 1999). However our code does not calculate ring self-gravity by direct summation of particle attractions. Instead, all ring objects are regarded as tracer particles whose motions trace streamlines within the ring. The ringlet is narrow, so all particles are close in the radial sense, and each streamline is perceived as a straight line of matter of linear density λ_j whose gravitational pull is $2G\lambda_j/(r_j - r)$. Summing over all streamlines j then provides the total gravity that the ringlet exerts on a tracer particle at r. This model also accounts for the central planet's oblateness, and satellite Mimas' gravity.

To account for viscous effects, the code calculates the viscous angular momentum flux $F_\nu = -\nu_s\sigma r^2(\partial\dot{\theta}/\partial r)$ (Lynden Bell & Pringle 1974). This is the torque-per-unit length that one streamline exerts on its neighbor. In the ringlet's interior, the tangential acceleration that is due to the ringlet's viscous friction is $A^\nu_\theta = -(\partial F_\nu/\partial r)/r\sigma$, while at the ringlet edges $A^\nu_\theta = \pm F_\nu/\lambda r$, with a minus sign at the ringlet's inner edge. A comparable scheme is also used to calculate the radial acceleration due to viscosity and pressure $p = c^2\sigma$, where c is the particles' dispersion velocity c. Toomre's stability parameter is set so that $Q = c\Omega/\pi G\sigma = 2$, which makes pressure weak in comparison to ring gravity. Additional details are given in Hahn et al (2009).

Nbody simulations

All simulations start with the Huygens ringlet in a circular orbit, so these models do not attempt to account for the ringlet's strong $m = 1$ shape whose amplitude is $R_1 = 28$ km. But this approach does simplify the analysis by allowing us to quickly assess the much smaller $m = 2$ disturbances. Most simulations use 8 streamlines, with each containing 16 tracer particles, or 128 particles total. The ringlet's initial width was two-thirds its current width, and they were evolved for 25 years (2×10^4 orbits), when they arrive at their current width due to viscous spreading with $\nu = 50$ cm^2/sec. Execution times on a desktop PC are one hour. Five simulations are performed for ringlets having a variety of surface densities. Outcomes separate into three types of evolution that are distinguished here as low-mass, intermediate-mass, and high-mass ringlet evolution.

evolution of a high-mass ringlet

High-mass ringlets have surface density $\sigma = 130$ and 400 gm/cm^2. The ringlet's self-gravity enforces similar behavior among all streamlines. There is lots of free epicyclic energy at the ringlet's inner and outer edges, with free/forced ~ 1.3. Figure 1 shows the ringlet endstate. These models are not not good candidates for the Huygens ringlet, due to the free epicyclic energy at the ringlet's inner edge.

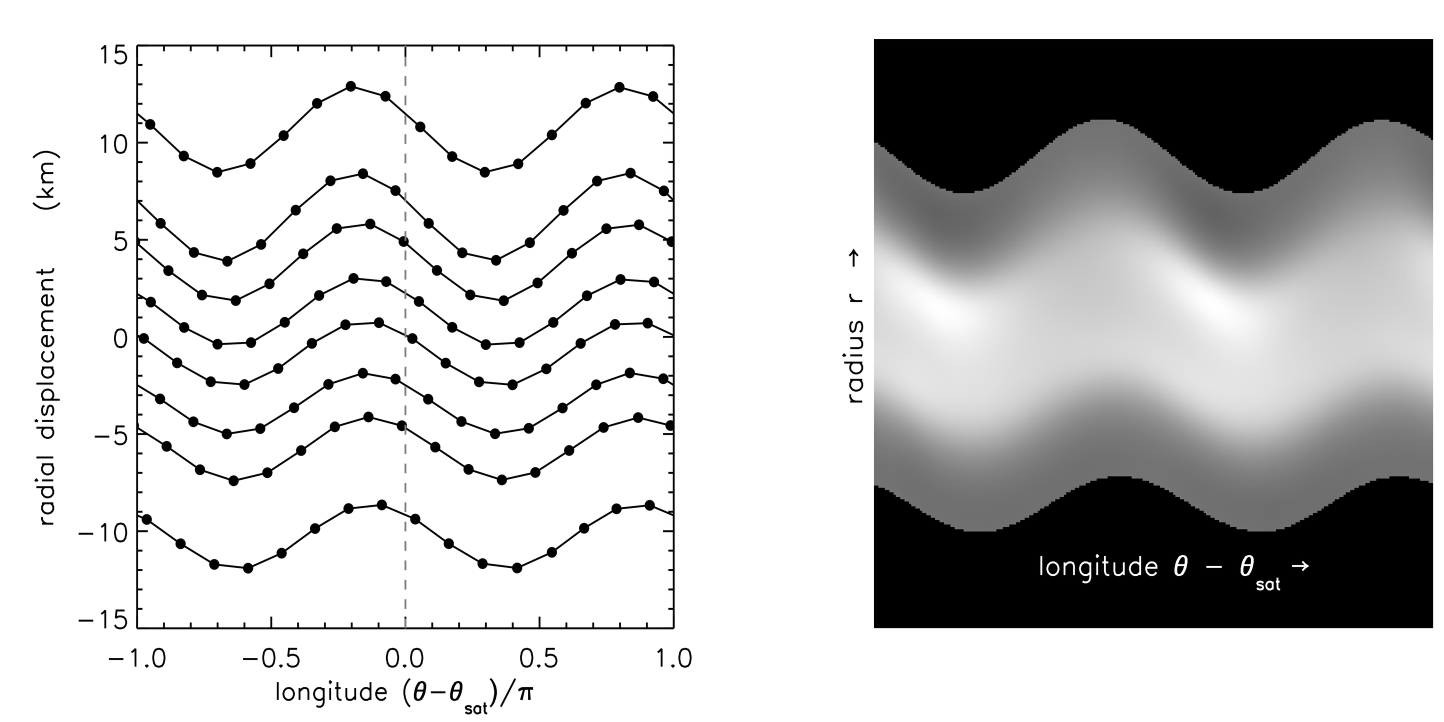

FIGURE 1: Nbody simulation of a high mass ringlet having surface density $\sigma = 130$ gm/cm^2. Left figure shows the locations of the 128 tracer particles, with curves connecting members of each of the 8 streamlines. Right figure shows the ringlet surface density, which varies by $\pm 20\%$.

evolution of a low-mass ringlet

A low-mass ringlet has a surface density $\sigma \lesssim 2$ gm/cm^2. In this case, ringlet self-gravity is too weak for streamlines to interact. Streamlines librate at rates that differ slightly from their neighbors, and an $m = 2$ kinematic spiral develops; see Fig. 2. That spiral pattern steadily winds up over time until streamlines start to cross, and the ringlet's subsequent evolution is unreliable. A low-mass ringlet is not a good candidate model for the Huygens ringlet.

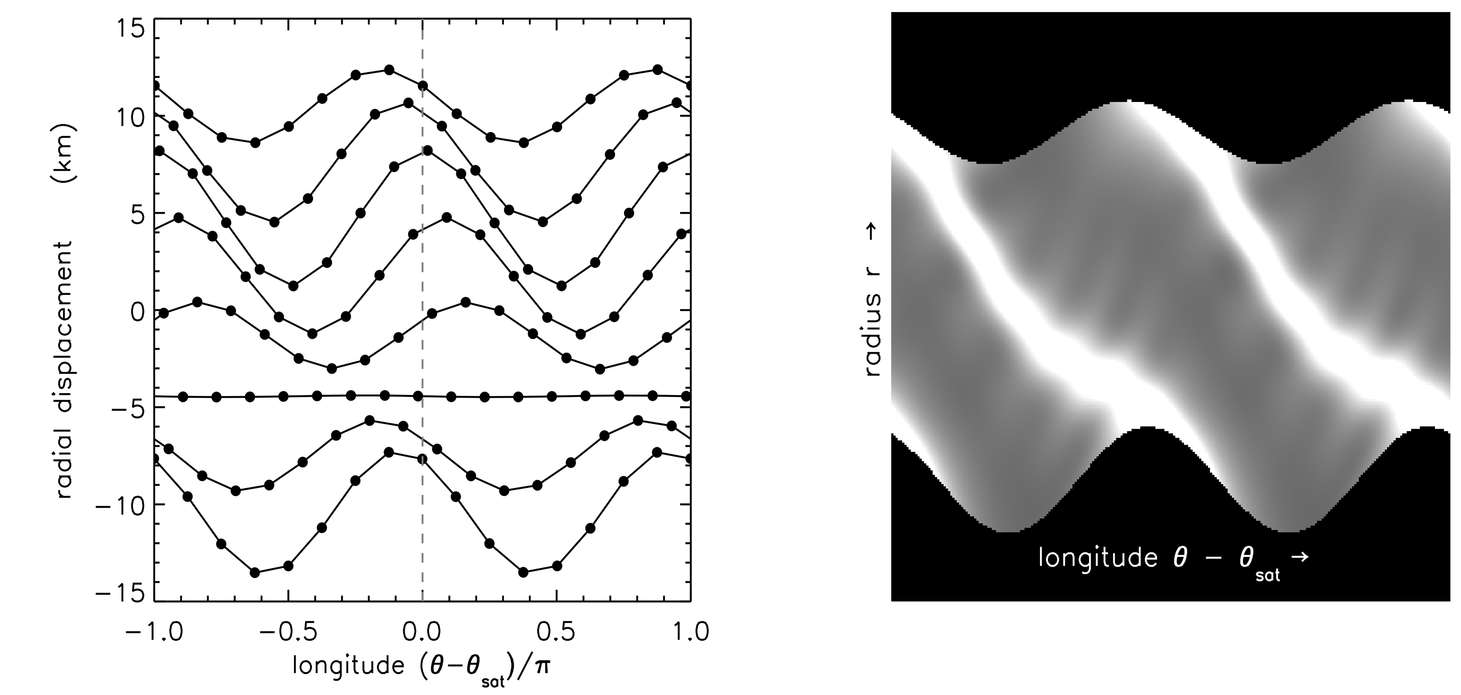

FIGURE 2: Nbody simulation of a low mass ringlet having surface density $\sigma = 2$ gm/cm^2. A kinematic spiral develops and winds up over time until adjacent streamlines cross and the subsequent evolution becomes unreliable.

intermediate-mass ringlet

An intermediate-mass ringlet has surface density $\sigma = 8$ to 40 gm/cm^2. In these simulations, the ringlet settles down such that its inner edge is dominated by its forced motion, with free/forced~ 0.4, while its outer edge is dominated by free epicyclic motion with free/forced~ 1.4. These relative amounts of free epicyclic energy at the ringlet's inner and outer edges is comparable to that observed in the Huygens ringlet. Figure 3 shows results from the $\sigma = 8$ gm/cm^2, which provides best agreement with the Huygens observations. The inferred ringlet masses are $m_r = (1 \text{ to } 5)\times10^{18}$ gm. If the Huygens ringlet is the disrupted remains of a satellite, then its diameter would have been $13 \lesssim d \lesssim 22$ km.

In all simulations shown here, the free mode drifts past the forced mode with relative angular velocity $\Delta\Omega = -1.3$ deg/day, in good agreement with the observed rate.

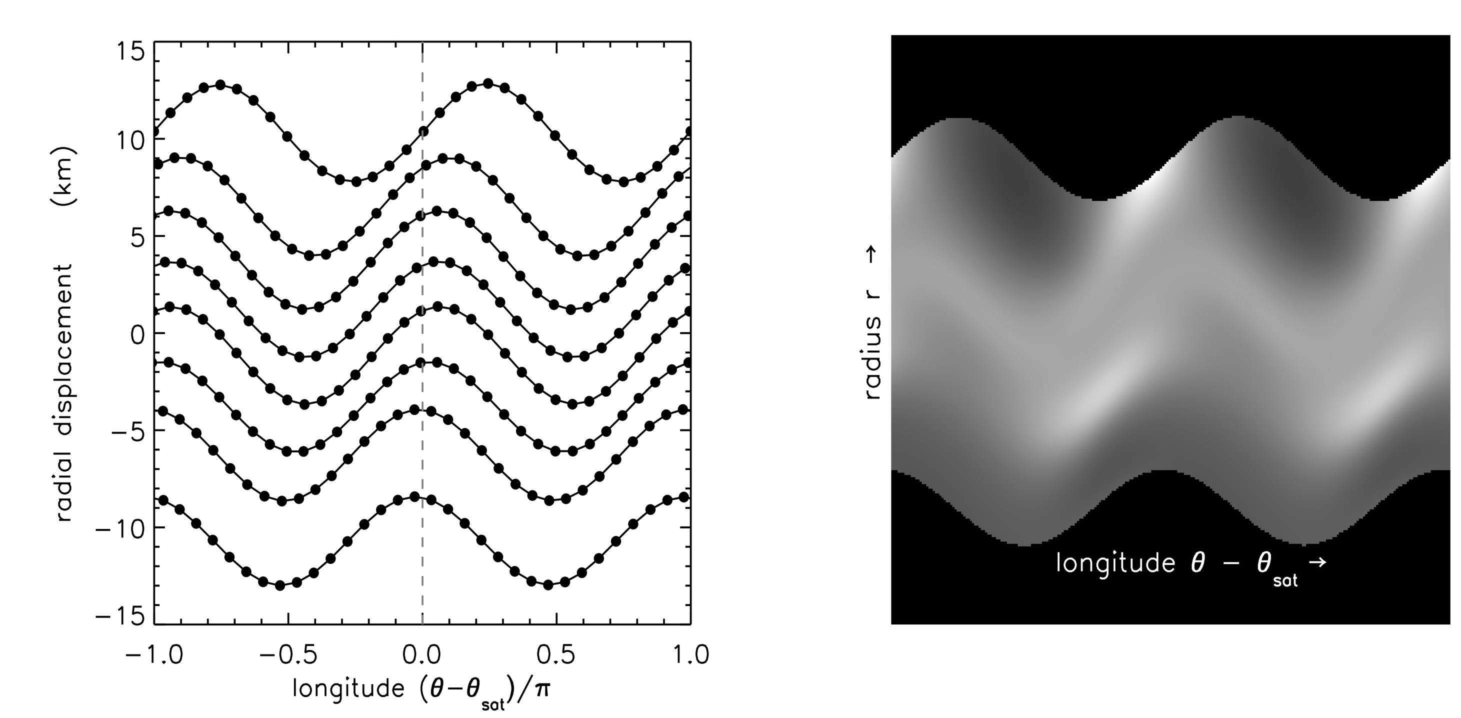

FIGURE 3: Nbody simulation of an intermediate mass ringlet having surface density $\sigma = 8$ gm/cm^2. The ringlet's inner edge is dominated by the forced mode (free/forced ~ 0.3), while the outer edge is dominated by the free mode (free/forced ~ 1.5).

References

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