Introduction

Secular gravitational perturbations can play a significant role in determining the global structure and long-term evolution of a disk-companion system. The circumstellar dust disk at β Pictoris is a well-known example, since its broad warp is thought to be due to secular perturbations exerted by an unseen planetary system (Mouillet et al. 1997). Eccentric planets can make a dust-disk appear lopsided (Wyatt et al. 1999), and inclined planets can also warp a more massive circumstellar gas disk, too (Lubov & Ogilvie 2001).

And if that planet inhabits a gap in the gas disk, then its secular perturbations can launch spiral density waves at the gap edge (Goldreich & Sari 2003). But if the disk is instead gravitationally-dominated, then the companion can launch spiral density and bending waves at its secular resonances in the disk, which also tends to damp the companion’s eccentricity and inclination (Ward & Hahn 1998, 2003).

The following will examine the secular evolution of a related system: of a small satellite that inhabits a narrow disk, which also tends to damp the companion’s eccentricity and inclination (Ward & Hahn 1998, 2003).

The rings model

The rings model of Hahn (2003) will be used to simulate the secular evolution of a satellite system. The model treats the disk as a set of N nested, gravitating ellipses of mass mj whose orientations are described by the usual orbit elements j = e, i, Ωj, ωj and thickness hj. The satellite is similarly represented here by a thin inclined ellipse. Because the masses of all perturbing bodies are smeared density and spiral bending waves.

The system’s evolution is described by the Lagrange planetary equations, and their solution is the familiar one for the secular evolution of N planets (Brouwer & Clemence 1961), except that the usual Laplace coefficients are softened by the ring’s fractional thickness h/a (e.g., Hahn 2003).

Pan and the Encke gap

The rings model is used to simulate the spiral waves that Pan can launch in Saturn’s rings. Pan is a 25km satellite that orbits in the 300-km-wide Encke gap in Saturn’s main A ring. Figure 1 shows the eccentricities that are associated with the spiral density wave that Pan’s secular perturbations launch at the outer edge of the Encke gap. A plot of the rings’ inclinations (not shown) is similar.

Figure 2 shows the ring’s longitudes of perihelion relative to the satellite’s longitude ωs. These curves represent an n = 1 armed trailing spiral density wave that slowly rotates at the rate δ, which is the satellite’s apse precession rate. Maxima in the rings’ surface density are 90° behind the δ curve. The lower figure shows the rings’ longitudes of ascending node v relative to the satellite’s node s. This is a one-armed spiral bending wave, and it rotates in a retrograde sense at the satellite’s precession rate δ. Maxima in the ring plane’s vertical displacement occur 90° ahead of δ.

Horizontal & vertical displacements

The waves that Pan can launch at the Encke gap edge are actually rather modest disturbances in the A ring. Figure 3 shows the fractional variation in the ring’s surface density due to Pan’s density edge–wave, which is only δs ≃ 0.5%. The initial wavelength is quite long, though, λ ≃ 500km, and gets shorter further downstream.

The ring’s vertical displacement due to the spiral bending wave launched at the gap edge depends on Pan’s inclination. Spitakile et al. (2006) report an upper limit of δ = 0.001° on Pan’s inclination. Adopting this upper limit means the

Eccentricity and inclination damping

The excitation of these spiral density and bending waves transfers angular momentum between the satellite and the nearby rings, which in turn damps the satellite’s eccentricity e and inclination i, see Fig. 4. This is important since the fate of the satellite’s e and i is somewhat uncertain.

Goldreich & Tremaine (1980) show that the satellite’s corotation resonances in the ring tend to damp the satellite’s ϵ, while Lindblad resonances excite e. And as long as the corotation resonances are unsaturated, e–damping is certain. But if that is not the case, then the Lindblad resonances will pump up the satellite’s ϵ over an e–fold timescale t ϵ ∼ 105Pωs/Pωs ∼ 10 yrs. Pan’s external vertical resonances in the ring also pump up its inclination over a comparable timescale (Bourrier et al. 1984, Ward & Hahn 2003).

However, Fig. 4 shows that the secular interactions damp Pan’s e and i at rates that are 40 times faster than the excitation due to Lindblad/vertical resonances. We suspect that these secular perturbations are responsible for circularizing Pan’s orbit and confining it to the ring plane.

Wave reflection, and standing waves

But if these spiral waves fail to damp downstream, they will reflect at the ring’s outer boundary and return to the launch site. The upper Fig. 5 shows that when the outward–bound long, trailing spiral density wave reflects at the ring’s outer edge, it returns as a short, leading density wave, a phenomenon also seen in Hahn (2003).

And if the long, leading spiral wave reflects at the outer ring edge, it returns as a long trailing wave; see lower Fig. 5. The superposition of the two wavetrains results in a standing wave, one where the nodes alternate 180° every half–wavelength.

If reflected waves do manage to return to the vicinity of the satellite, they will communicate some of their angular momentum back into the satellite’s orbit, which then stalls further eccentricity or inclination damping.

Towards an analytic theory

Lastly, the Lagrange planetary equations will be used to derive analytically the numerical results presented here. Our goal is to use these equations to derive the amplitudes of these spiral density and bending waves, their dispersion relations, and the rates at which this wave–action damp the satellites eccentricity and inclination.