Planet Migration Via Numerous Stochastic Scattering Events

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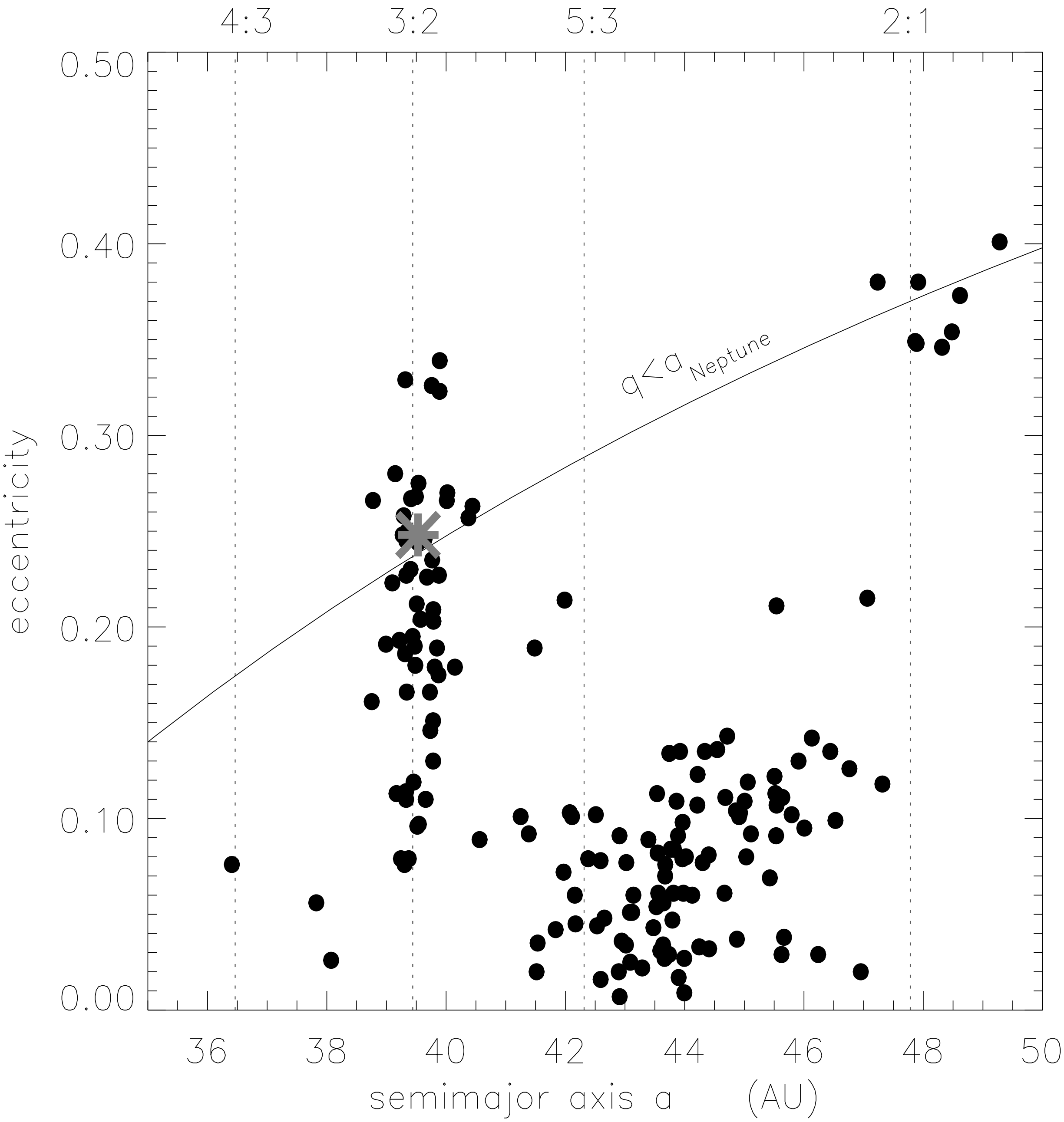
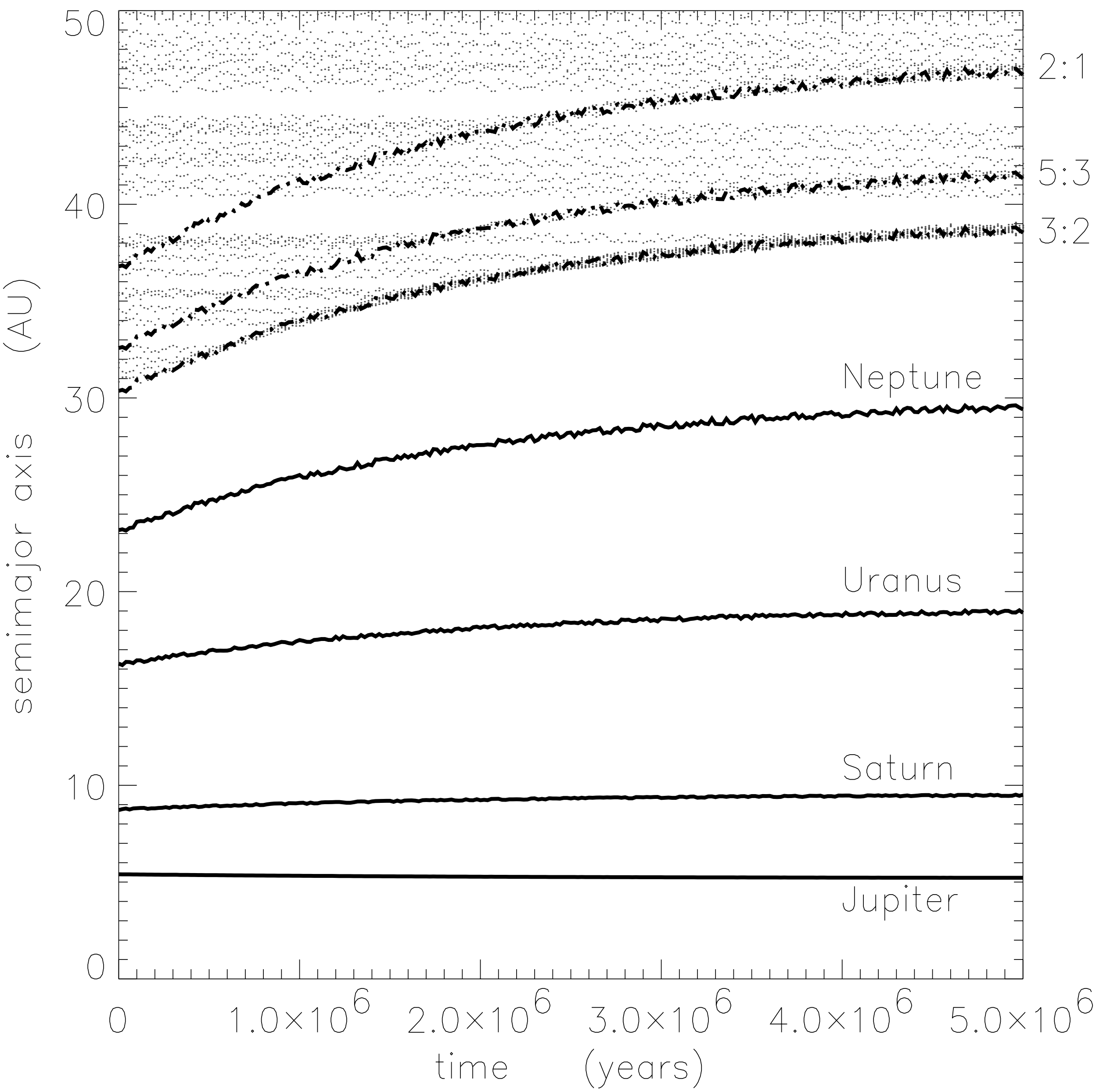
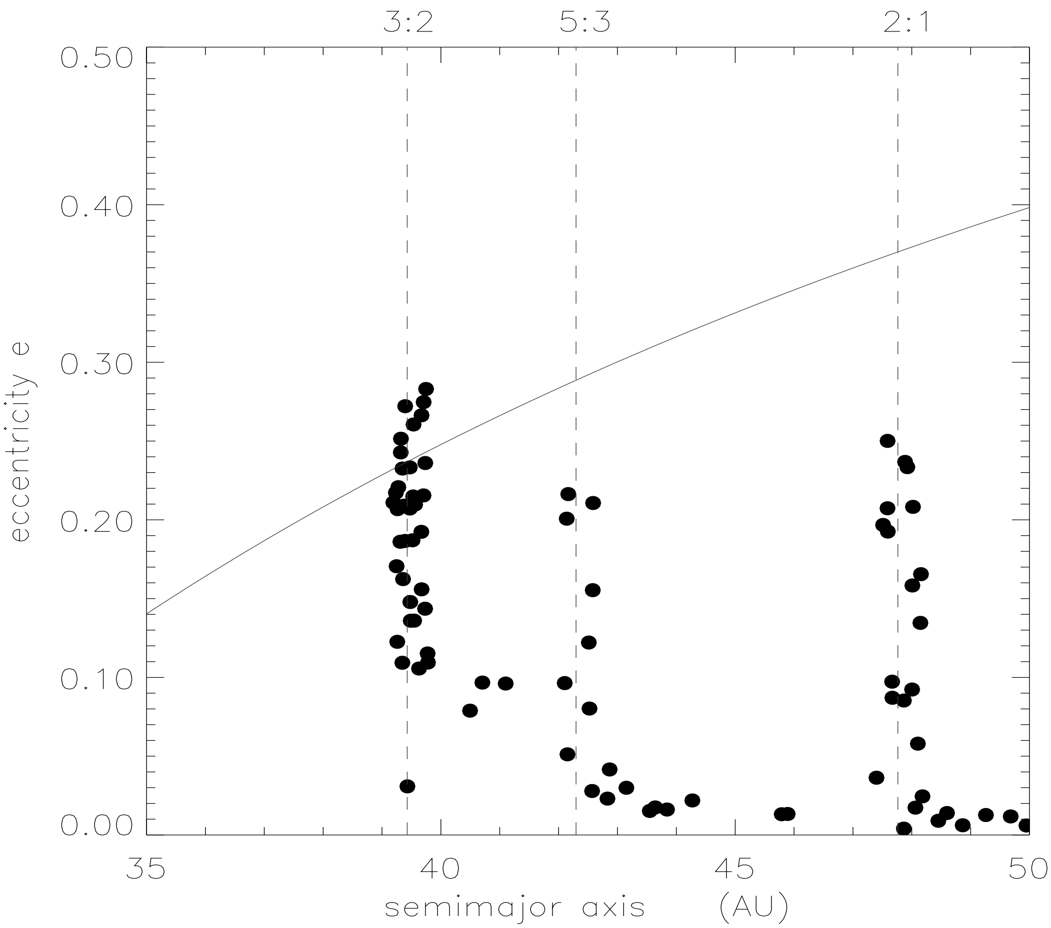


The distribution of KBO orbit elements suggests a prior history of outward-migration by Neptune.



Neptune migrates outwards,
resonances sweep through Kuiper Belt
(e.g., Malhotra 1993, 1995).



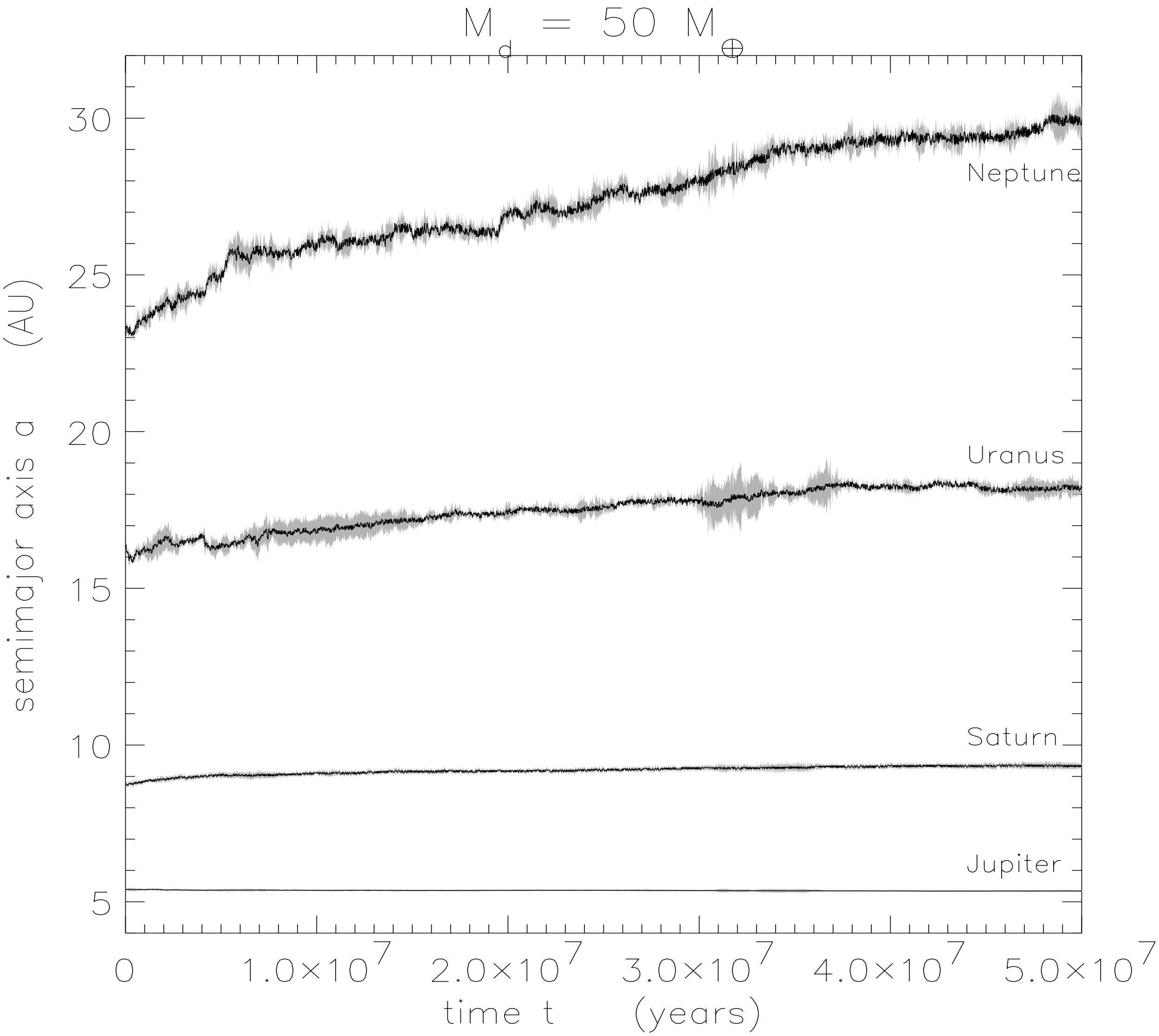
Since $B = a[(m+1)\sqrt{1-e^2} - m]^2 =$ constant for particles at an $m+1 : m$ resonance,

$$e^2_{\text{max}} = 1 - \left(\frac{m + \sqrt{a_{init}/a_{final}}}{m+1} \right)^2$$

Since KBOs at 3:2 have $e_{\text{max}} \simeq 0.3$ and $m = 2$

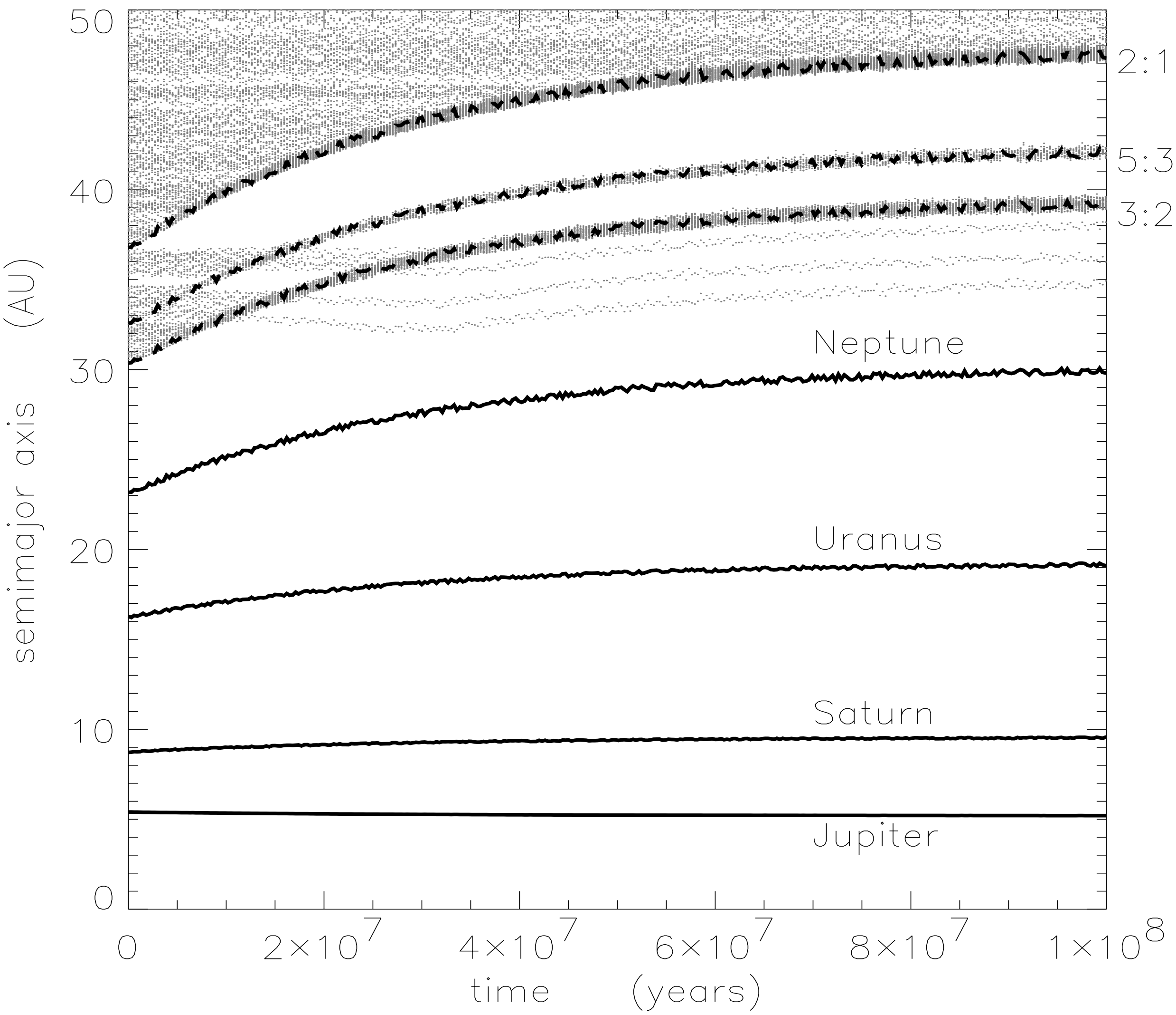
$\Rightarrow$ $\Delta a_{\text{KBO}} \simeq 9$ AU and
$\Delta a_{\text{NEPTUNE}} \simeq 7$ AU.

Why would Neptune migrate outwards?



$N = 1000$-body simulation of 4 giant planets embedded in a *trans-Saturnian* Kuiper Belt of mass $M_d = 50$ $M_\oplus$ at $10 < r < 50$ AU (Hahn and Malhotra 1999).

$\Rightarrow \Delta a_{\text{NEPTUNE}} \simeq 7$ AU over $\tau \sim 3 \times 10^7$ years,
but no resonance trapping!

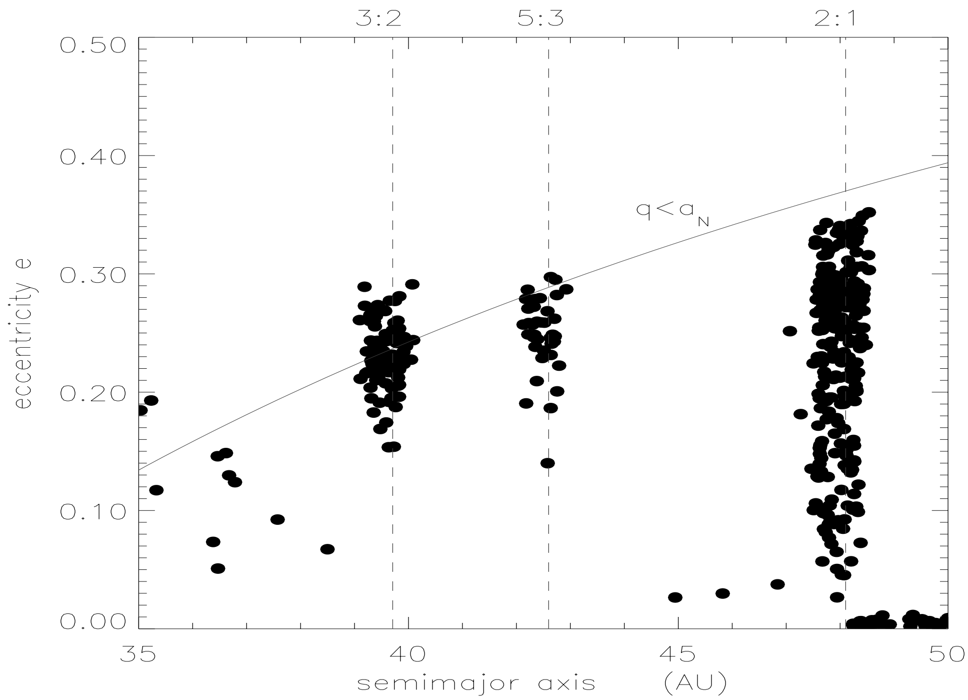


Instead, embed the giant planets in a *massless* particle disk, and drive migration via a *smooth* external torque

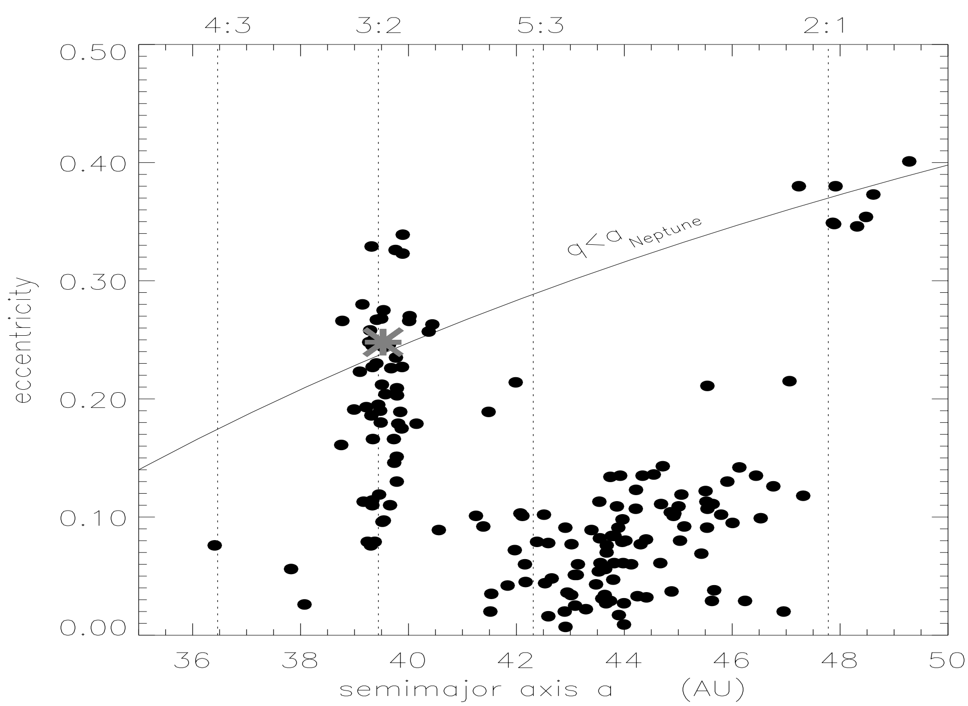
$$T_0 = \frac{\Delta a_i}{2\tau}\sqrt{\frac{GM_\odot}{a_i}}e^{-t/\tau}$$

where $\Delta a_{\mathsf{NEPTUNE}} = 7$ AU and $\tau = 3 \times 10^7$ years.

Simulated e versus a



Observed e versus a



The trapping by 2:1 is too efficient,
which precludes the formation of the 'Classical Disk'

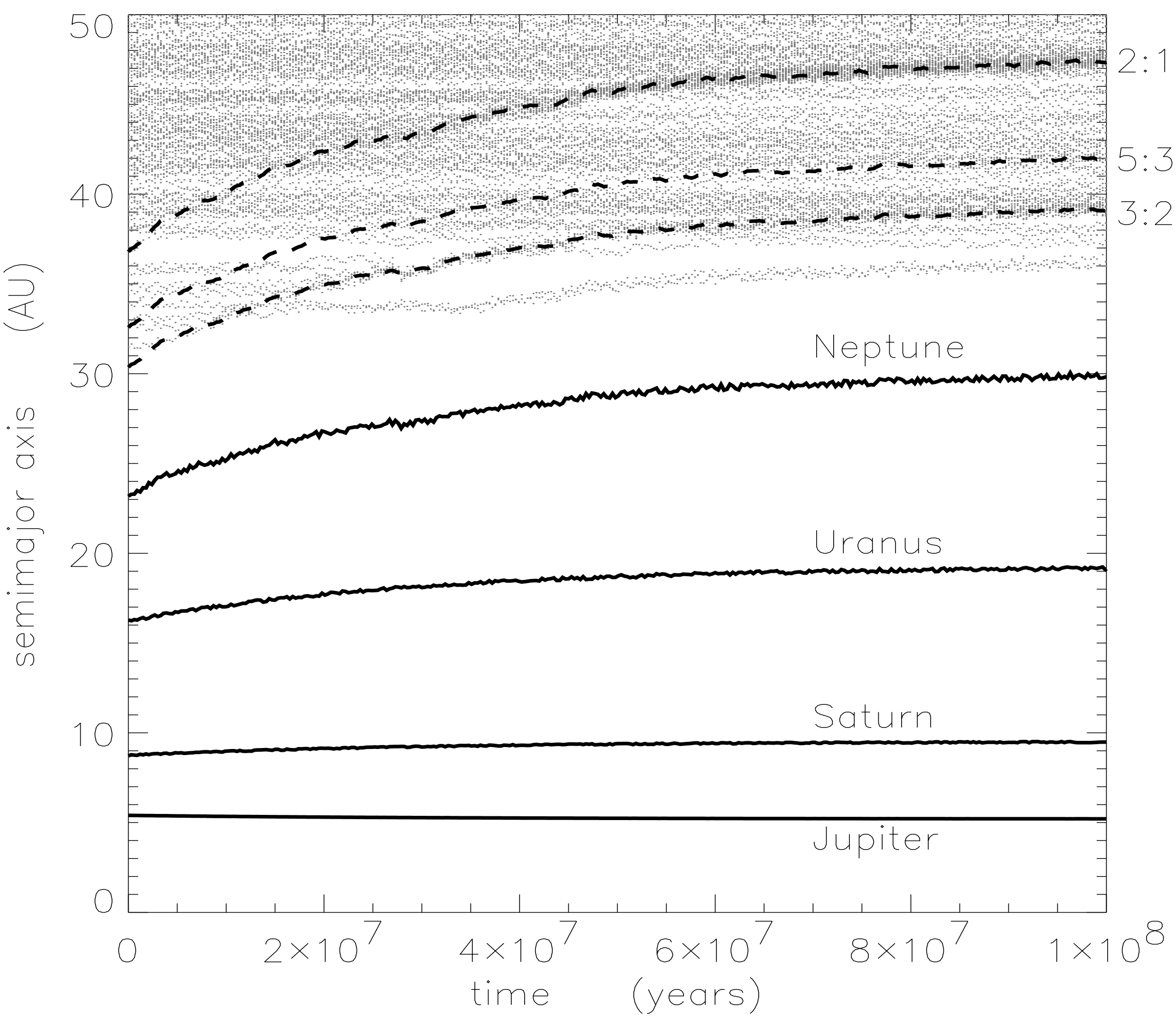
- Recall that planet–migration is due to *stochastic* scattering of planetesimals by the giant–planets.

- To effect this, add 'jitter' to the planet–migration torque:

$$T = (1 + r_\sigma) T_0$$

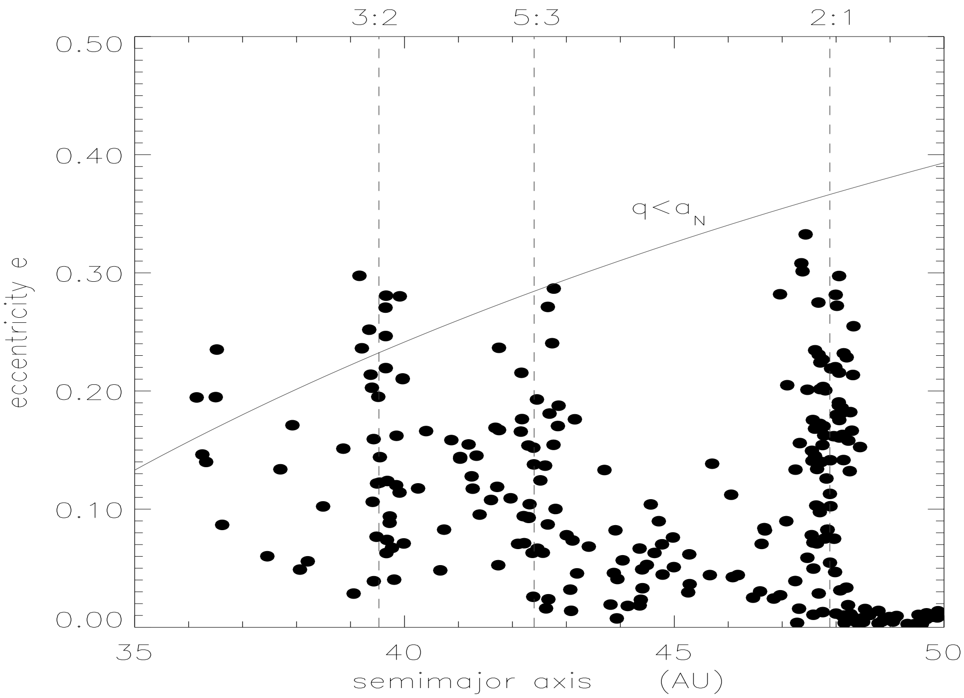
where r_σ is a random number distributed about zero with standard deviation σ.

- Results:
  - Resonance trapping is surprisingly insensitive to jitter!
  - Simulations with $\sigma \lesssim 10$ are indistinct, and allow efficient trapping of KBOs at resonances.
  - Trapping occurs at reduced efficiency when $\sigma \sim 25$ to 75.
  - $\sigma \gtrsim 100$ depletes the KB interior to $r \simeq 45$ AU, trapping still occurs at the 2:1.

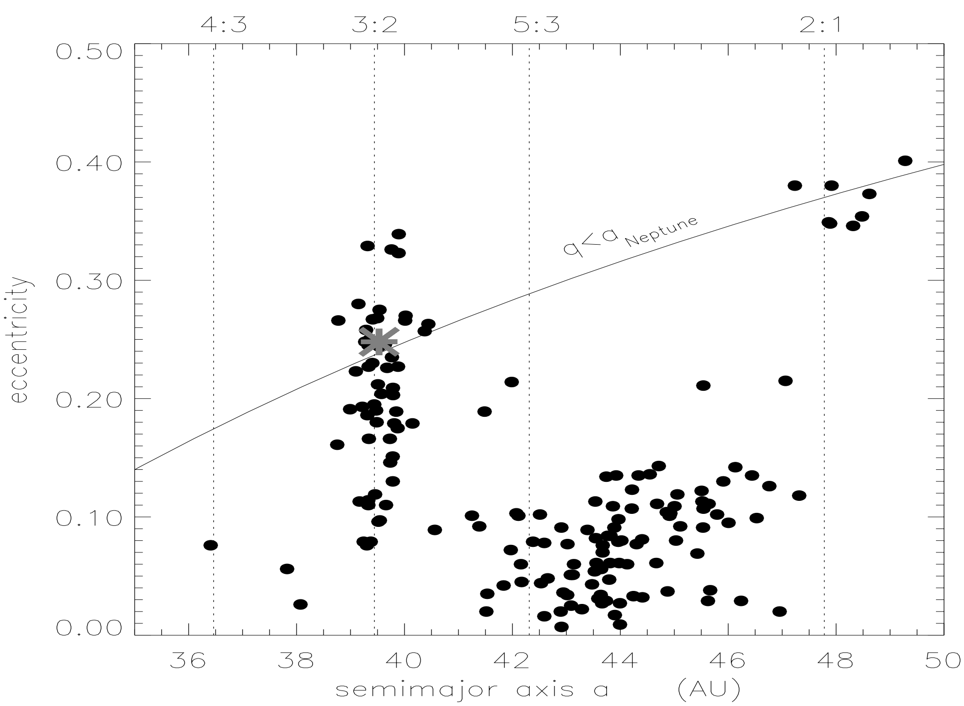


Nominal simulation has $\sigma = 50$
evolved for $t = 3 \times 10^8$ years, or 7% of the solar age!

Simulated (intrinsic) e versus a



Observed (apparent) e versus a



We must deal with observational selection effects before comparing apparent & intrinsic orbit elements!.

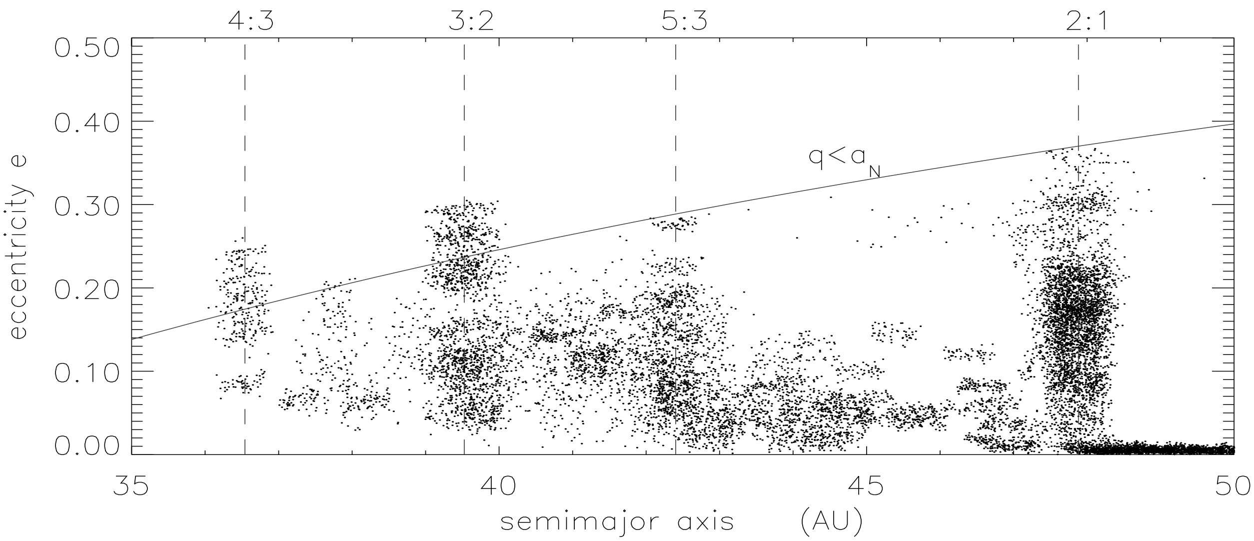
Use a Monte Carlo model to translate from intrinsic $\Rightarrow$ apparent orbit elements.

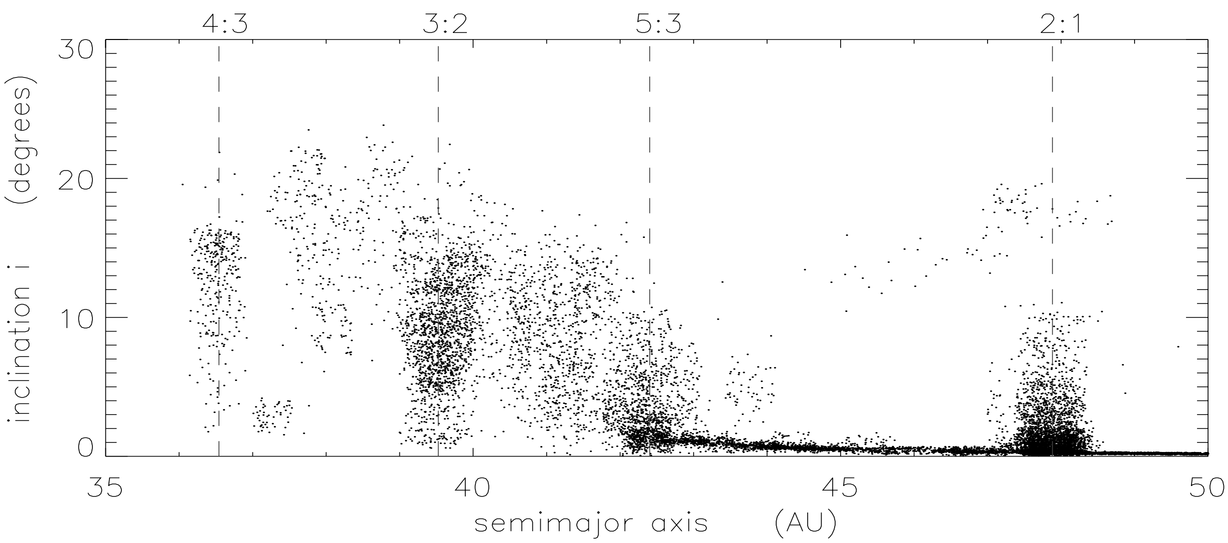
From my set of 250 KBO orbit elements, generate 10^6 Monte Carlo KBOs having:

- same orbits but random mean–anomalies
  (this destroys phase relationship between Neptune & resonant KBOs).

- observed size distribution $dN/dR \propto R^{-3.6}$.

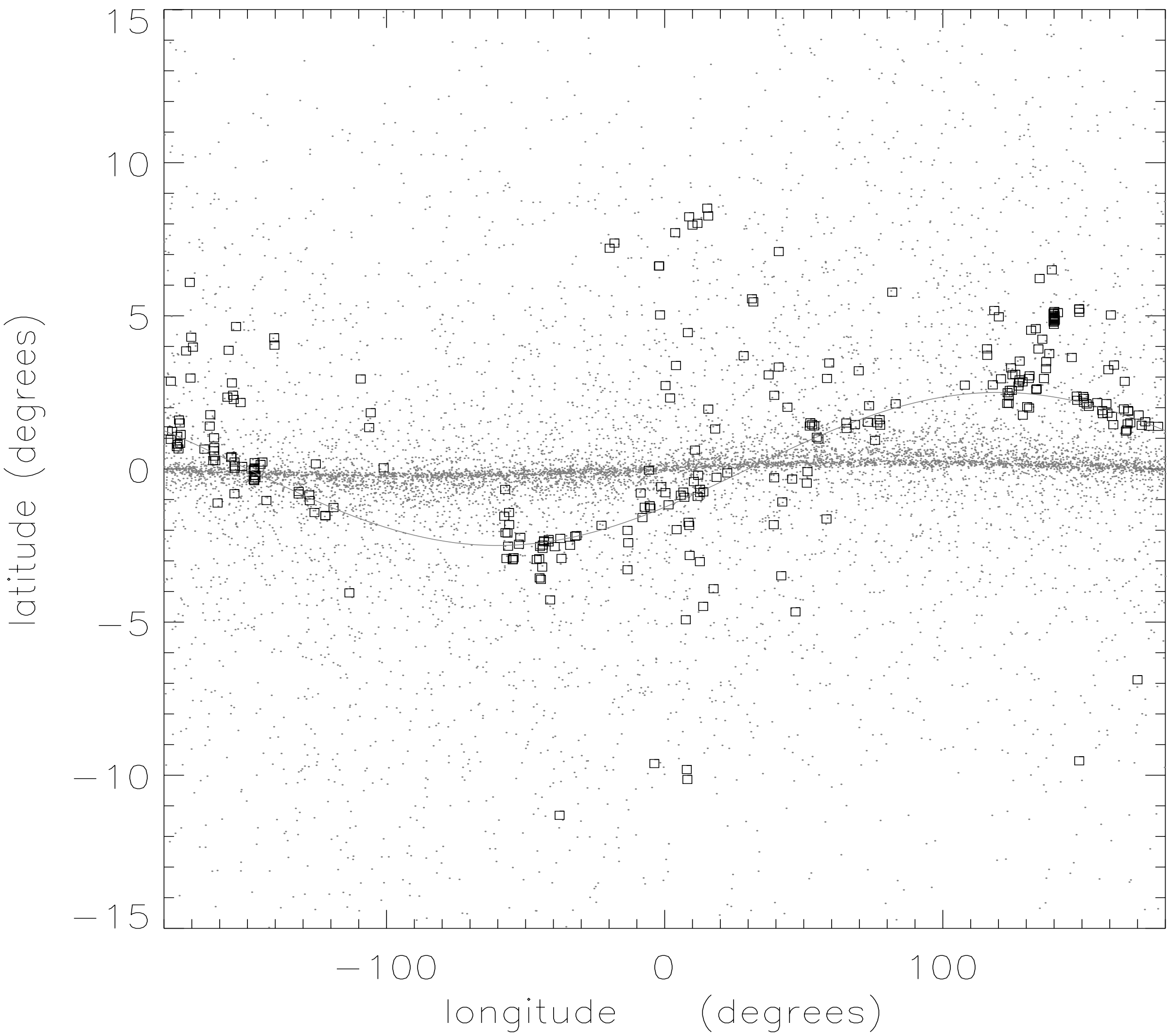
- assume maximum KBO radius R decreases with distance r as $R_{\text{max}}(r) \propto r^{-3}$
  (this is inferred from multi–zone KBO accretion model of Davis *et al.* 1999 LPSC XXX abstract).





Orbit elements used by the Monte Carlo model.

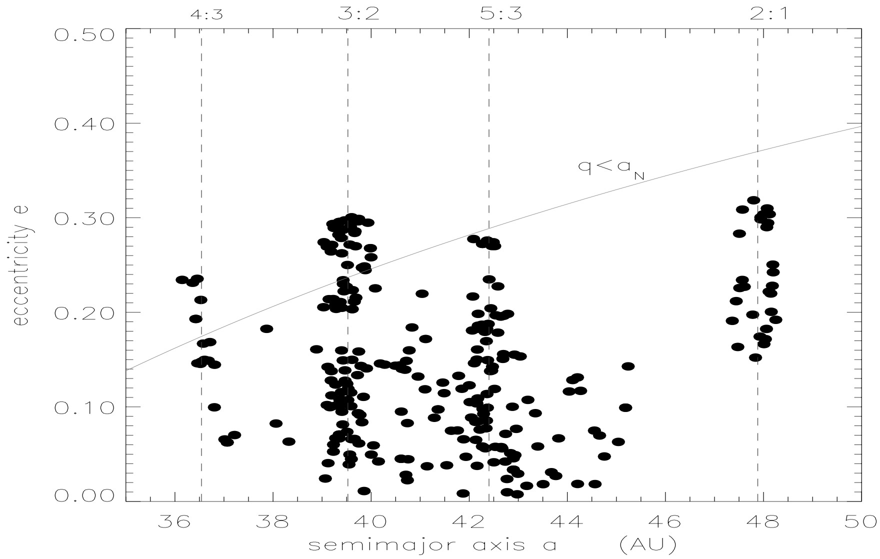
Lines–of–Sight into the Kuiper Belt



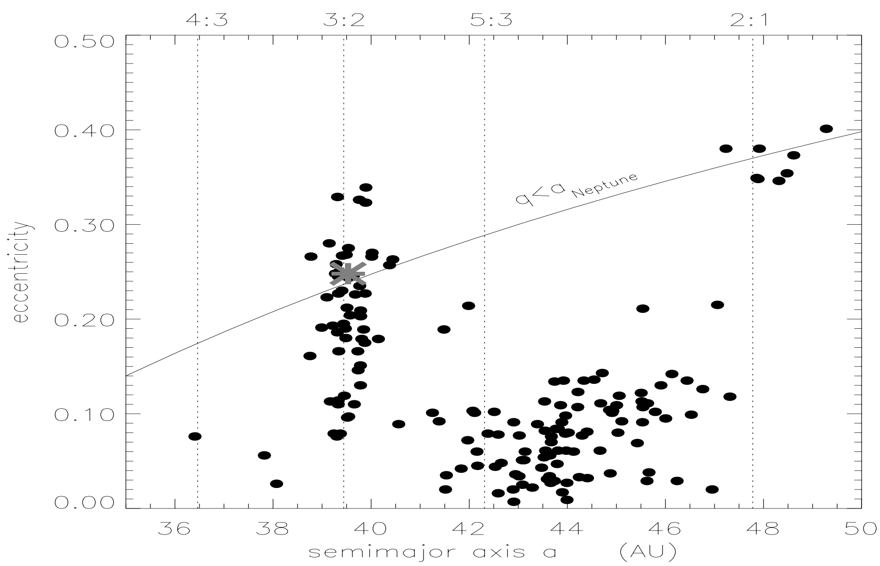
High–latitude KBOs are in/near the 3:2,
and low–latitude KBOs are in the Classical Disk and beyond.

Note that KBO astronomers observe largely along the ecliptic, which might be inclined relative to the more distant, denser part of the Kuiper Belt.

Simulated (apparent) e versus a



Observed (apparent) e versus a

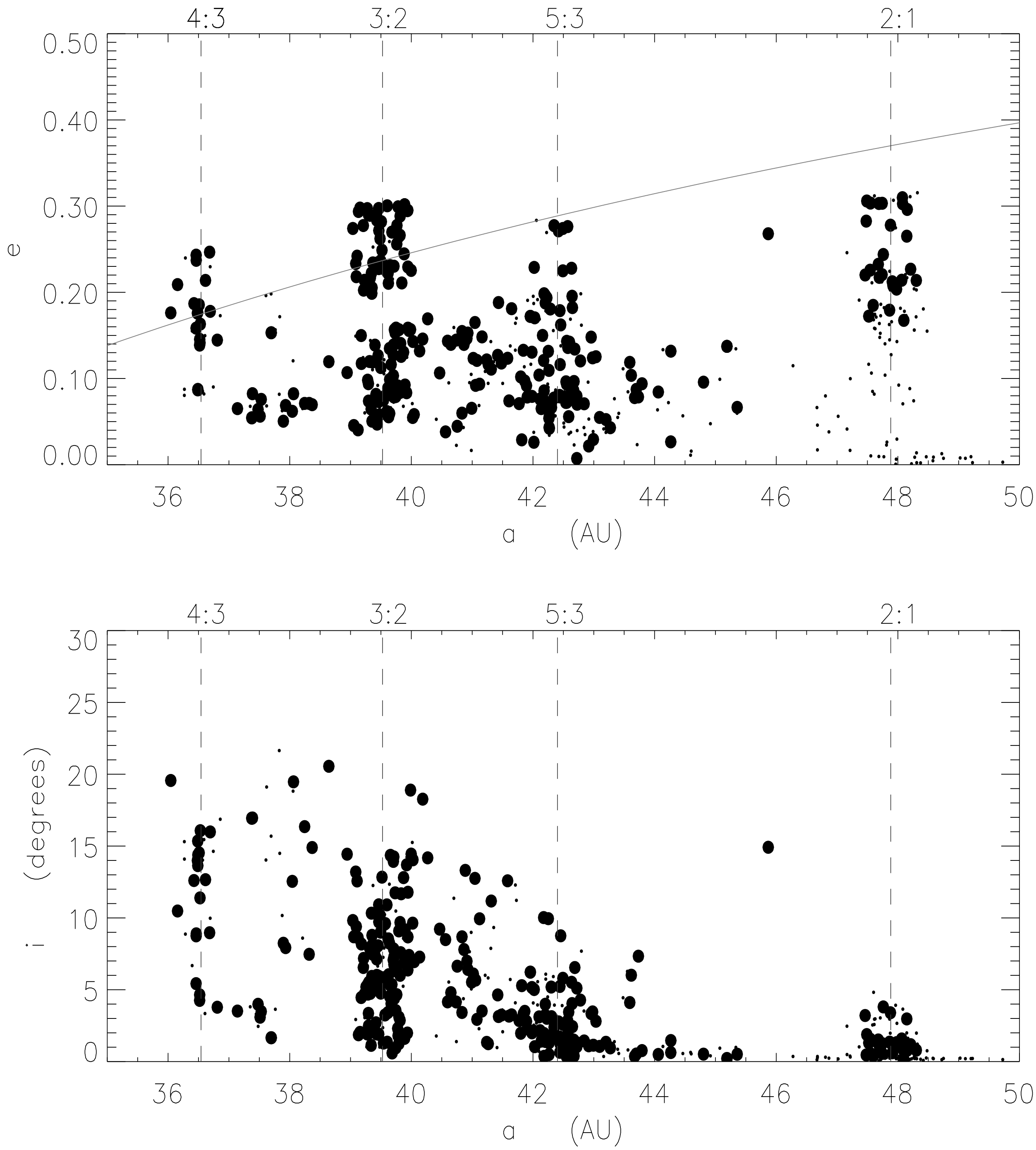


Simulate numerous shallow, wide–angle surveys along same LOS by observing the model KB to magnitude m=23 (typical of the *published* KB surveys...) over solid angle $\Omega = 1$ degree^2.

- qualitative agreement & disagreement between model & observations.
- shallow surveys are insensitive to i_{ec}.

Conclusions

- jitter reduces the trapping efficiency of a migrating planet.
  - a $\sigma \sim 50$ allows some KBOs to slip thru Neptune's 2:1 and enter the Classical Disk with $e \sim 0.1$.
  - this implies that the fractional asymmetry in the total planet–migration torque is as low as $1/\sigma \sim 2\%$.
- based upon a youthful ($t = 3 \times 10^8$ years) KB model, and an overly simplistic treatment of selection effects,
  - I see qualitative agreement between the planet–migration scenario and the KB observations provided $R_{\text{max}}(r_{\text{init}}) \propto r_{\text{init}}^{\text{-3 or so}}$.
- If Neptune is the sole source of the (e, i) excitation seen in the Kuiper Belt, then there should be lots of low–i KBOs at Neptune's 2:1 resonance, and perhaps a *razor–thin* disk KBOs beyond $a > 48$ AU.
- Firmer conclusions will be reported once the N–body models have evolved $> 10^9$ years and the 'observing' is modeled more realistically.



$t = 3 \times 10^8$ years