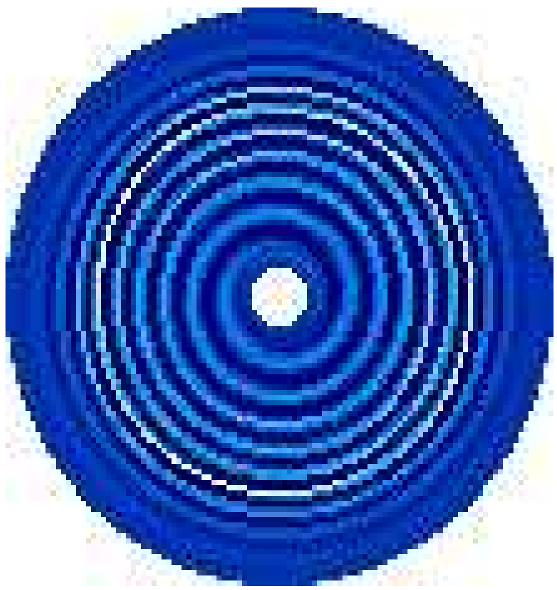
Spiral Bending Waves Launched at a Vertical Secular Resonance



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I. Examine secular forcing exerted between a perturber (e.g., planet or satellite) and a self–gravitating disk (e.g, planetesimal disk, solar nebula, planetary ring, stellar disk).

A. Secular resonances are sites in the disk where

1.

$$g_p \equiv \frac{d\tilde{\omega}_p}{dt} = g_{disk}$$

which excites eccentricities and can launch spiral *density* waves (Ward and Hahn 1998).

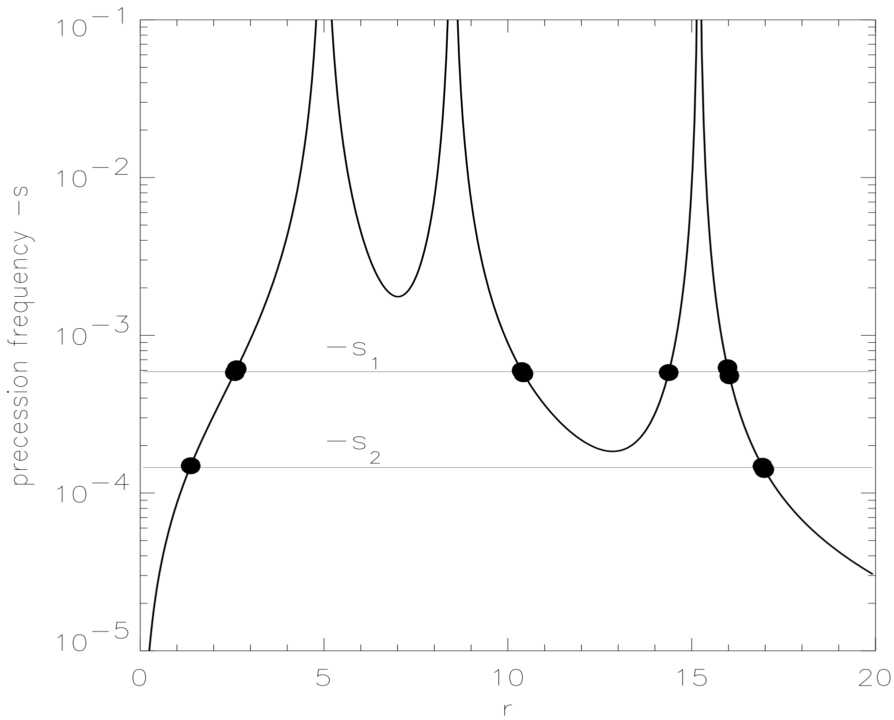
2.

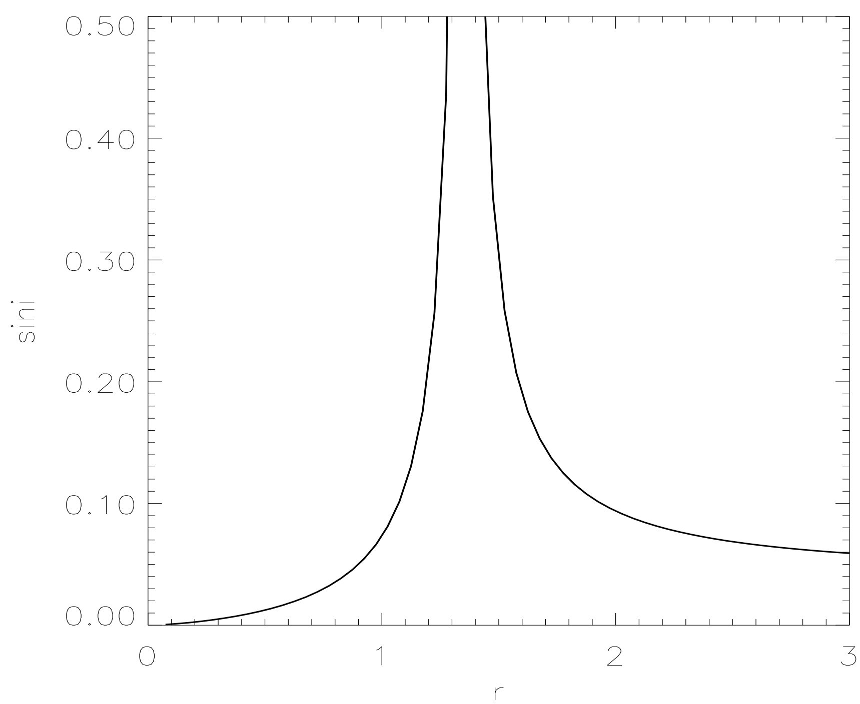
$$s_p \equiv \frac{d\Omega_p}{dt} = s_{disk}$$

which excites inclinations and can launch spiral *bending* waves.

B. Review: Planets embedded in a zero–mass disk.

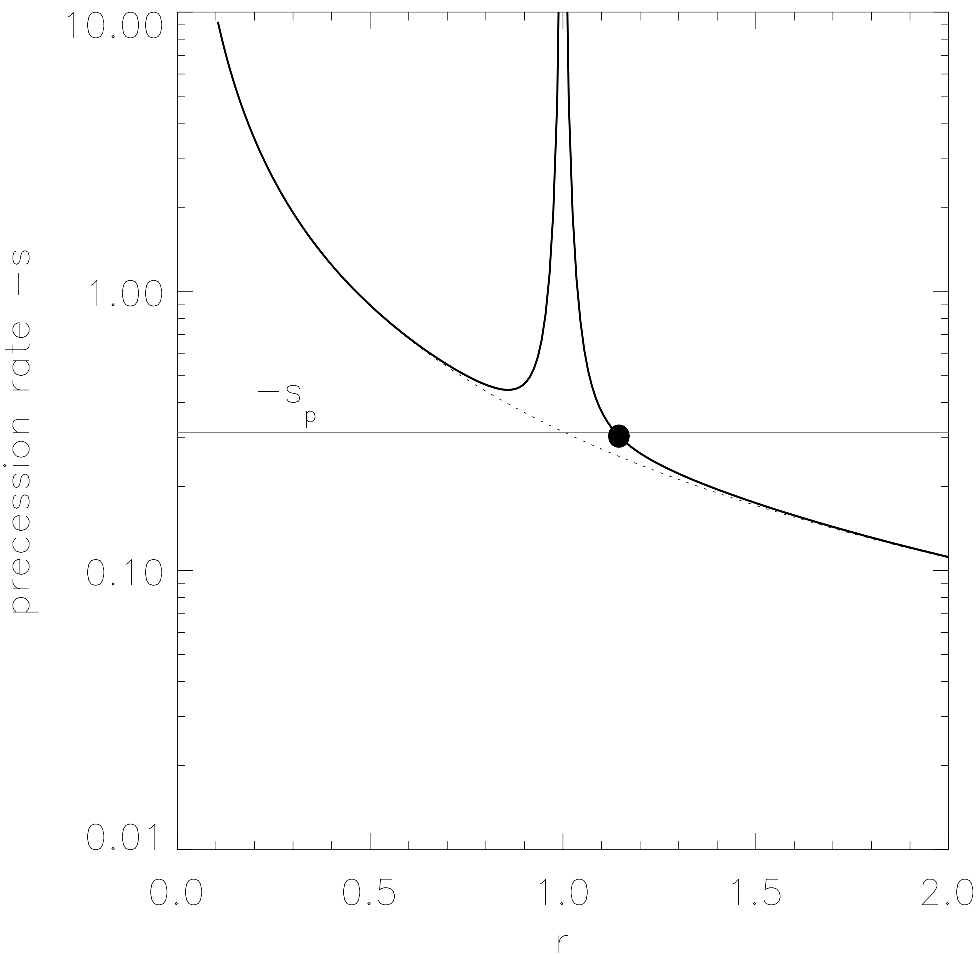
1. The planets' gravities drive the disk's nodal regression ($i.e.$, $s < 0$).





C. In a self–gravitating disk,
the disk gravity drives the regression of the nodes:

$$s_{disk} = -\frac{1}{4}\mu_p\beta^2 b_{3/2}^{(1)}(\beta)\Omega - \frac{2\pi G\rho_{disk}}{\Omega}$$

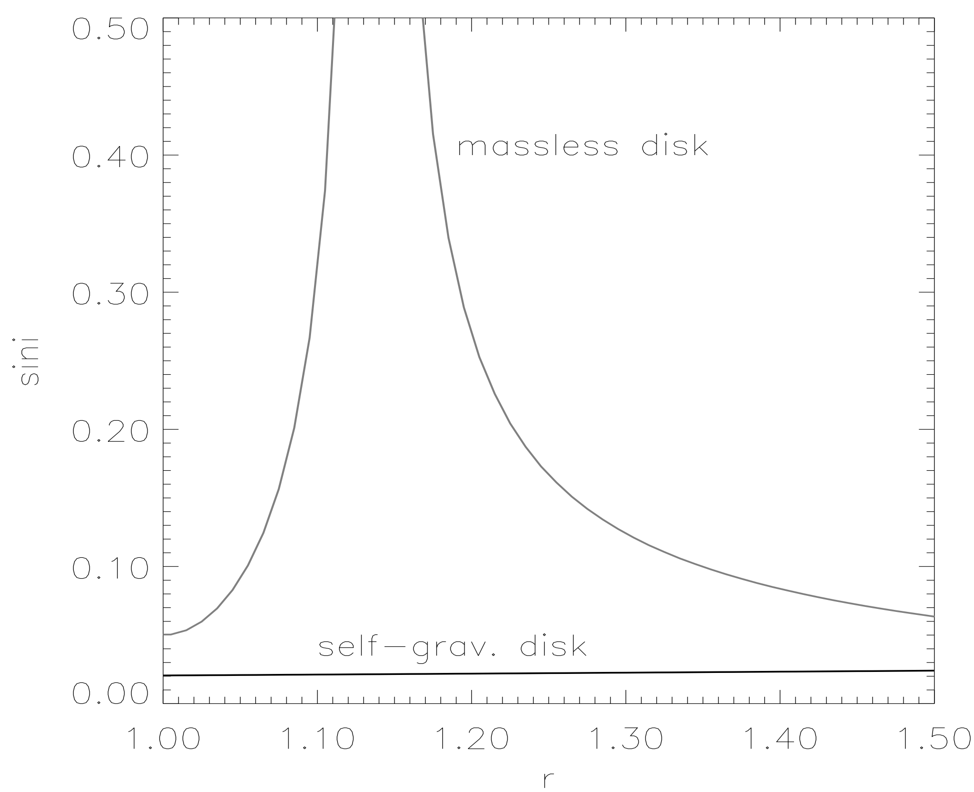


Massive disks having $\pi\sigma_{disk}r^2 \sim M_p$ have secular resonance near the planet.

D. A planet excites inclinations at the secular resonance. Disk self–gravity transports that disturbance away as a spiral bending wave with amplitude (Shu, Cuzzi, Lissauer 1983)

$$\sin i_d = \sqrt{\frac{\pi}{8\mu_d}} \frac{\mu_p \beta^2 b_{3/2}^{(1)}(\beta)}{\left| r^2 \frac{d}{dr}\left(\frac{s}{r\Omega}\right)\right|^{1/2}} \sin i_p$$

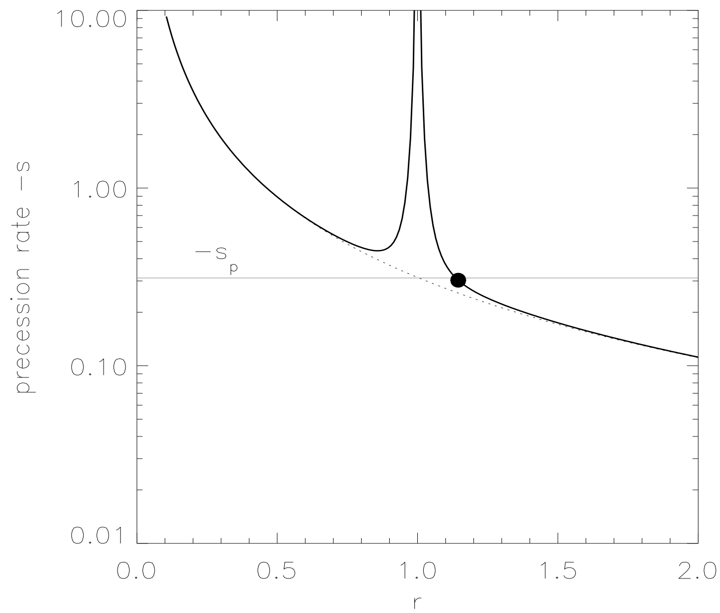
where $\mu_p \equiv M_p/M_\star$ and $\mu_d = \pi\sigma r^2/M_\star$.



Note that $i_{\text{self–grav.}} \ll i_{\text{test–part.}}$

II. Resonance location & wave amplitude depends on the disk structure.

Scenario A. Consider a planet embedded in a continuous disk, **no gap**.



$$\Delta r \simeq \left(\frac{Q\mu_p}{3\pi}\right)^{1/3} r_p \qquad \sim 0.007Q^{1/3}\ \text{AU}$$

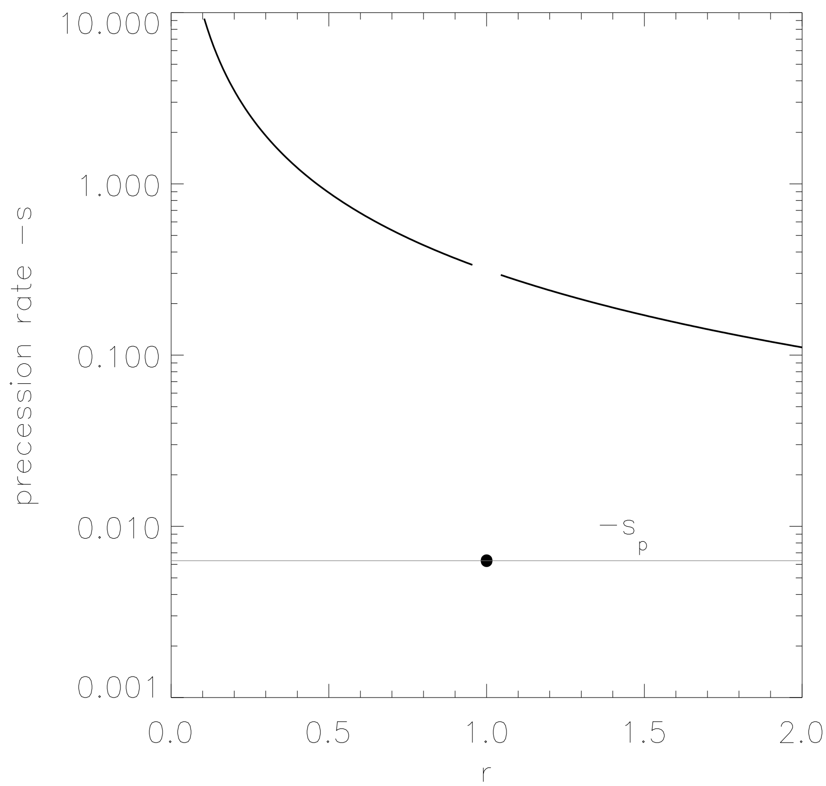
$$\lambda \simeq \sqrt{3Q\mu_d}\, r_p \qquad \sim 0.01Q^{1/2}\ \text{AU}$$

$$\sin i_d \simeq \left(\sqrt{\frac{\pi}{Q}}\frac{\mu_p}{3}\right)^{1/3} \frac{\sin i_p}{\sqrt{\mu_d}} \sim 2Q^{-1/6}\sin i_p$$

where RHS is evaluated for an Earth–mass protoplanet at 1 AU in a minimum–mass *planetesimal disk*.

Scenario B. Planet opens gap in planetesimal disk
$w \sim 2\sqrt{3}R_H$ (width of feeding zone).

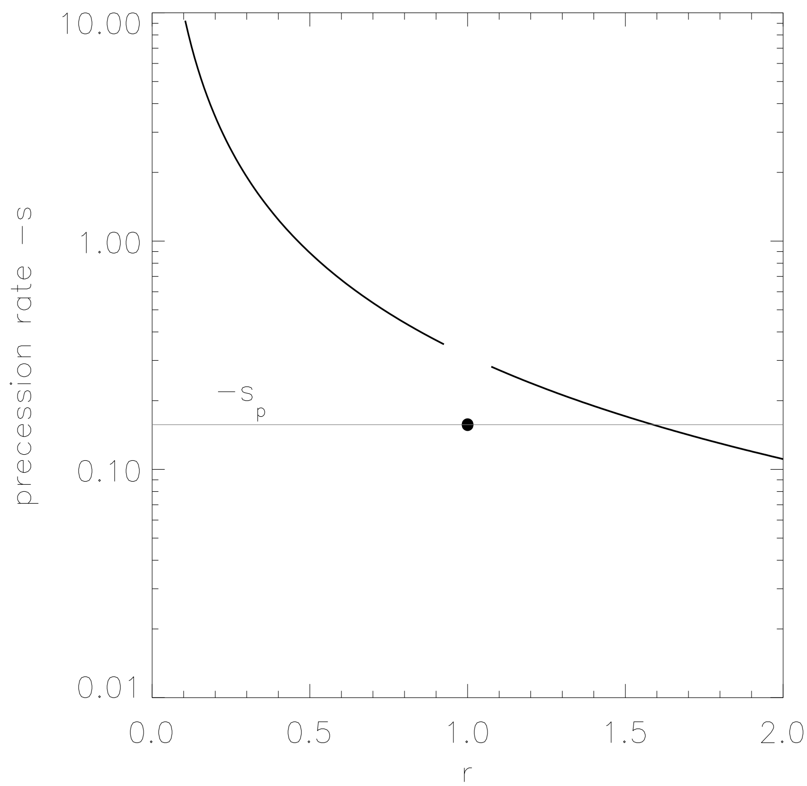
$$|s_p| \ll |s_{disk}|.$$



- planet's node *slowly* regresses,
- resonance lies far away,
- little (if any) wave excitation.

Scenario C. Giant planet opens gap in nebula gas disk $w \sim h$ (width $\sim$ disk scale height).

$$|s_p| \simeq \frac{1}{2}|s_{disk}|$$



- $r_{res} \simeq 1.6 r_p$ (near $m = 1$ OLR).
- $\lambda \simeq 3.6\sqrt{h r_p} \sim 5$ AU.
- $\sin i_d \simeq 2\sqrt{\frac{h}{r}\frac{\mu_p}{\mu_d}} \sin i_p \sim 0.1 \sin i_p$.

For Jupiter embedded in a minimum–mass nebula with $h \sim 0.07r$.

III. The Planet's Response to the Spiral Wave.

- The mutual attraction of the disk and planet causes in–plane $\vec{L}$ to flow from the planet$\rightarrow$resonance, which propagates away via the bending wave.

- Consequently the planet's inclination is damped at the rate (c.f., Borderies, Goldreich, & Tremaine 1984):

$$\frac{1}{i_p}\frac{di_p}{dt} = -\frac{\pi}{8}\frac{\mu_p\mu_d\beta^3[b_{3/2}^{(1)}(\beta)]^2}{\left|r^2\frac{d}{dr}(\frac{s}{r\Omega})\right|}\Omega_p \equiv \tau_i^{-1}$$

which is similar to $\dot{e}_p$ obtained by Ward & Hahn 1998 due to apsidal density waves.

Scenario A. For an Earth–mass planet embedded in a minimum–mass *planetesimal* disk, no gap

$$\tau_i \simeq \left(\frac{Q\mu_p}{3\pi}\right)^{1/3} \frac{3}{\mu_d \Omega_p} \sim 100 Q^{1/3} \text{ orbits.}$$

For an Jupiter–mass planet embedded in a minimum–mass nebula *gas* disk,
with $Q \sim 20$ and no gap

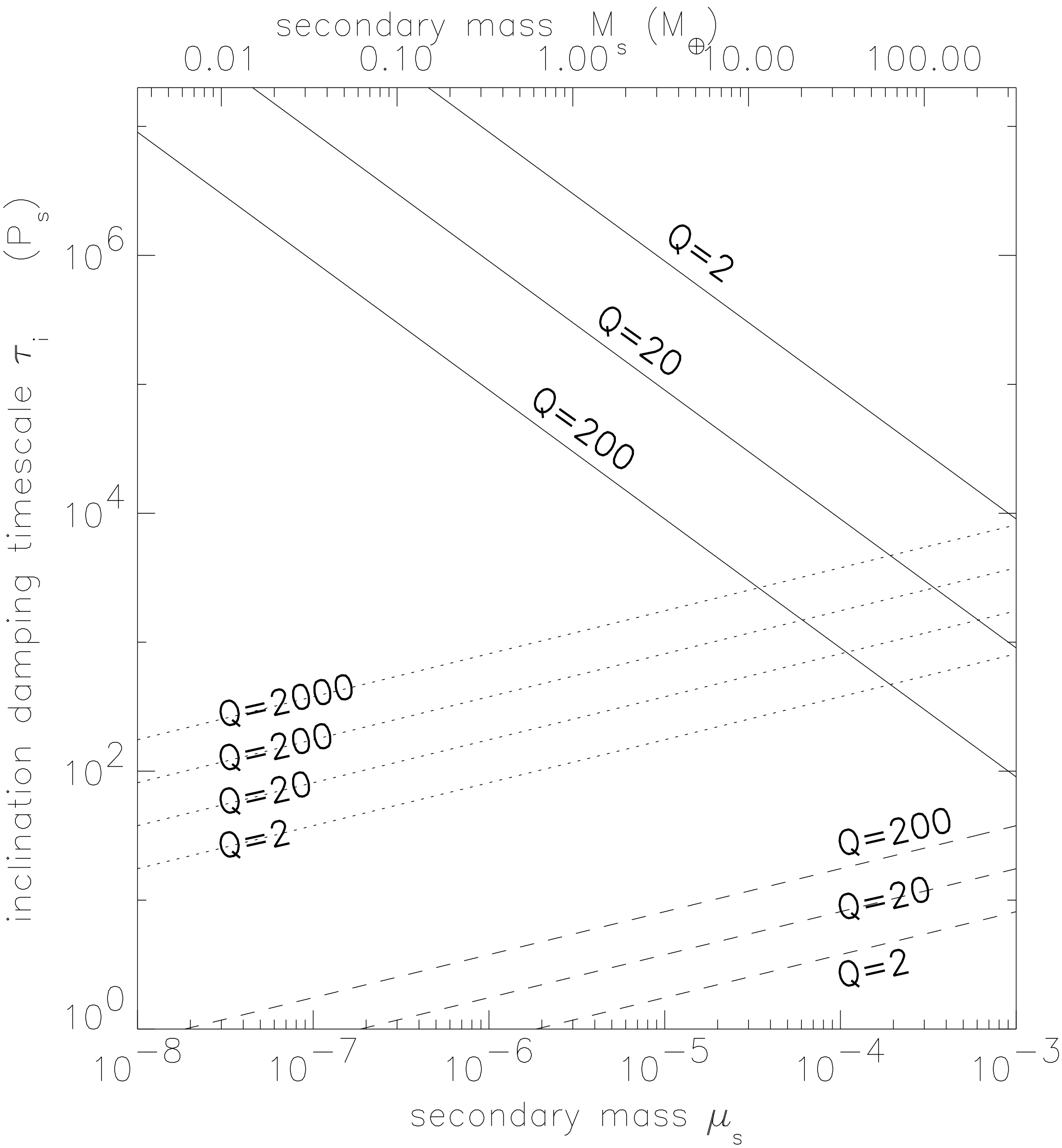
$$\tau_i \sim 20 \text{ orbits.}$$

Scenario B. For a terrestrial planet embedded in a planetesimal disk with a gap width $w \sim 2\sqrt{3}R_H$,

- τ_i is very long since the resonance lies far from the planet.

Scenario C. For a Jupiter–mass planet embedded in a minimum–mass gas disk with a gap width $w \sim h$

$$\tau_i \simeq \frac{0.4}{Q\mu_s\mu_d\Omega_p} \sim 10^3 \text{ orbits.}$$


secondary mass M_s ($M_\oplus$)
0.01
0.10
1.00
10.00
100.00
inclination damping timescale τ_i (P_s)
10^6
10^4
10^2
10^0
Q=2
Q=20
Q=200
Q=2000
Q=200
Q=20
Q=2
Q=200
Q=20
Q=2
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
secondary mass μ_s

IV. Summary of Results.

- An inclined planet will excite i at a vertical secular resonance.

- Disk self–gravity transports that disturbance radially away as a spiral bending wave.

- In a typical planet–forming environment having $\mu_p \sim \mu_{disk}$:

  - The disk's gravity drives the planet's nodal regression s_p & determines the location of the resonance, which can lie quite near the planet.

  - However gap formation slows the planet's node and pushes the resonance away, weakening the disk–planet interaction.

  - $i_{\text{self–grav.}} \ll i_{\text{non–grav.}}$

- The excitation of spiral bending waves damps i_p:
  - For terrestrial protoplanets & no gap, $\tau_i \sim 100$ orbits.
  - However $\tau_i \to$ very long if a gap forms in the planetesimal disk.
  - For giant planets embedded in the solar nebula, $\tau_i \sim 20$ orbits (no gap) and $\tau_i \sim 1000$ orbits if gap width $w \sim h$.
  - this i_p–damping mechanism will have important implications for the planet's accretion history,
  - and may be responsible for the giant planets' low mutual inclination of $\sim 1^\circ$.

- Are there other applications for nodal bending waves?
  - launched by small moonlets embedded in Saturns rings?
  - launched by satellite galaxies in stellar disks?

- Animations of N–ring simulations of spiral bending waves.