

Introduction

Asymmetries are often observed in circumstellar dust-disks like β Pictoris, ϵ Eridani, HD107146, etc. Two phenomena are often invoked to account these asymmetries:

- 1) Planetary perturbations, and
- 2) Asymmetric scattering of star-light by dust

Further, a disk's asymmetric appearance is wavelength dependent:

- At infrared wavelengths, a pericenter glow is observed due to stellar heating of dust at periaapse (Wyatt *et al.*, 1999)
- At optical wavelengths, an apocenter enhancement is observed due to star-light reflected by dust that is loitering at apoapse (Marsh *et al.*, 2005)

The aim of our research is to reproduce the inner $\sim 3^\circ$ warp seen in the β Pictoris dust-disk (Fig. 3). Since it is assumed that this warp is due to the presence of embedded planets, our goal then is to infer their masses and orbital parameters.

Planetary Perturbations

A broad circumstellar dust-disk that harbours a planetary system tends to be disturbed by the planets' *secular* perturbations. To assess this, it is convenient to first transform a grain's orbital elements $(e, \varpi, i, \Omega) \rightarrow (h, k, p, q)$ where

$$h = e \sin \varpi \quad k = e \cos \varpi \quad p = i \sin \Omega \quad q = i \cos \Omega$$

where p and q can be interpreted as projections of the angular momentum vector onto the equatorial plane. The Lagrange planetary equations govern how a grain's orbit evolves due to perturbations from N planets. What makes the secular planetary equations extremely attractive is the analytical form of the solutions. In general these solutions can be broken into a free and forced component, e.g.:

$$p(t) = i_{free} \sin(Bt + \gamma) + p_{forced}(t)$$

with

$$p_{forced}(t) = \sum_{j=1}^N C_j \sin(f_j t + \gamma_j)$$

where the C_j depends upon the masses and semi-major axes of the planets, f_j is the precession rate for planet j while B is the free precession rate. Similar expressions also exist for h , k and q . The i_{free} component in the expression above is due to the grain's nascent dispersion velocity, while the forced component is induced by the planets. So any radial or vertical structure in a disk is governed by the forced component of the dust grains' eccentricities e_{forced} and inclinations i_{forced} , respectively.

At this point we can quickly transform $(h, k, p, q) \rightarrow (x, y, z)$ for each dust grain, from which we can generate a synthetic image of the disk.

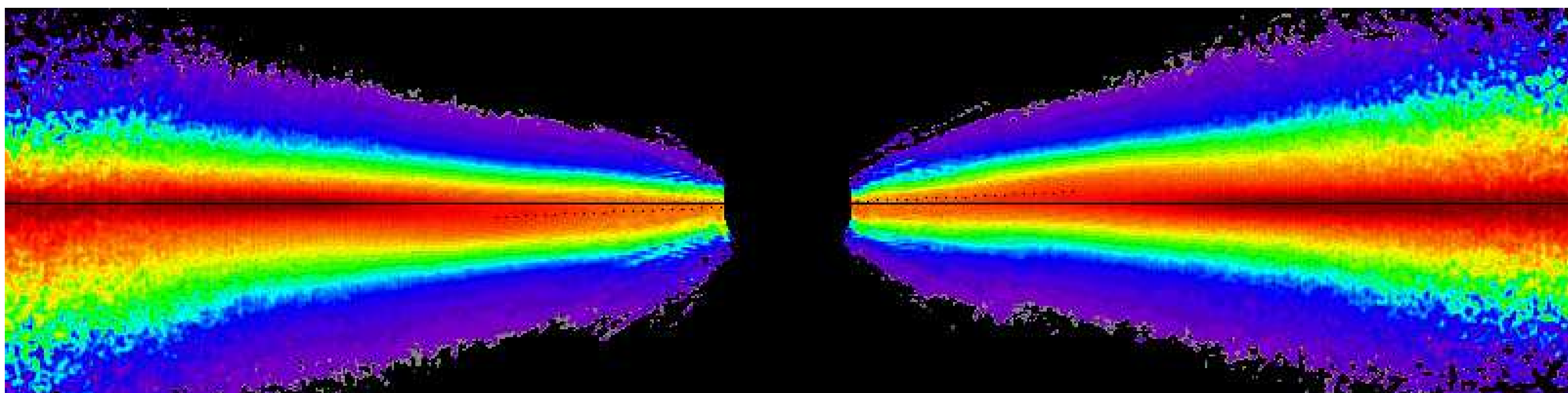


Figure 3: 400×100 AU of β Pictoris seen edge-on at optical wavelengths (Heap *et al.* 2000). The solid black line is the mid-plane of the disk for the star, and the dotted line inclined at 2.5° indicates the tilted inner $r \lesssim 100$ AU disk's mid-plane.

Asymmetric Light Scattering by Dust

Dust can also asymmetrically scatter star-light, which can impart a brightness asymmetry in the disk. Light scattered by interplanetary dust is usually described with a Henyey-Greenstein phase function:

$$\Psi(\phi) = \frac{1 - g^2}{4\pi(1 - 2g \cos \phi + g^2)^{3/2}}$$

where ϕ is the scattering angle and g is the scattering parameter (Hong, 1985). Where $g > 0$ indicates forward-scattering, while $g < 0$ indicates back-scattering. We will assume that circumstellar dust behaves like dust in the solar system, which is largely forward scattering (i.e. $g > 0$).

Caveats

Since this simple model is valid in the limit of negligible radial drift, we tacitly ignore:

- Radiation pressure
- Poynting-Robertson drag

This implies:

- The dust sizes considered here are relatively large e.g. $r_{dust} \gtrsim 1.0 \mu m$, below which radiation pressure would remove them from the system
- The dust-dust collision timescale is shorter than the PR drag timescale, which appears to be the case for the β Pictoris system

Models

Due to the analytical form of the solutions to the secular evolution equations, this allows us to:

- Rapidly generate synthetic images of dusty planetary systems
- Investigate a large range of the possible parameter space

This is significant since this problem is highly degenerate, however not all combinations of the parameters are plausible. Previous work by Mouillet *et al.*, 1997 demonstrated that a single planet was sufficient to explain the inner warp seen in β Pictoris (Fig. 3). While the initial conditions chosen for Mouillet's simulations are physical, they are not very plausible.

Since the progenitors of the dust grains (e.g. unseen, Kuiper belt-like objects) are expected to be distributed about the system's Laplace plane (i.e. the plane in which an orbit precesses), one would expect the dust grains to be similarly distributed about the same Laplace plane. However for Mouillet's single planet model, the Laplace plane is just the planet's orbital plane. Hence it is not feasible to produce a warp with just a single planet, at least *two* planets in non-coplanar orbits are required.

We now compare a new two planet model where the dust grains are initially placed in the system's Laplace plane, and a single planet model using initial conditions similar to Mouillet *et al.*, 1997. For both models, we use the following:

- $M_* = 1.8 M_\odot$
- A system age of 2.0×10^8 years
- The dust grains are symmetric light scatterers, i.e. $g = 0$
- 10^6 dust grains are initially distributed in semi-major axes with a broken power-law

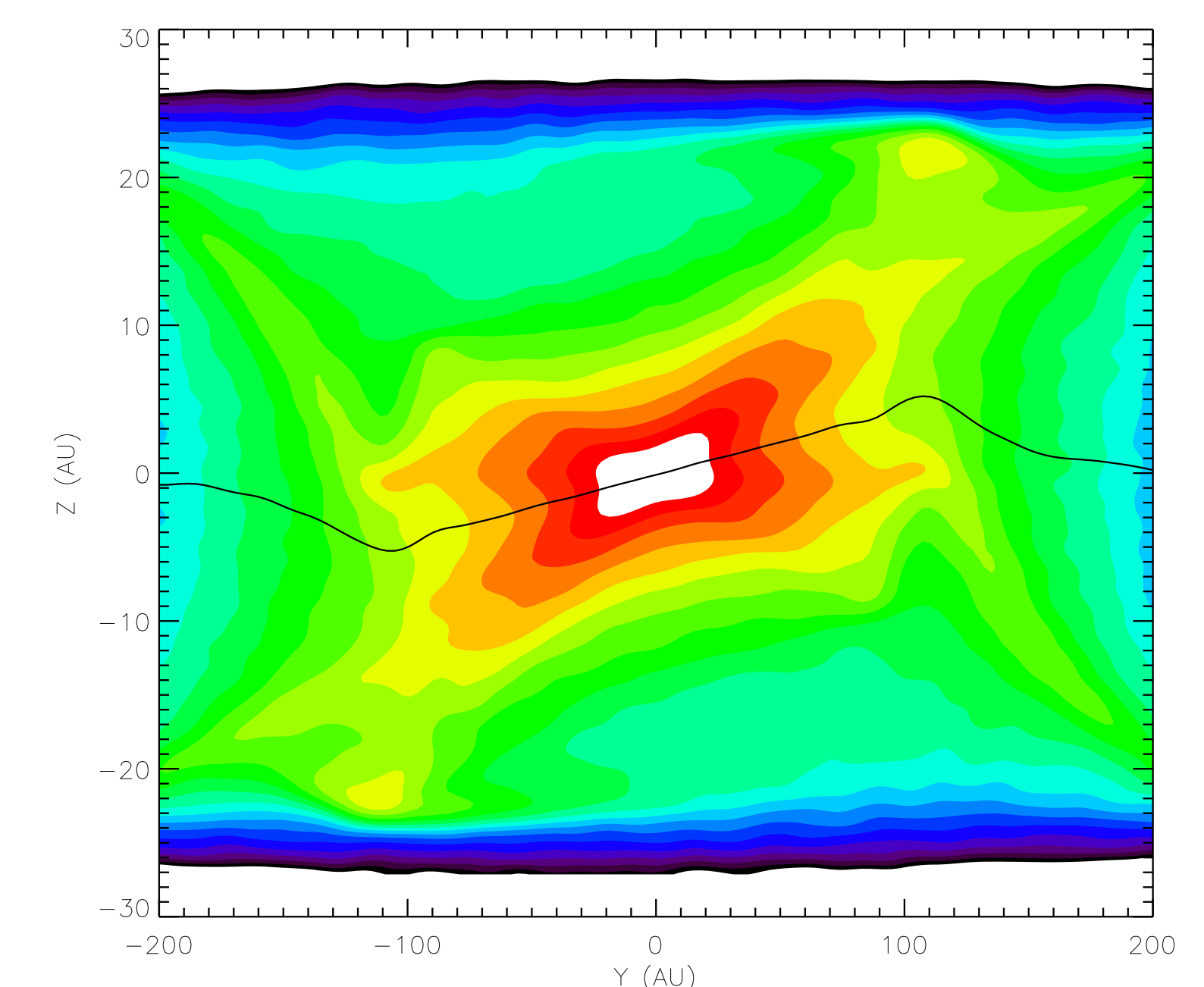


Figure 1: Column density map for a single planet model using initial conditions similar to Mouillet *et al.*, 1997. The dust grains are initially distributed about the equatorial plane on circular orbits, and given an aspect ratio $H/r = 0.1$ or inclination of 5.7° . The planetary parameters used in this simulation are: $M_{pl} = 2.5 M_J$, $a_{pl} = 5$ AU, $e_{pl} = 0.1$, and $i_{pl} = 3^\circ$. The black line is the apparent disk mid-plane. The data is plotted on a logarithmic scale.

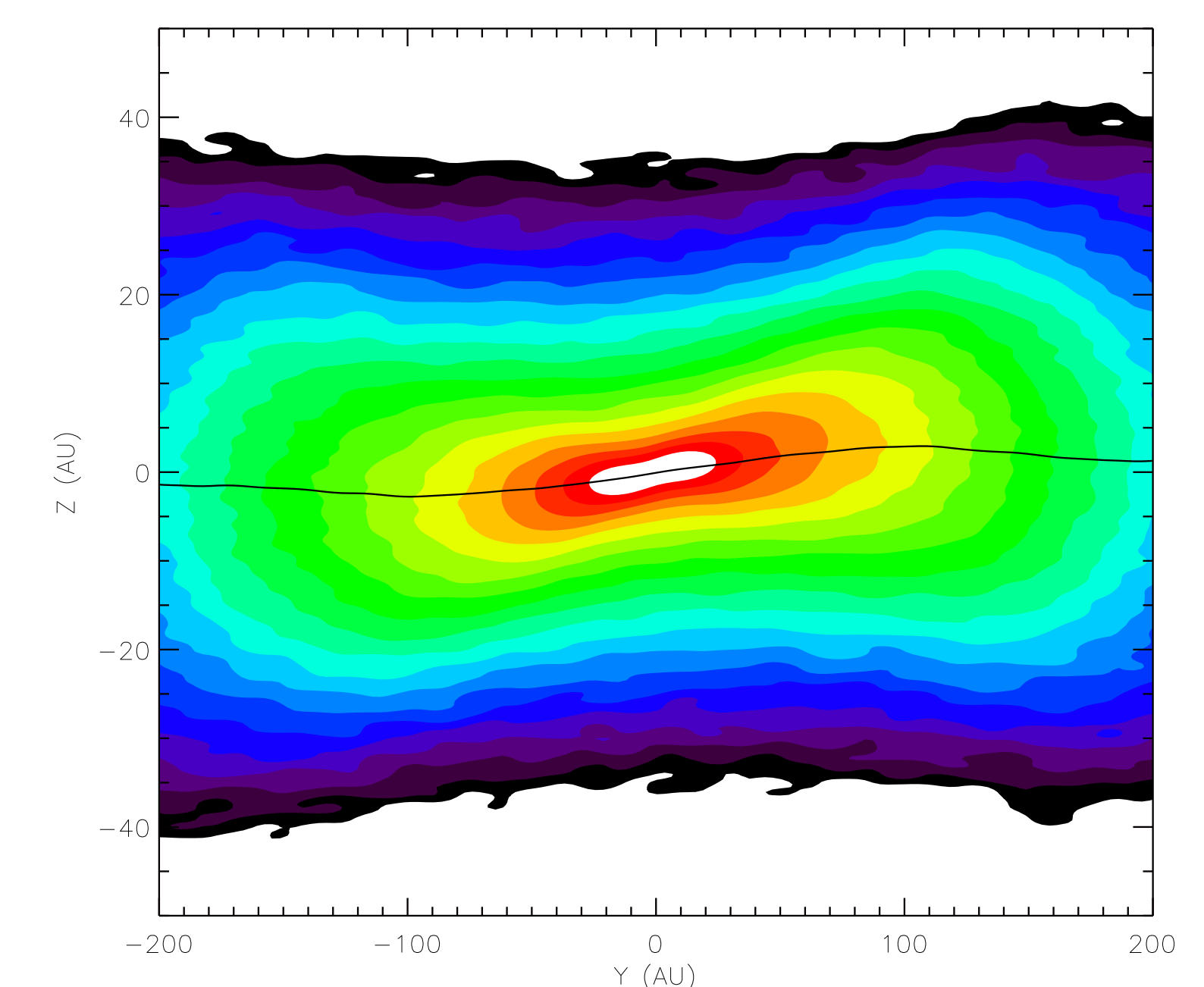


Figure 2: Column density map for a two planet model. The dust grains are initially distributed about the system's warped Laplace plane, which is due to the i_{forced} induced by both planets. The dust grains are also given eccentricity and inclination dispersions with median values of $e = 0.21$ and $i = 6.1^\circ$, respectively. The planetary parameters used in this simulation are: $M_{pl} = (5 M_J, 2.5 M_J)$, $a_{pl} = (70 \text{ AU}, 200 \text{ AU})$, $e_{pl} = (0.1, 0.01)$, and $i_{pl} = (3^\circ, 0.3^\circ)$.

Findings and Future Activities

Both planetary perturbations and asymmetric light scattering may be contributing to the asymmetries seen in dusty circumstellar disks such as β Pictoris. With proper initial conditions and our secular model, we can accurately model such systems quickly and easily.

By taking into account more plausible initial conditions, it necessary to include two planets (Fig. 2) to reproduce a system similar in appearance to β Pictoris (Fig. 3). While the Mouillet model does reproduce the warp seen around $\sim 80 - 100$ AU in β Pictoris, this is an artifact of the artificial initial conditions.

Our next goal will be to develop a fitting routine and a robust parameter searching algorithm to localize the best fitting models for β Pictoris and other such systems.