The Secular Evolution of the Primordial Kuiper Belt

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1 Introduction

The Kuiper Belt is a vast swarm of comets orbiting at the Solar System's outer edge. This Belt is comprised of debris that was left over from the epoch of planet formation, and this swarm's distribution of orbit elements preserves a record of events that had occurred when the Solar System was still quite young. One goal of this study is to decipher this Kuiper Belt record, which to date remains quite open to interpretation...

Figure 1 shows the orbits of the known Kuiper Belt Objects (KBOs), and reveals the Belt's three primary populations:

- the Plutino's which inhabit Neptune's 3:2 resonance
- the Main Belt KBOs between the 3:2 and the 2:1
- the Scattered Disk near the Neptune–crossing curve

Evidently, some process has stirred up the KBO orbits since the time of formation. Scattering by Neptune is likely responsible for lofting KBOs into eccentric, inclined orbits in the Scattered Disk (Duncan and Levison 1997), and the outward migration by Neptune will capture numerous KBOs at its 3:2 and 2:1 resonances (Malhotra 1995). However perturbations from a migrating Neptune are unable to account for the KBO's high inclinations of $i \sim 15°$.

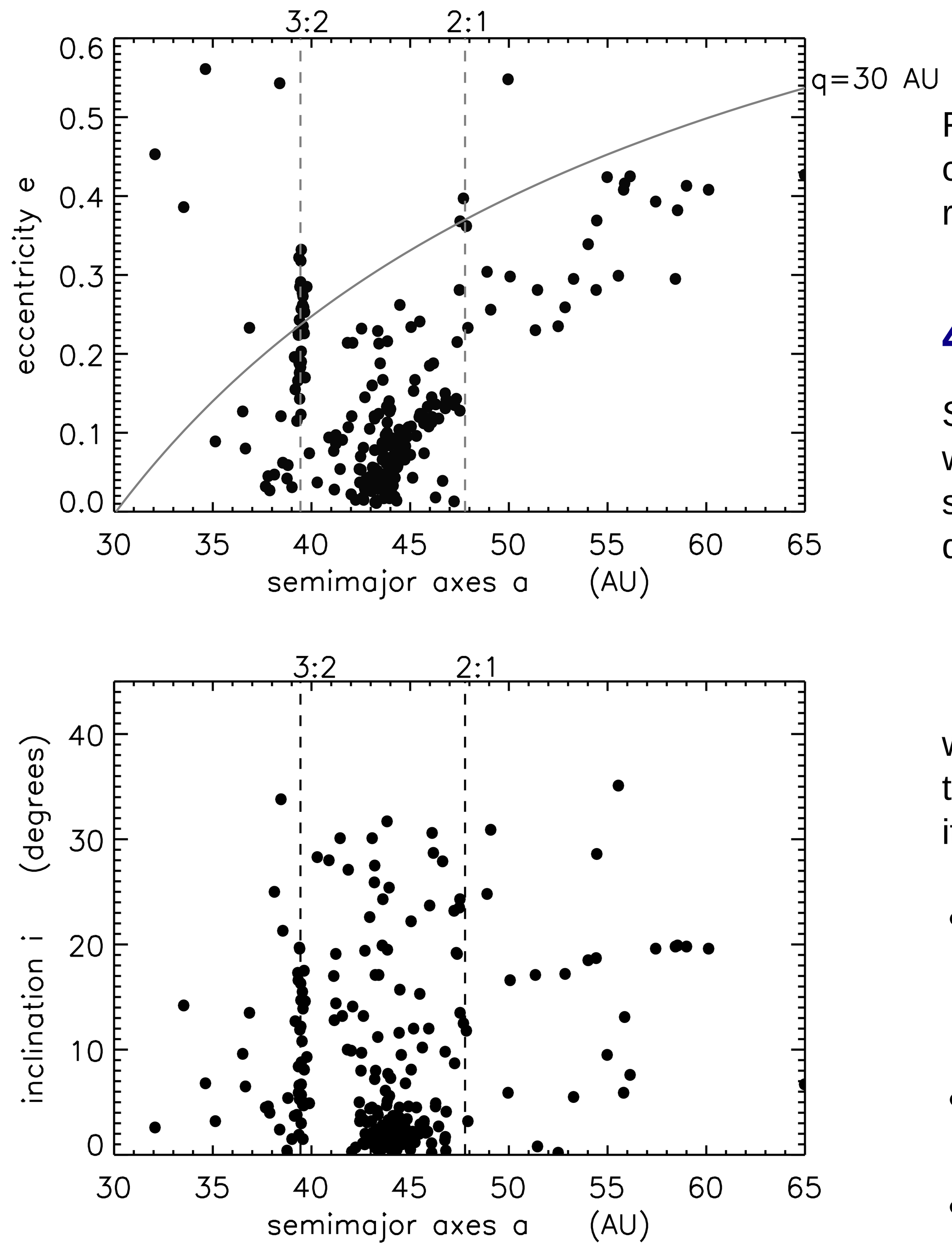


Figure 1: KBO eccentricities e and inclinations i versus semimajor axes a reported by the Minor Planet Center. Dashed lines are Neptune's 2:1 and 3:2 resonances, and orbits above the $q = 30$ AU curve are Neptune–crossing.

2 Secular Perturbations

The following shall consider the giant planets' secular perturbations, and asses their role in stirring up the Belt.

- secular perturbations are the slowly–varying gravitational forces that drive the precession, or rotation, of orbits
- secular resonances are sites in a disk where a perturber can excite large e's and i's
- in a *gravitating* particle disk, this e–disturbance can propagate away from resonance as a spiral density wave [aka, apsidal wave (Ward and Hahn 1998)]
- the i–disturbance can propagate away as a spiral bending (or nodal) wave (Ward and Hahn 2003)

3 The Rings Model

Spiral waves in the Belt will be studied using a *rings model*

- treat the Belt as a nested set of interacting rings of mass m_j, orbits $(a_j, e_j, i_j, \tilde{\omega}_j, \Omega_j)$ and thickness h_j due to their particles dispersion velocities $c_j = h_j n_j$. See Figure 2
- the planets are thin $h_j = 0$ rings
- evolve the system as per the Lagrange planetary eqn's
  - apply the well known Laplace–Lagrange solution (Murray & Dermott 1999) to obtain the system's evolution
  - however note that the rings' finite thickness h softens their gravitational potential, which also softens the solution's Laplace coefficients $b_s^{(m)}(\alpha)$ over the scale h/a.

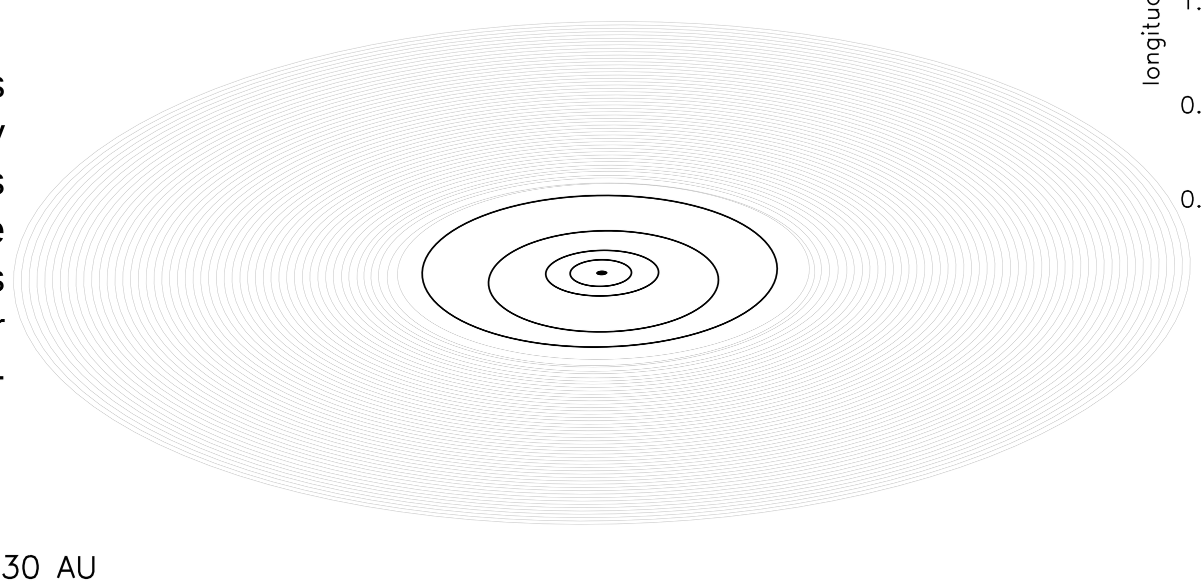


Figure 2: A typical disk–planet configuration. The heavy curves are the giant planet's orbits with the Kuiper Belt represented by 500 rings

4 Spiral Wave Theory

Solutions to this system's Lagrange equations are spiral waves, which have properties that are governed by dispersion relations, also derived in Hahn (2003). For apsidal density waves, the dispersion relation is

$$\omega_e(k) = \frac{\pi G \sigma}{n} e^{-|hk|}|k| \quad (1)$$

where ω_e is the pattern speed, k is the wavenumber, σ is the disk surface density, and n is the disk's angular velocity. These waves have the following properties:

- there are two wave solutions:
  - a long wavelength solution $\lambda_L \simeq 2\pi(\frac{\pi\sigma a^2}{M_\odot})|\frac{n}{\omega_e}|a$
  - a short wave solution $\lambda_S \lesssim 2\pi h$
- apsidal waves only propagate in sufficiently thin disks where $h < h_Q$ where $h_Q \simeq 0.3(\frac{\pi\sigma a^2}{M_\odot})|\frac{n}{\omega_e}|a \simeq \lambda_L/24$
- apsidal waves will *reflect* if they encounter a site where $h = h_Q$; this site is also called a Q–barrier

The nodal bending waves have a dispersion relation

$$\omega_i(k) = -\frac{\pi G \sigma}{hn}(1 - e^{-|hk|}) \quad (2)$$

- there is only a *single* wave solution λ_L
- nodal waves *stall* as they approach a site where $h \simeq 3h_Q$

5 Simulations of Waves in the Kuiper Belt

The rings model is used to simulate the propagation of apsidal and nodal waves in an early, massive Kuiper Belt. The Belt shown below is composed of 500 rings having a total mass $M_{KB} = 15$ M$_\oplus$ distributed over a thin $h/a = 0.01$ disk of radii $36 < a < 70$ AU. Animations provided on a PDA show spiral density and bending waves being launched at the Belt's inner edge, reflecting at the disk's edges, and forever sloshing about the Kuiper Belt. Figure 3 shows a snapshot of this system at time $t = 2$ million years.

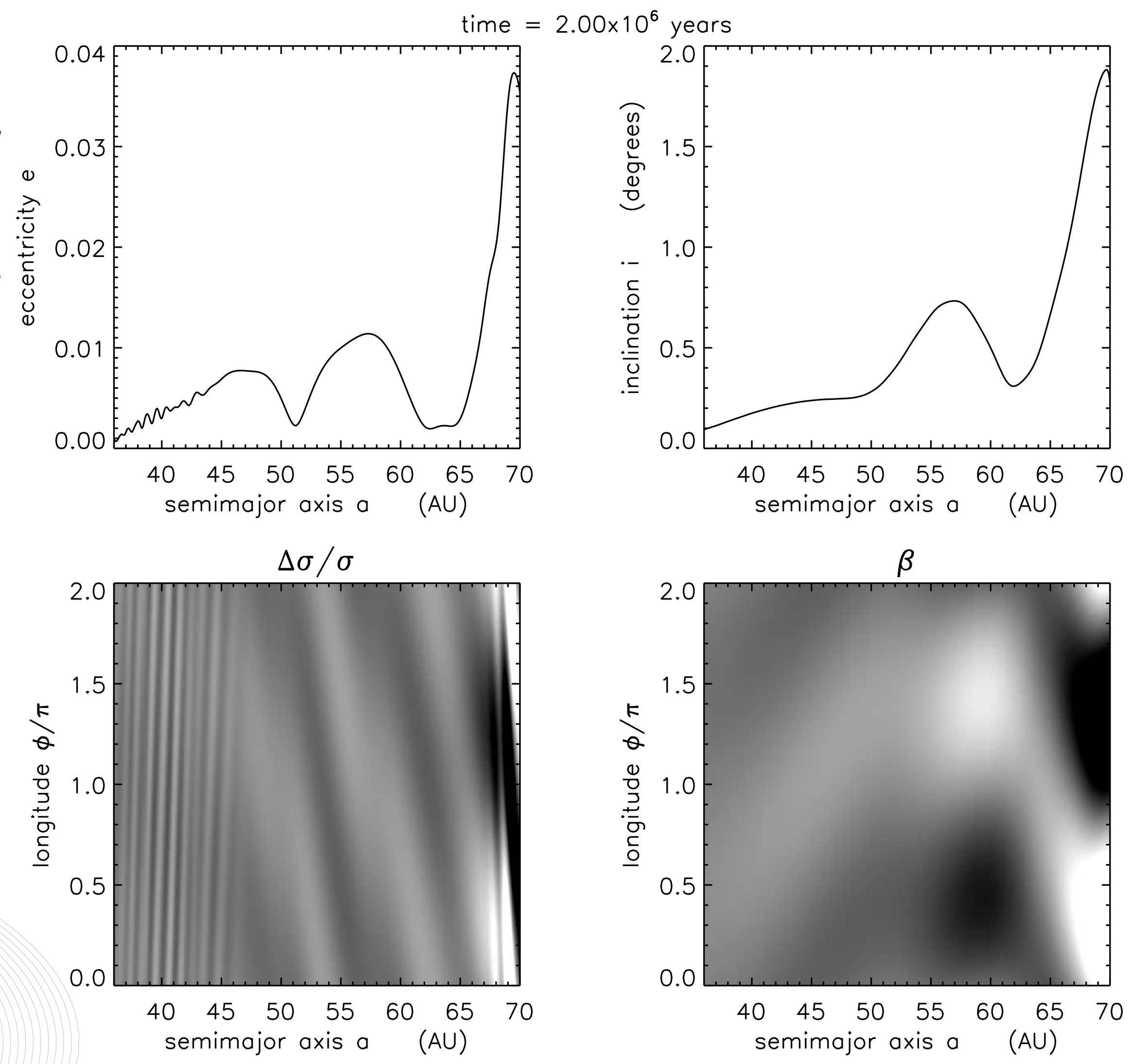


Figure 3: The upper curves show the disk's e and i versus a, while the lower greyscales show the fractional changes in surface density $\Delta\sigma/\sigma$ due to the density waves, as well as the disk's latitude variations $\beta = z/a$ due to the spiral bending waves. By time $t = 2$ million years, these waves have just reflected at the disk's outer edge. Both long and short density waves are evident in the left greyscale, with the reflected short wave being nonlinear, *i.e.*, $\Delta\sigma/\sigma > 1$.

6 Variations with Kuiper Belt Mass

The rings model is also used to simulate waves propagating in thin Kuiper Belt's having masses ranging over $M_{KB} = 30$ M$_\oplus$ (the Belt's primordial mass) down to $M_{KB} = 0.08$ M$_\oplus$ ($\sim 40\%$ of the Belt's current mass). Results are displayed in Figure 4, with the main findings being:

- the giant planets deposit $\sim 1\%$ of their e-AMD and $\sim 10\%$ of their i-AMD into the disk in the form of spiral waves
- thus lower–mass disk's experience larger e's and i's
- Figure 4 suggests that waves in a $M_{KB} = 0.2$ M$_\oplus$ might account for the Belt's excited state—but probably not—see Section 8

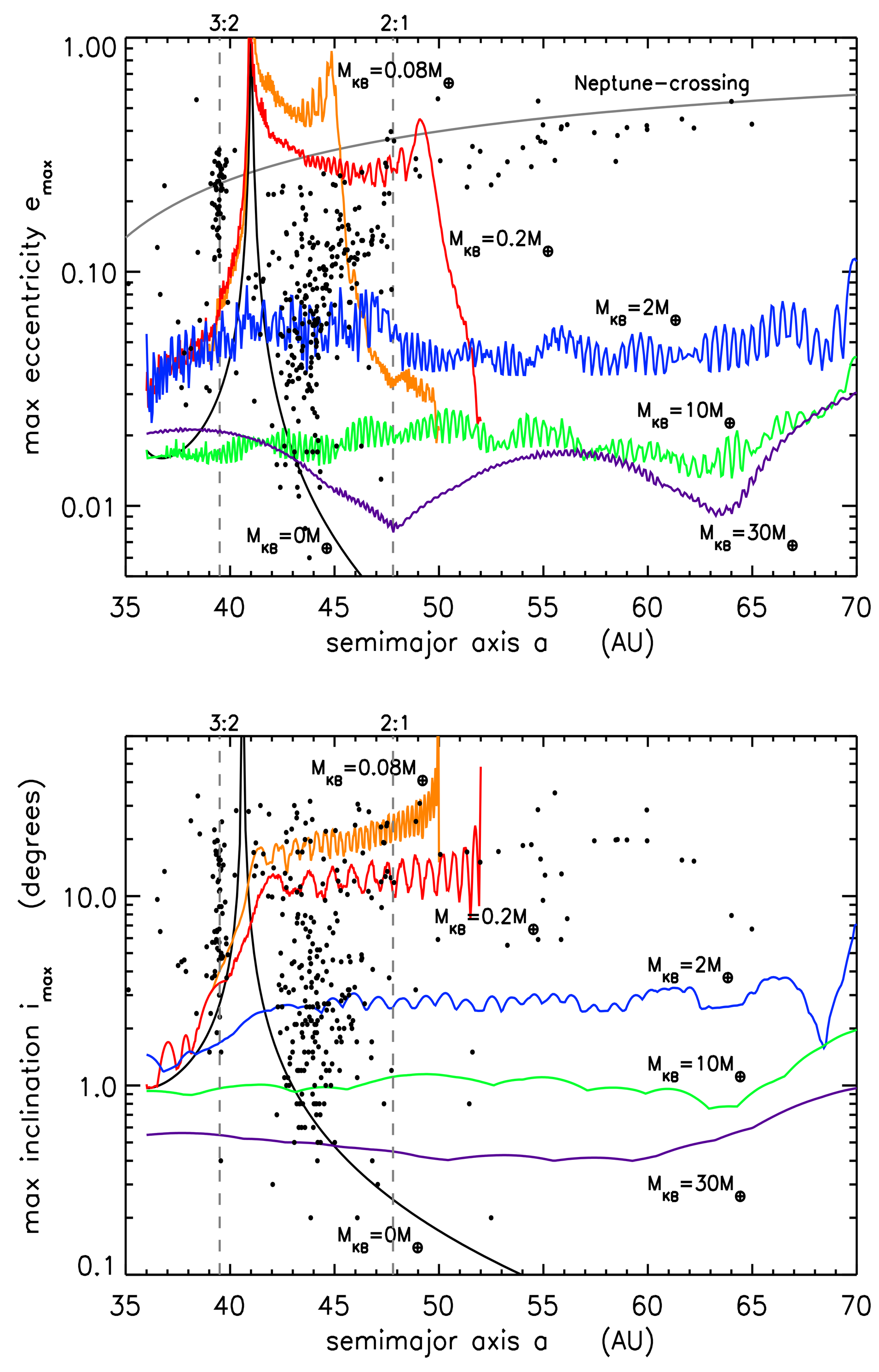


Figure 4: The amplitudes of the spiral waves propagating in thin $h/a = 0.002$ Kuiper Belts having the indicated masses. Dots are the observed KBO orbit elements.

7 Stalling Spiral Bending Waves

Nodal bending waves have an interesting property—they *stall* as they approach a site in the disk where $h \simeq 3h_Q$. This phenomenon is illustrated in Figure 5

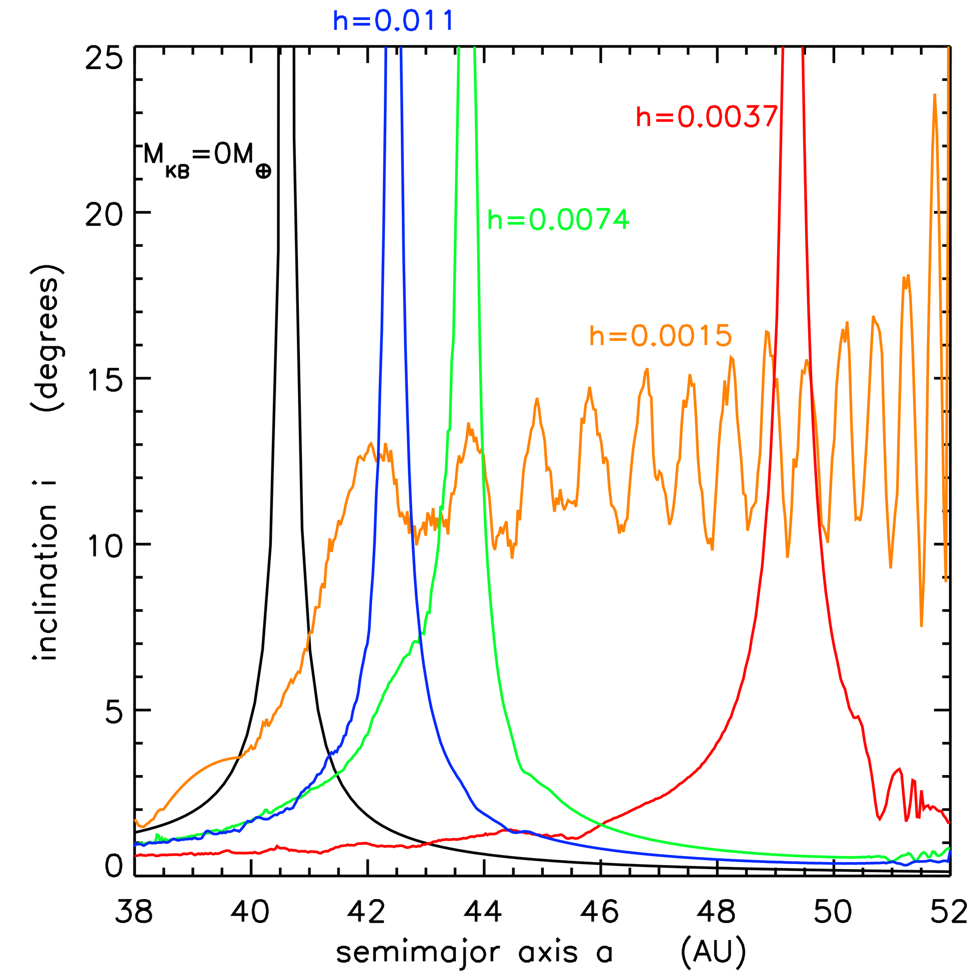
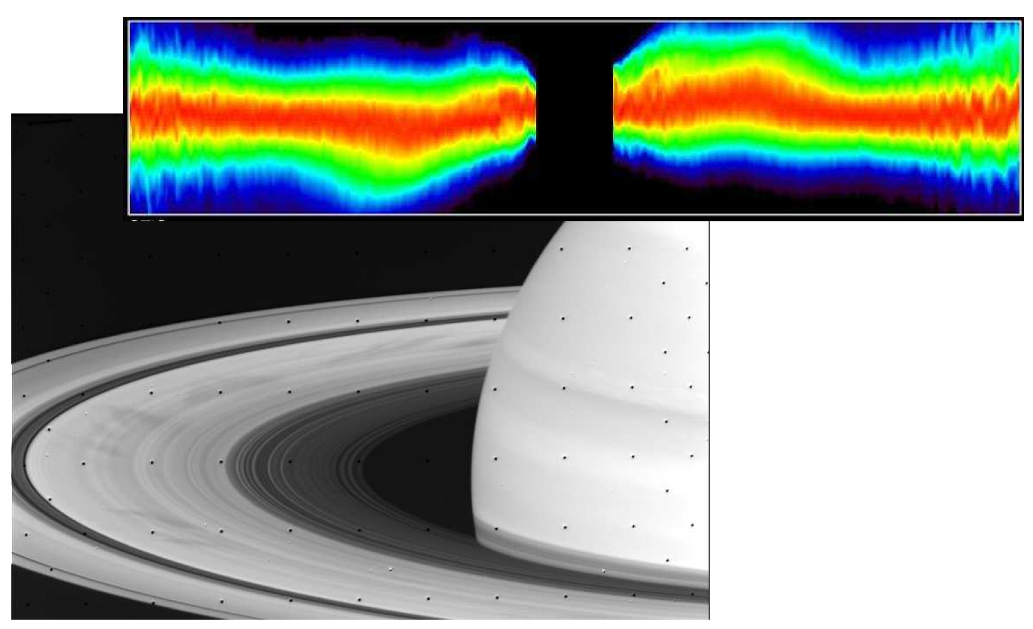


Figure 5: Spiral bending waves are launched in thin Kuiper Belt's of mass $M_{KB} = 0.2$ M$_\oplus$ having the indicated thickness h in units of a. This disk has a critical thickness $h_Q = 0.002a$, and the orange curve shows the amplitude of waves as they propagates everywhere since $h < h_Q$. The other curves are for thicker disks having $h > h_Q$, with the peaks indicating the sites where the waves stall and dump their angular momentum to the disk.

8 Implications for the Kuiper Belt

- wave–action is possible only in thin disks obeying $h \lesssim h_Q$
- the primordial Kuiper Belt had a mass $M_{KB} \sim 30$ M$_\oplus$ and $h_Q \sim 0.1a$ and thus could easily easily sustain waves
- however the formation of large $R \sim 100+$ km KBOs triggers collisions that erode the Belt down to $M_{KB} \rightarrow 0.2$ M$_\oplus$ (Kenyon & Luu 1999) which reduces $h_Q \rightarrow 0.001a$
  - formation of large KBOs likely pumped the disk thickness above the critical limit $h > h_Q$ and shut off waves
- this epoch of wave–action likely lasted during the first $\sim 10^7$ years (when the large KBOs first formed) and $\sim 5 \times 10^8$ years (when collisions eroded the Belt's mass)

9 Other Applications of the Rings Model



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