On The Mathematical Depiction of The Solar Wind Speed-Solar Magnetic Field Relationship

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SOLAR WIND

Parker 1958
solar wind model
above a reference height, radially directed
solar wind totally
controls the magnetic
field

determine this reference height to quantitatively model background IMF & solar wind speed

SOLAR WIND

supersonic expansion of the solar corona out into the heliosphere

300 < speed < 450 km/s: slow solar wind, originating from the vicinity of closed magnetic field regions

450 > speed < 850 km/s: fast solar wind, originating from coronal holes — open magnetic field regions

faster speed detected — associated with CMEs

SOLAR WIND ORIGIN

Fast wind > 450 km/s:
coronal holes open magnetic field region
Slow < 450 km/s:
near streamers – closed magnetic field

Y. M. Wang & N. R. Sheeley, 1990s
All the solar wind originate from coronal holes
fast wind – center
slow wind – near the boundaries

SOLAR WIND ORIGIN

solar wind speed α 1/fte

$$fte = \left(\frac{R_{phot}}{R_{ss}}\right)^2 \frac{B_{r(phot)}}{B_{r(ss)}}$$

fte – flux tube expansion factor – between photosphere and source surface;

R_{phot}; R_{ss} – radii of photosphere & source surface

$B_{r(phot)}$; $B_{r(ss)}$ – magnetic field

SOLAR WIND ORIGIN

WSA: Arge & Pizzo, JGR, 105, 2000

$$v = 265.0 + (1.5/(1+f_s)^{1/2.5}) * (5.8 - 4.0 * \exp(\vartheta_b 2.5)^2))^3$$

(from McGregor et al., JGR, **113**, 2008)

f_s - flux expansion factor
ϑ_b - angular distance of the magnetic field footpoint from the nearest coronal hole boundary

WSA-ENLIL

ENLIL: state-of-the-art space weather prediction model of NOAA - Space Weather Prediction Center

WSA provides ambient solar wind at the inner boundary of ENLIL

1 - 4 day advance warnings of geomagnetic storms caused by earth-directed CMEs & quasi-recurrent solar wind structures

Error: 1-2 days

WSA-ENLIL

major single source:

WSA background solar wind, due to intrinsic flaws in PFSS model (Pizzo et al., Space weather, 2012)

other:

quality of the photospheric synoptic map - input to the PFSS model

reduce error & improve inner boundary conditions of ENLIL

Why should one care about space weather?

Why is space weather forecast important?

SPACE WEATHER

changing conditions in the interplanetary medium causing disruptions to technological systems on Earth and nearby space

SPACE WEATHER

system of connected physical processes manifesting as a multitude of near-Earth disturbances

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includes studies on: **solar activity/solar wind, aurorae, geomagnetic disturbances, …,**

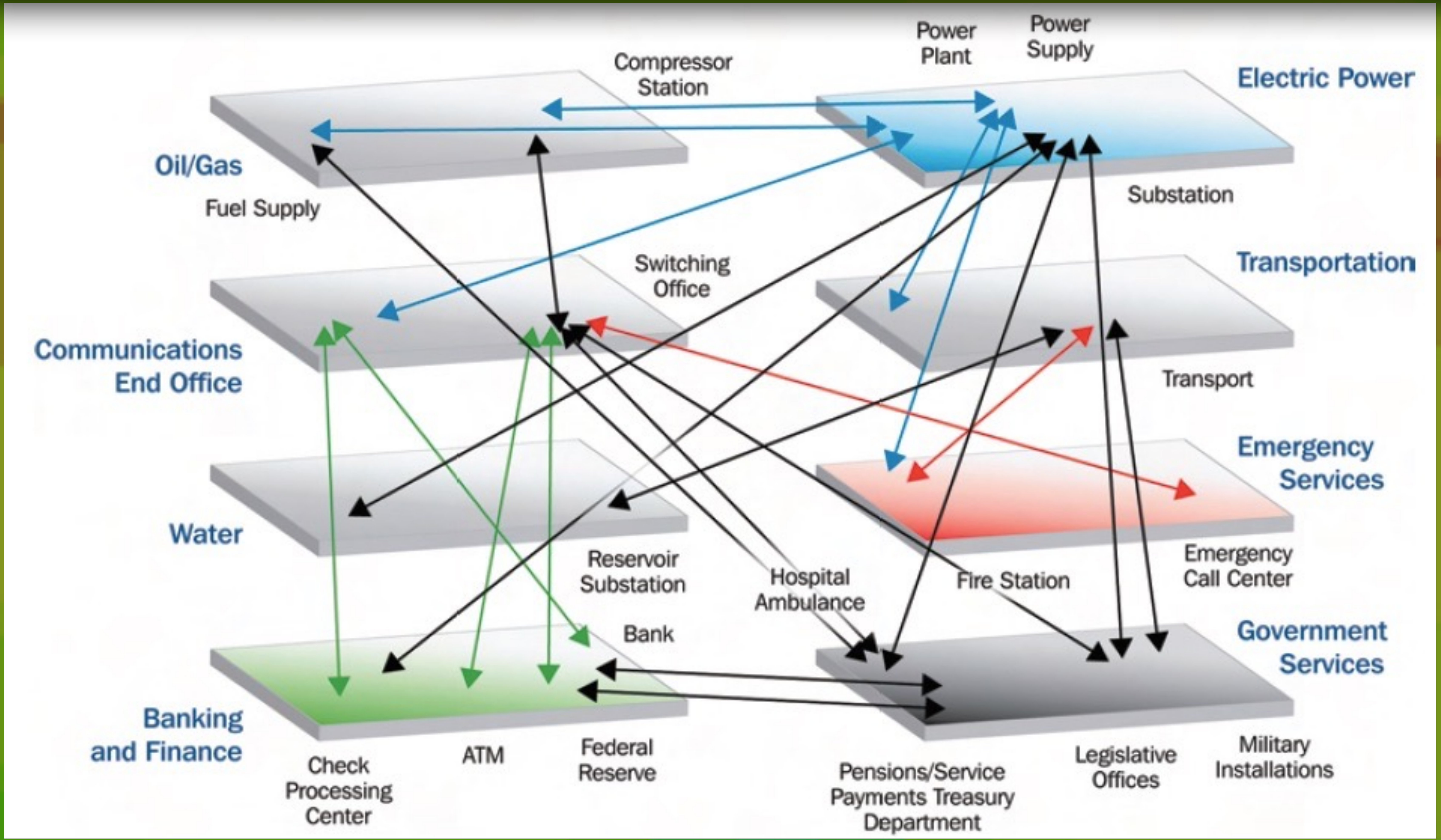
and their interrelationships

SOCIOECONOMIC IMPACT

Impact Area	Customer (examples)	Action (examples)	Cost (examples)
Spacecraft (Individual systems to complete spacecraft failure; communications and radiation effects)	• Lockheed Martin • Orbital • Boeing • Space Systems Loral • NASA, DoD	• Postpone launch • In orbit - Reboot systems • Turn off/safe instruments and/or spacecraft	• Loss of spacecraft ~$500M • Commercial loss exceeds $1B • Worst case storm - $100B
Electric Power (Equipment damage to electrical grid failure and blackout conditions)	• U.S. Nuclear Regulatory Commission • N. America Electric Reliability Corp. • Allegheny Power • New York Power Authority	• Adjust/reduce system load • Disconnect components • Postpone maintenance	• Estimated loss ~$400M from unexpected geomagnetic storms • $3-6B loss in GDP (blackout)
Airlines (Communications) (Loss of flight HF radio communications) (Radiation dose to crew and passengers)	• United Airlines • Lufthansa • Continental Airlines • Korean Airlines • NavCanada (Air Traffic Control)	• Divert polar flights • Change flight plans • Change altitude • Select alternate communications	• Cost ~ $100k per diverted flight • $10-50k for re-routes • Health risks
Surveying and Navigation (Use of magnetic field or GPS could be impacted)	• FAA-WAAS • Dept. of Transportation • BP Alaska and Schlumberger	• Postpone activities • Redo survey • Use backup systems	• From $50k to $1M daily for single company

Severe Space Weather Events: Understanding Societal and Economic Impacts: A Workshop Report —The National Academies Press (22 May 2008)

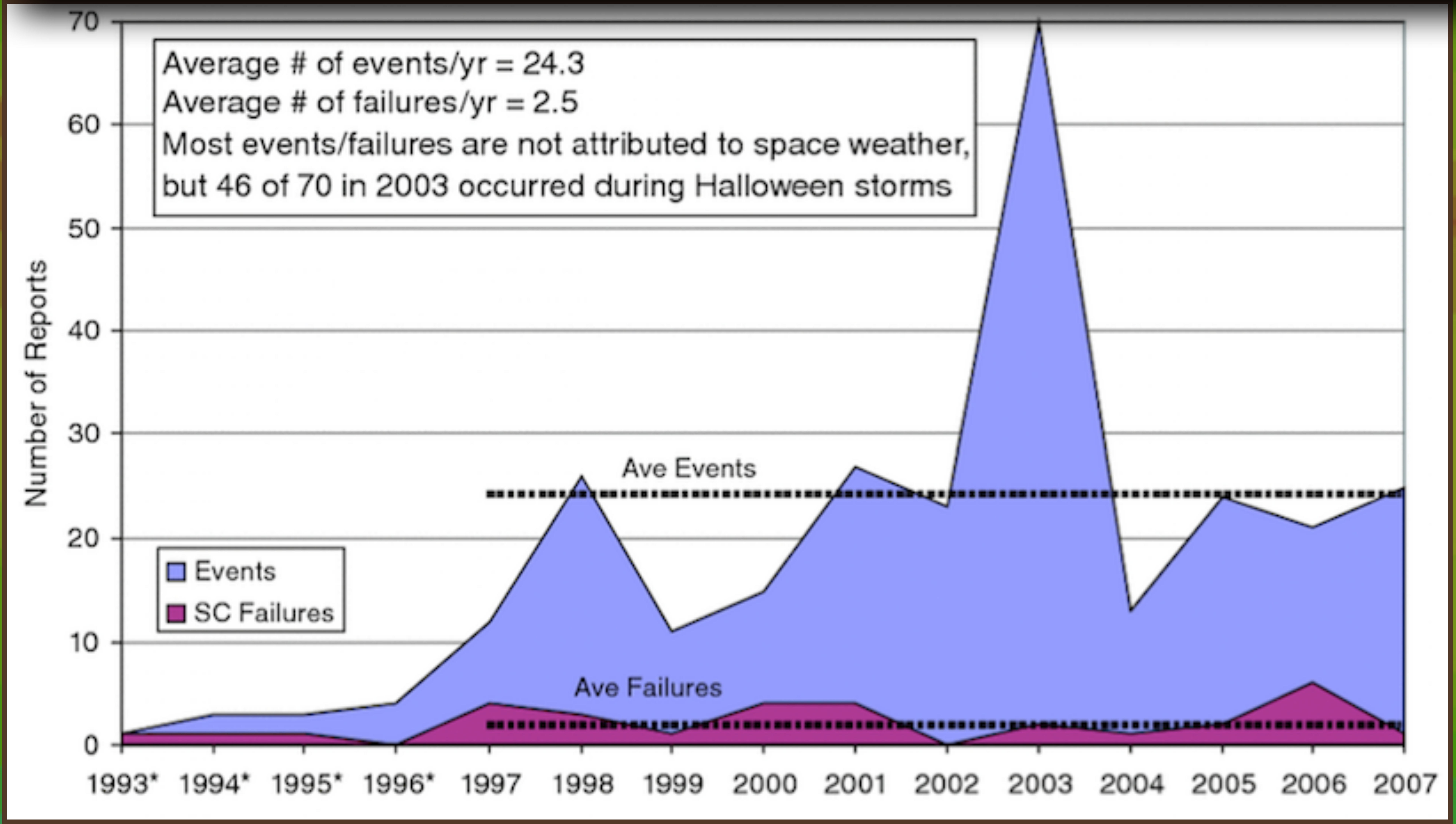
SOCIOECONOMIC IMPACT



interconnected infrastructures & their qualitative dependencies and interdependencies

Source: Department of Homeland Security, National Infrastructure Protection Plan (http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm).

SOCIOECONOMIC IMPACT



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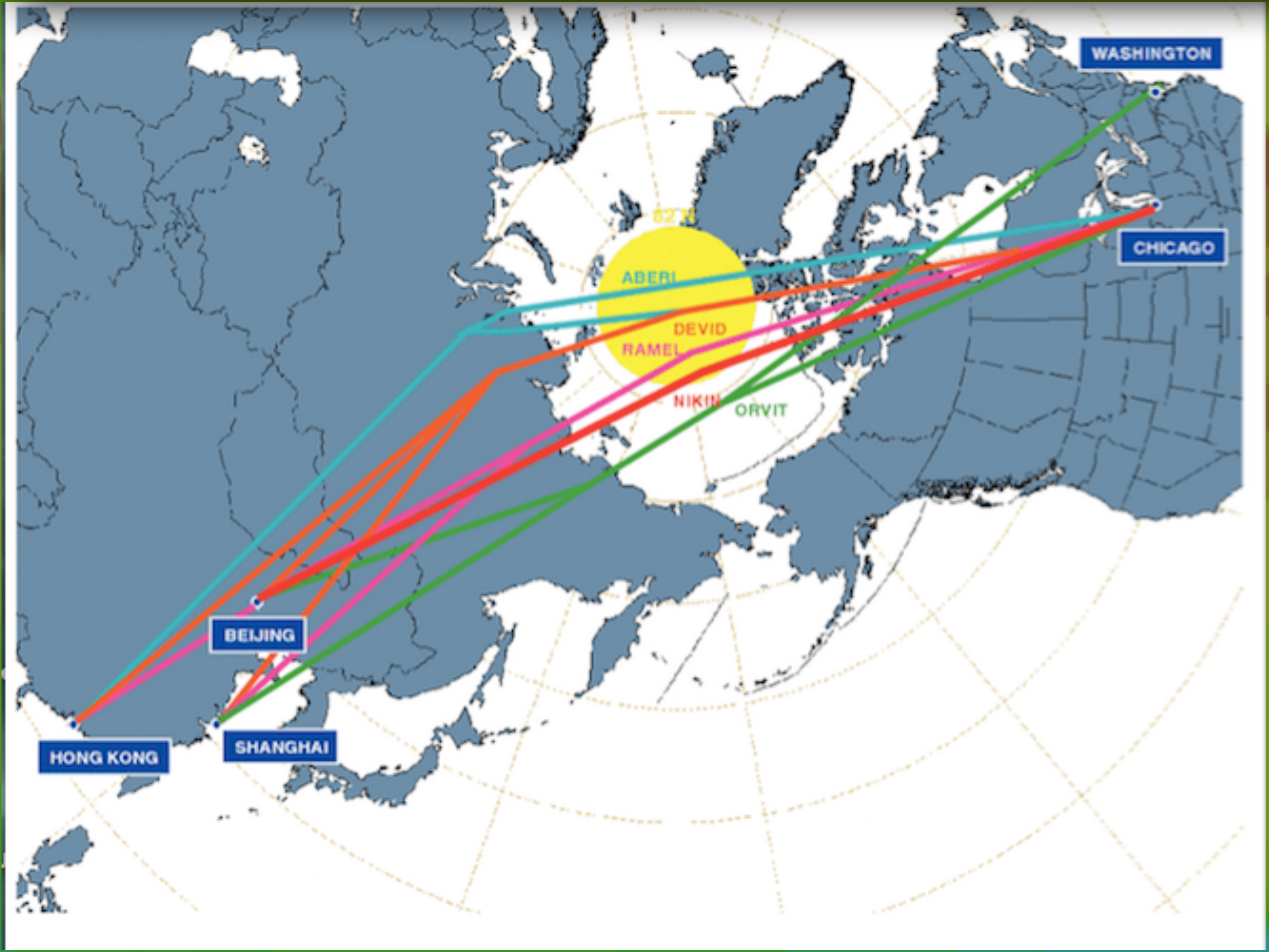
SOCIOECONOMIC IMPACT



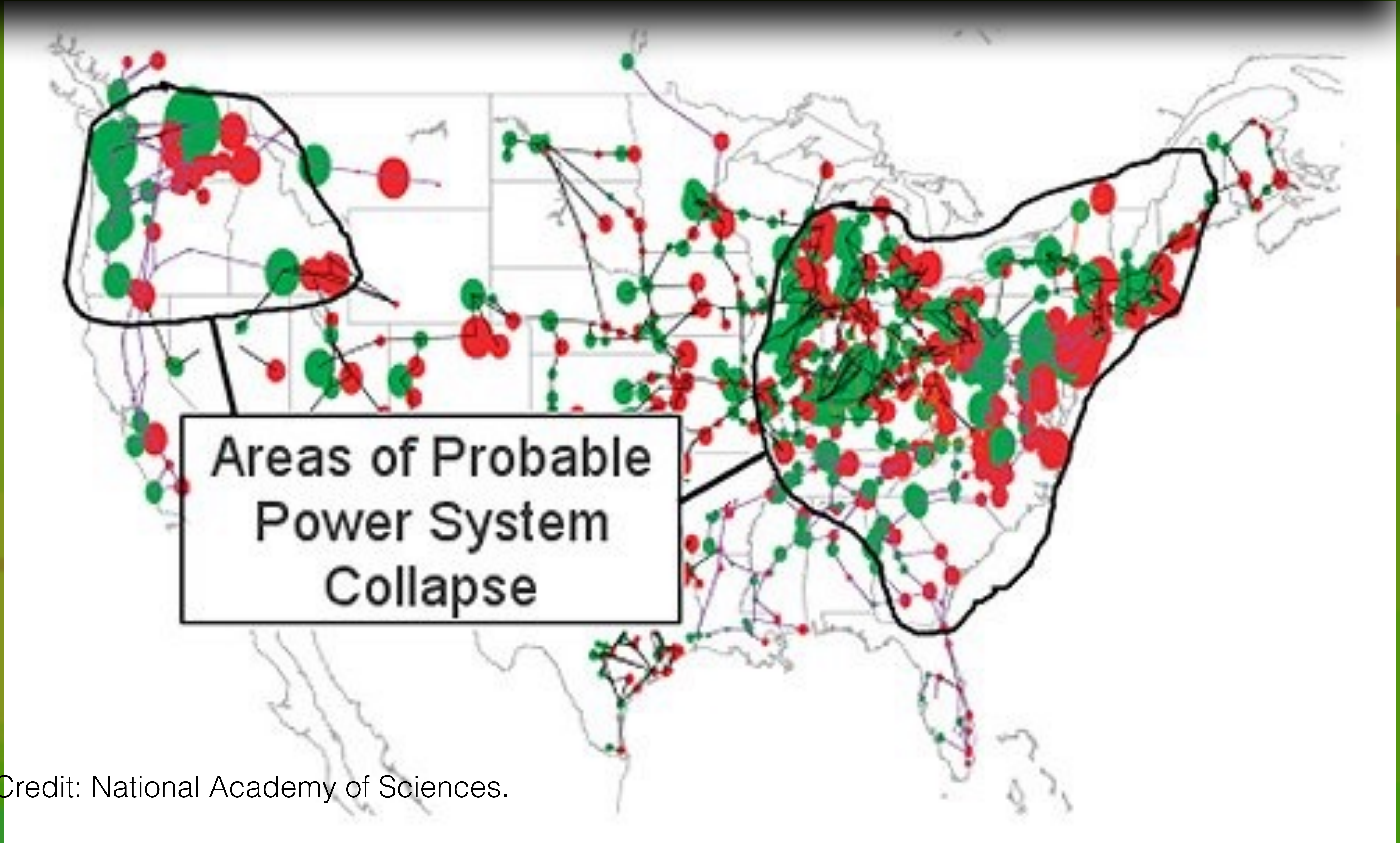
transpolar flights rely on HF radio communications

magnetic storm/polar cap absorption (PCA) — cause ionospheric density disturbances interfere with HF, VHF, UHF radio communications and navigation signals from GPS satellites

Severe Space Weather Events: Understanding Societal and Economic Impacts: A Workshop Report —The National Academies Press (22 May 2008)

SOCIOECONOMIC IMPACT

installation of supplemental transformer neutral ground resistors to reduce GIC flows — inexpensive & low engineering trade-offs — produce 60-70% reductions of GIC levels for storms of all sizes



Credit: National Academy of Sciences.

Severe Space Weather Events: Understanding Societal and Economic Impacts: A Workshop Report —The National Academies Press (22 May 2008)

RECALL THAT:

solar wind speed α 1/fte

$$fte = \left(\frac{R_{phot}}{R_{ss}}\right)^2 \frac{B_{r(phot)}}{B_{r(ss)}}$$

fte – flux tube expansion factor – between photosphere and source surface;

R_{phot}; R_{ss} – radii of photosphere & source surface

$B_{r(phot)}$; $B_{r(ss)}$ – magnetic field

WHY CORONAL MODELS?

Direct observations of coronal magnetic field challenging and limited (e. g. using CoMP: Dove et al., ApJ, 731, 2011; Bak-Steslicka et al., ApJL, 770, 2013)

Models that extrapolate observed photospheric magnetic field into the corona and beyond.

CORONAL MODELS

- Potential Field Source Surface (PFSS) model
- NonLinear Force Free (NLFFF) model
- Current Sheet Source Surface (CSSS) model
- Magnetohydrodynamic (MHD) models

CORONAL MODELS

Potential Field Source Surface (PFSS) model

Schatten et al., 1969; Altschuler & Newkirk, 1969

coronal magnetic field - computed from scalar potential obeying LaPlace's law

Current Sheet Source Surface (CSSS) model

Zhao & Hoeksema, 1995

PFSS MODEL

popular – addresses a variety of problems

Schrijver & DeRosa, 2003;
Luhmann et al., 2009,
Wang & Shelley, 1990, 1992, 1995, etc..

CSSS MODEL

BOGDAN & LOW 1986
obtained solution to magnetostatic equilibrium — electric currents flowing perpendicular to gravity ($1/r^2$) everywhere

$$J = \frac{1}{\mu_0 r}[1-\eta(r)]\left[\frac{1}{sin(\theta)}\frac{\partial^2\phi}{\partial\phi\partial r}\hat{\phi} - \frac{\partial^2\phi}{\partial\theta\partial r}\hat{\phi}\right] \quad (1)$$

and

$$B = -\eta(r)\frac{\partial\phi}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} - \frac{1}{sin(\theta)}\frac{\partial\phi}{\partial\phi}\hat{\phi} \quad (2)$$

where, μ_0 is the magnetic permeability, $\eta(r) = 1 + (a/r)^2$ and $\phi(r,\theta,\phi)$ is a scalar function determined by the boundary conditions at the photosphere and corona (Zhao and Hoeksema, 1995).

Bogdan & Low
ApJ 306, 271-283, 1986

CSSS MODEL

using spherical harmonic expansion & source surface technique

Zhao & Hoeksema (JGR, 100, 99, 1995) developed CSSS model

- volume & sheet currents

- source surface

CSSS MODEL - GEOMETRY

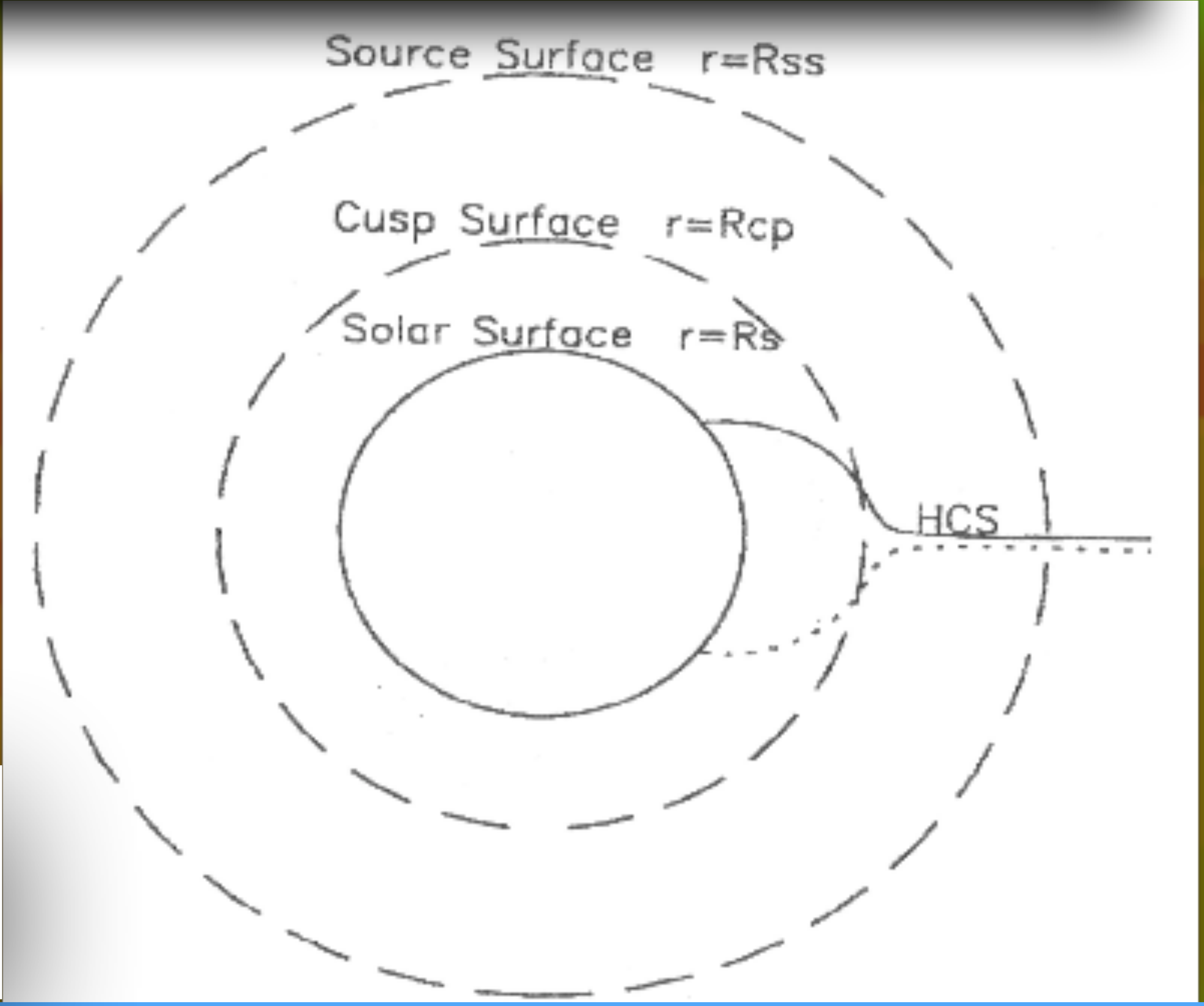
inner region

$$\Phi = \sum_{n=1}^{N_\odot} \sum_{m=0}^{n} R_n^\odot(r) P_n^m(\cos\theta)(g_{nm}^\odot \cos m\phi + h_{nm}^\odot \sin m\phi) \quad (3)$$

$$R_n^\odot(r) = \frac{R_\odot(1+a)^n}{(n+1)(r+a)^{n+1}} \quad (4)$$

middle region

$$\Phi = \sum_{n=0}^{N_c} \sum_{m=0}^{n} R_n^c(r) P_n^m(\cos\theta)(g_{nm}^c \cos m\phi + h_{nm}^c \sin m\phi)$$



outer region: extrapolate B out to heliosphere because

$$B_\theta(R_{ss}, \theta_{ss}, \phi_{ss}) = B_\phi(R_{ss}, \theta_{ss}, \phi_{ss}) = 0$$

CSSS MODEL

<u>potential field - over simplification</u>

because corona not strictly current free

large-scale plasma structures above 1.5 R_{sun} indicate magnetic field-electric currents interaction

PFSS limitations:

cause uncertainties in footpoint locations of solar wind source regions — a few tens of degrees in longitude (Poduval & Zhao, JGR 109, 2004)

COMPARISON OF MODELS

PFSS

- source surface – 2.5 R_{sun}
- magnetic field at SS: open & constrained to be radial
- Coronal magnetic field: latitudinally structured
- Predicts polarity, but strength in terms of total unsigned flux crossing SS

CSSS

- Free to vary: 14–15 R_{sun}
- Open at cusp surface 2.5 R_{sun} — not radial until SS
- uniform - no lat/lon dependence —> consistent with observations (Smith & Balogh 1995, 2003; Acuña, 2008)
- Can predict HMF strength & polarity

PRESENT WORK

- **used CSSS and PFSS models to compute FTE**
- **used the speed-FTE relationship of Wang-Sheeley to obtain a mathematical description between them**
- **used this mathematical relationship to predict solar wind speed near the Sun**
- **compared the predictions of the two models**

DATA

daily averaged solar wind data 1996-2010

OMNI data

solar cycles 23 - 24

Photospheric synoptic maps

WSO

5° lat/lon

SOHO/MDI

1° lat/lon

NO MDI data exist outside this period

NSO/Kitt Peak

1° lat/lon

NSO/SOLIS

1° lat/lon

HMI

1° lat/lon

METHOD - STEP 1

Step 1: map observed solar wind back to corona

$$\varphi_0 = \varphi_R + \frac{R\Omega}{V_R} \qquad \& \qquad \vartheta_0 = \vartheta_R$$

ϑ_0, φ_0 – latitude & longitude at source surface

ϑ_R, φ_R – at a distance R from Sun

Ω – angular rotation of the Sun

V_R – the solar wind velocity at R – we used the daily averaged value

METHOD - STEPS 2-4

Step 2

map coronal location back to photosphere along open field lines using CSSS & PFSS models

Step 3

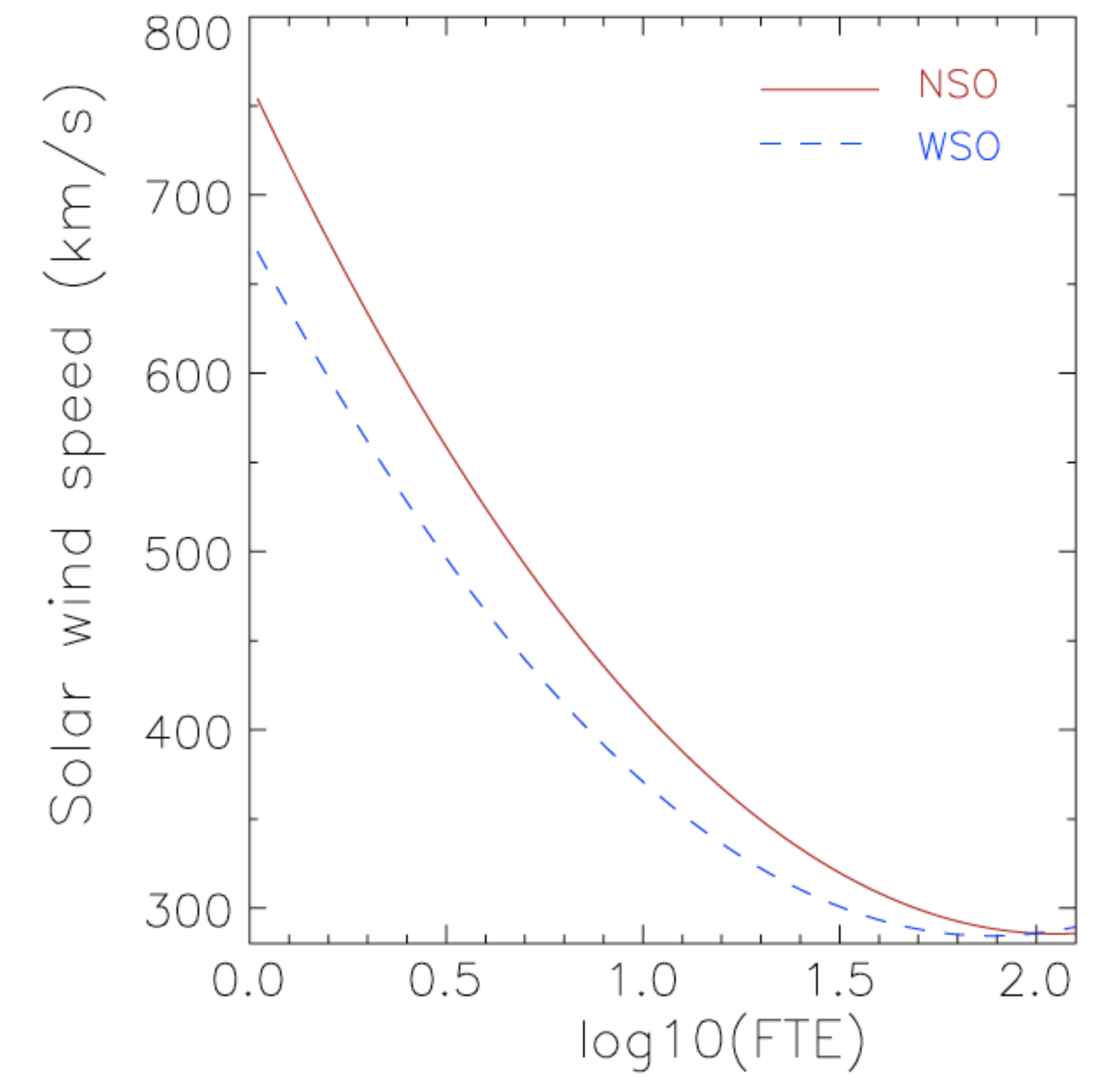
compute FTE at each solar wind source

Step 4

predict solar wind speed using WS inverse relationship

Speed	FTE
> 750	< 4.5
650 – 750	4.5 – 8.0
550 – 650	8.0 – 10.0
450 – 550	10.0 – 20.0
< 450	> 20.0

QUADRATIC FUNCTION



WSO
a = 110.3
b = -416.0
c = 676.6

NSO/Kitt Peak
a = 113.9
b = -466.6
c = 763.4

Poduval & Zhao 2014: ApJ, 782, L22

SOLAR WIND PREDICTION

- the fitted quadratic function — used for all the subsequent solar wind speed predictions

- used the same function for both PFSS & CSSS models

METRICS OF ACCURACY

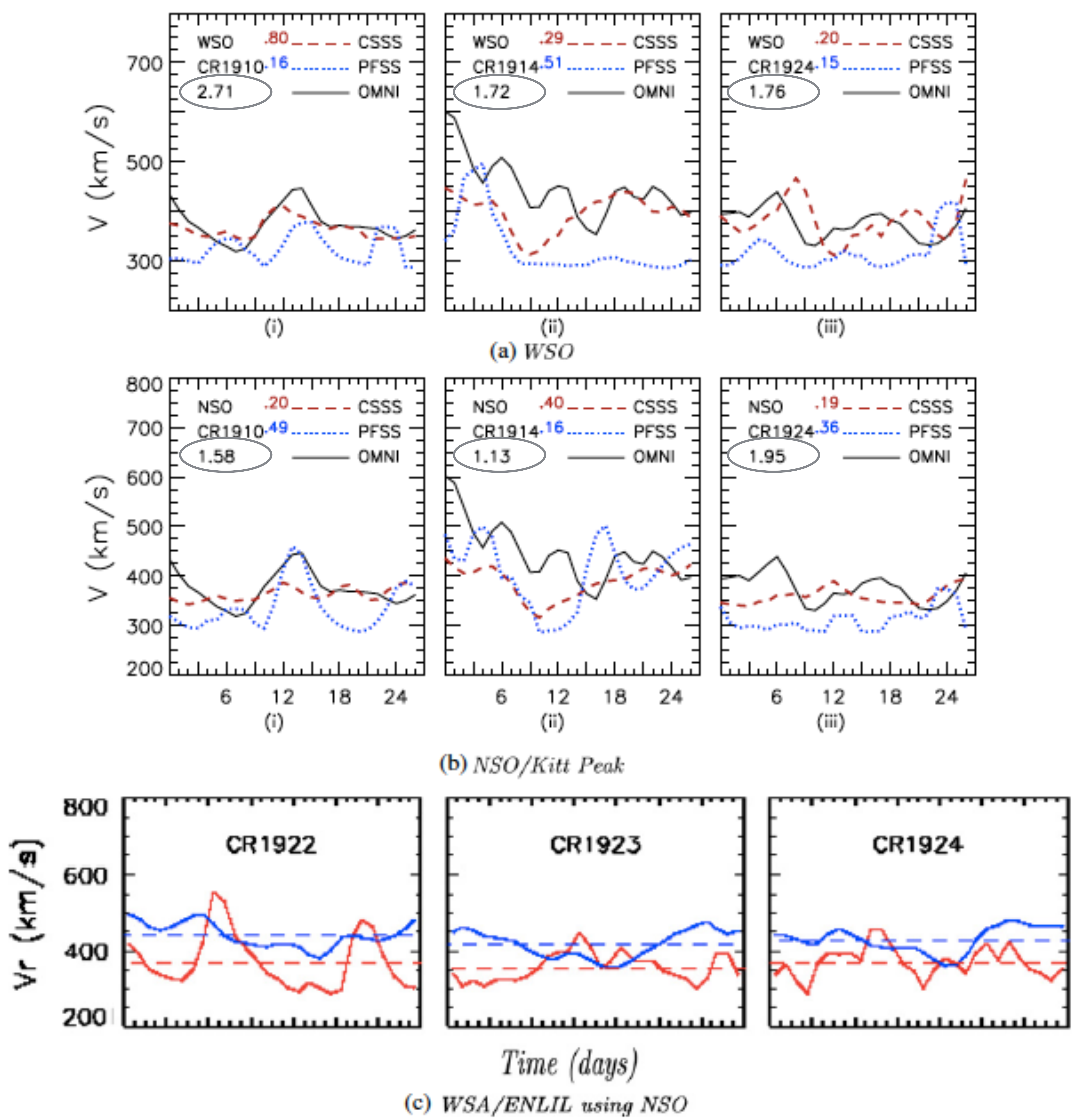
RMS ERROR

correlation coefficient – inadequate:
Good correlation not necessarily imply causality.
Correlation does not capture scaling differences between observed and predicted quantities.

Evaluate predictive capabilities of PFSS & CSSS models

Root Mean Square Error (RMSE) between observed & predicted speeds

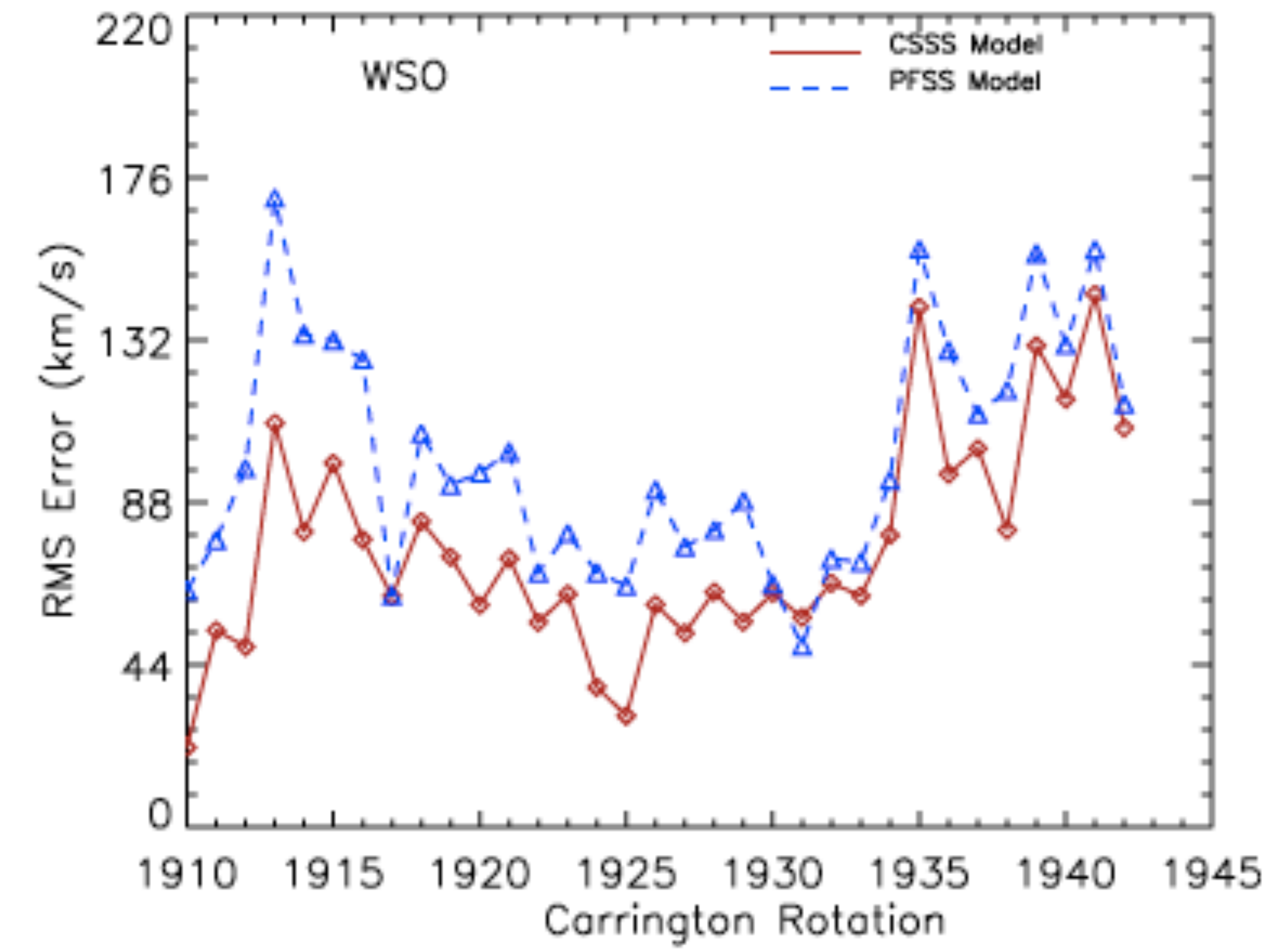
$$\mathrm{RMSEratio} = \frac{\mathrm{RMSE}_{\mathrm{PFSS}}}{\mathrm{RMSE}_{\mathrm{CSSS}}}$$



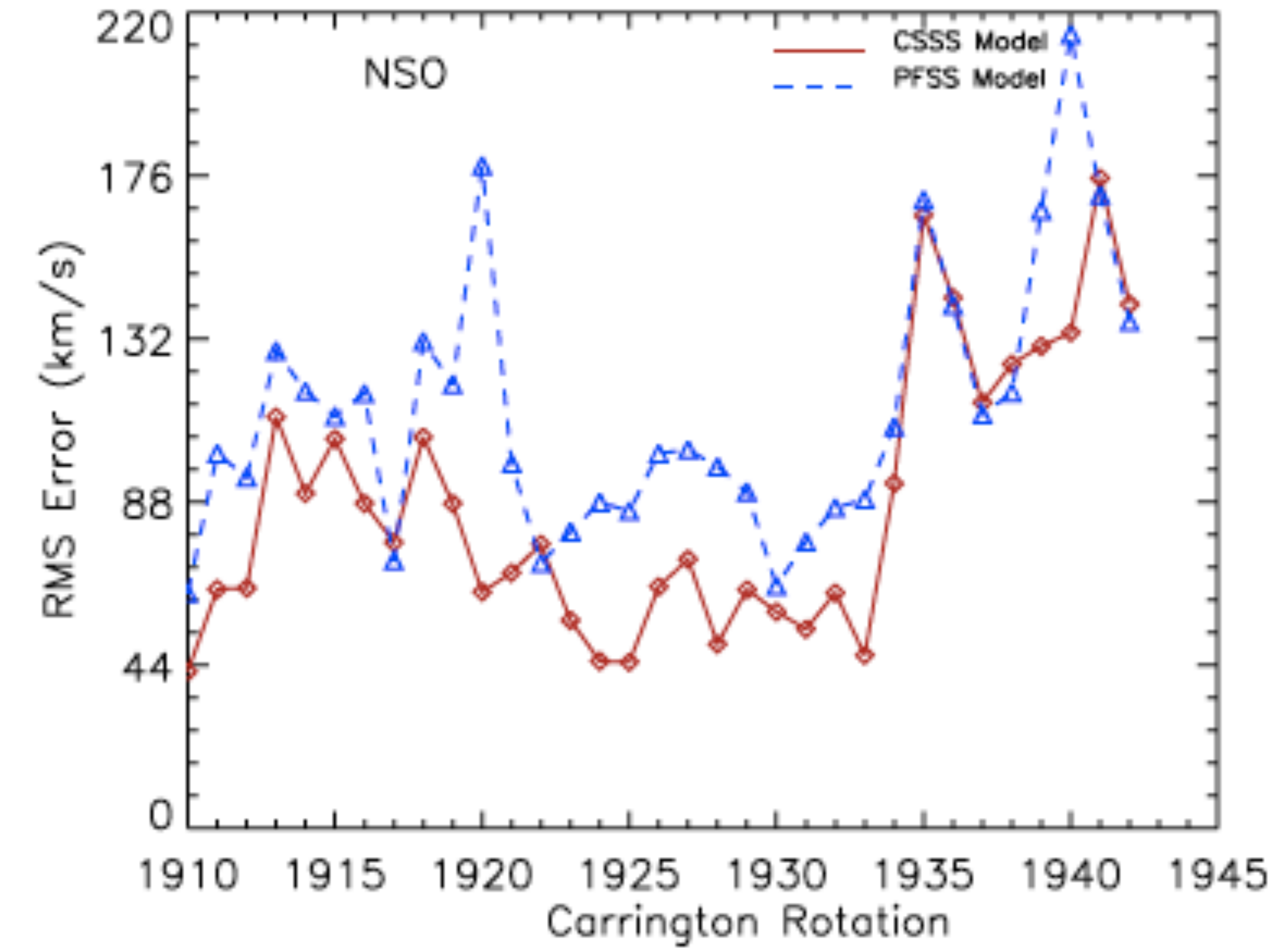
Observed solar wind projected back to the Sun

Predicted solar wind speed using PFSS & CSSS models

Poduval & Zhao 2014: ApJ, 782, L22



(a) *WSO*



(b) *NSO/Kitt Peak*

RMSE increases as solar cycle progresses

(1) increasing complexity of the solar magnetic field makes it more difficult to model

(2) need to optimize free parameters:

(i) R_{ss} = 15 R_{sun} or closer?

(ii) R_{cp} = 2.5 R_{sun}?

height of the cusp varies over a wide range during a solar cycle (Cranmer et al., 2007; Zhao & Hoeksema, 1995)

COMPARISON

correlation coefficient – inadequate:
good correlation not necessarily imply causality

		WSO	NSO
COR COFT > 0.5	CSSS	24%	24%
	PFSS	15%	15%
MEAN COR COFT	CSSS	0.15	0.23
	PFSS	0.12	0.13
MEAN RMSE RATIO	WSA-ENLIL/CSSS	-	1.9
MEAN RMSE RATIO	PFSS/CSSS	1.3	1.6
RMSE > 1.3		32%	55%

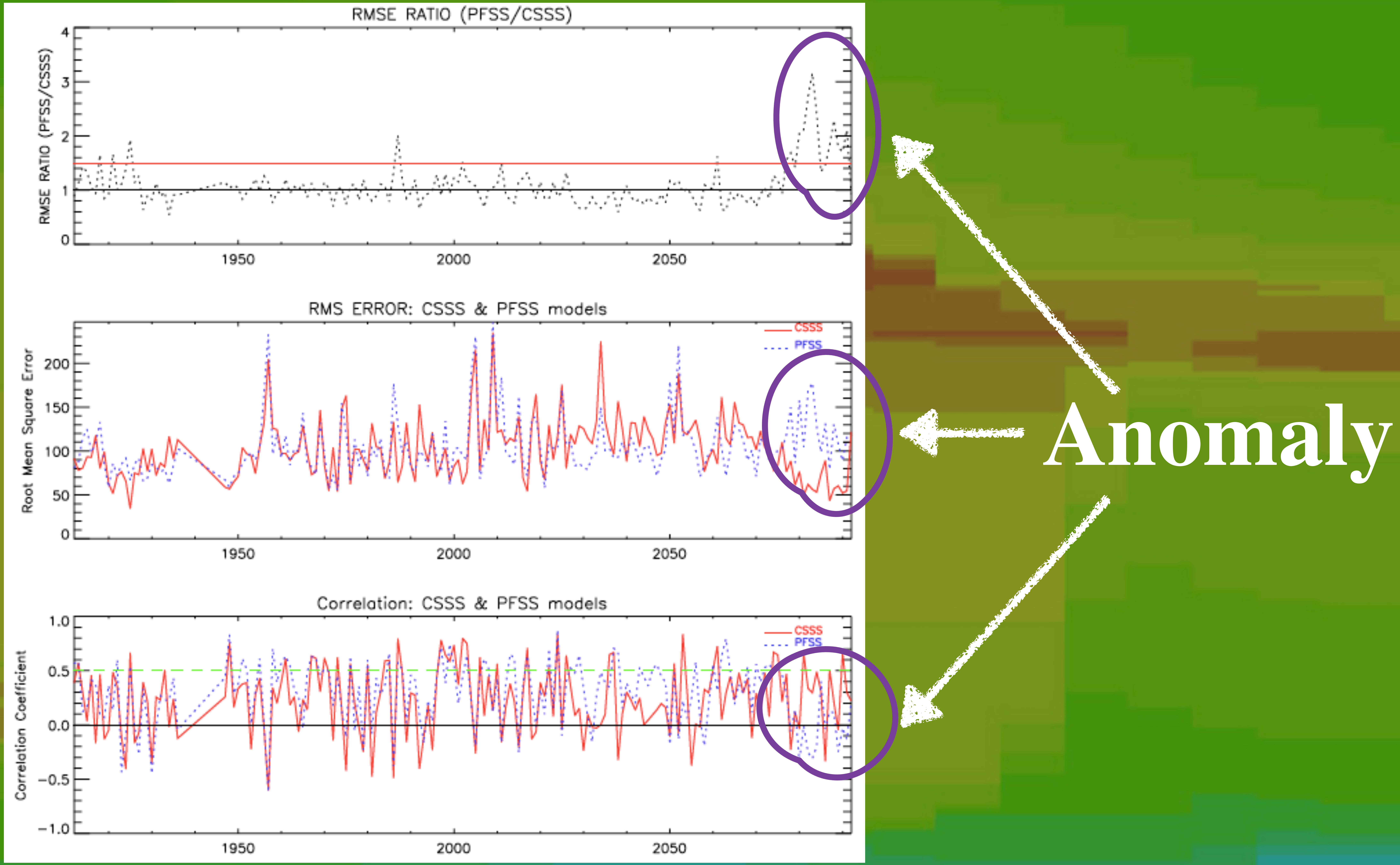
82% with RMSE >= 1.0

—>

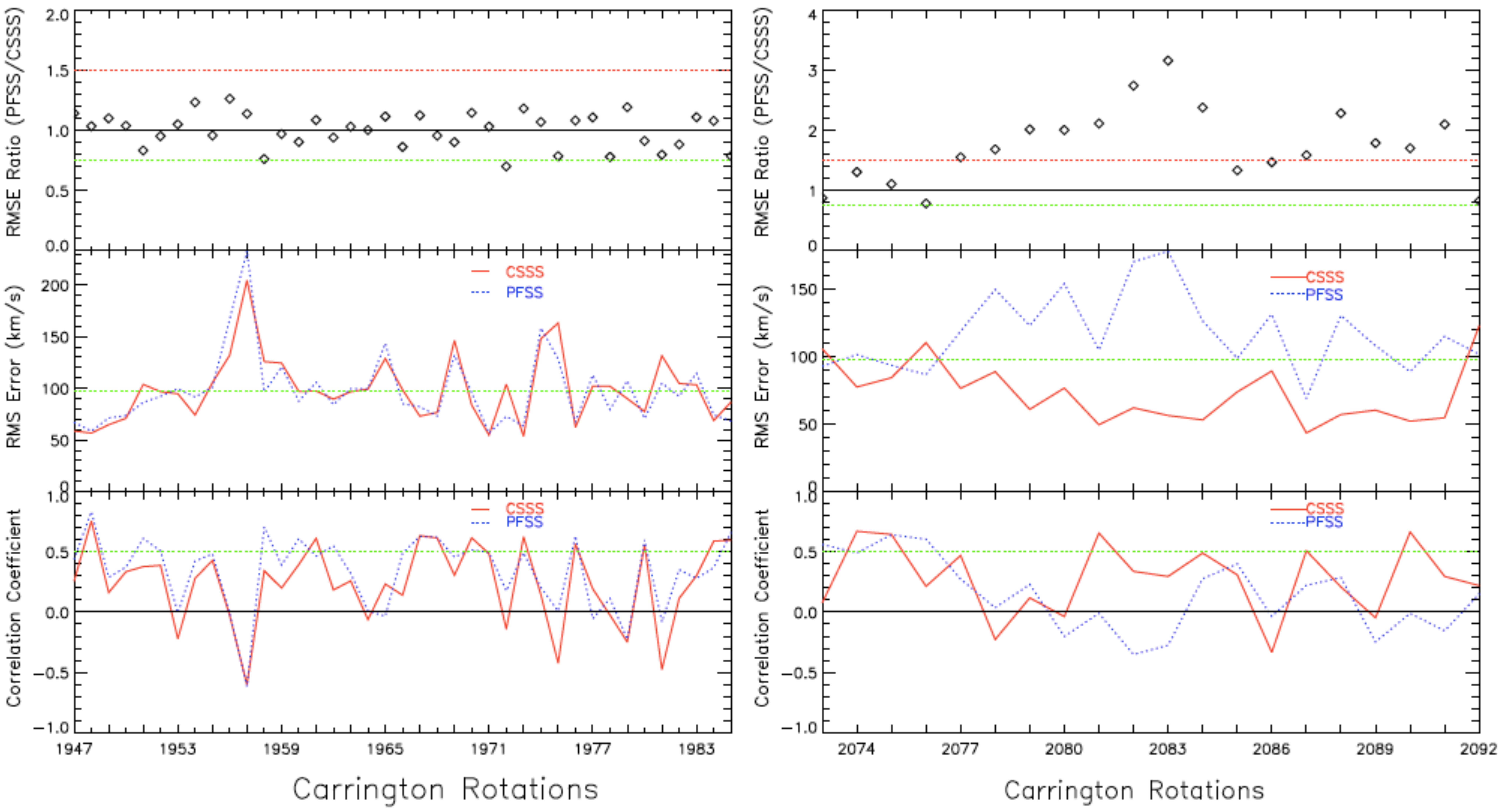
CSSS predictions are comparable to or better than PFSS predictions

AN ANOMALY …

MDI, SOLIS, WSO

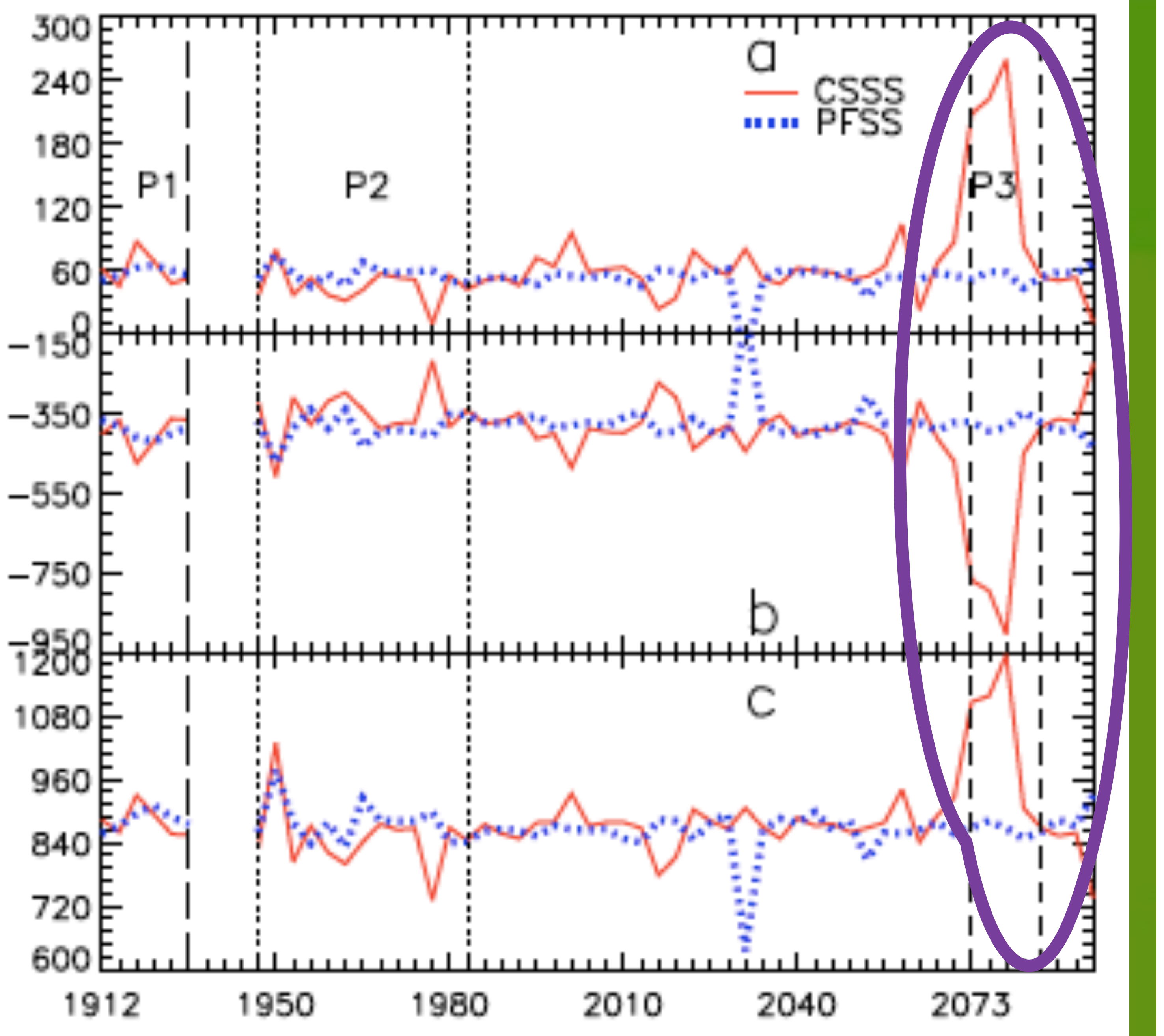


RMSE RATIO (PFSS/CSSS)
RMSE RATIO (PFSS/CSSS)
1950
2000
2050
RMS ERROR: CSSS & PFSS models
Root Mean Square Error
CSSS
PFSS
Correlation: CSSS & PFSS models
Correlation Coefficient
Anomaly

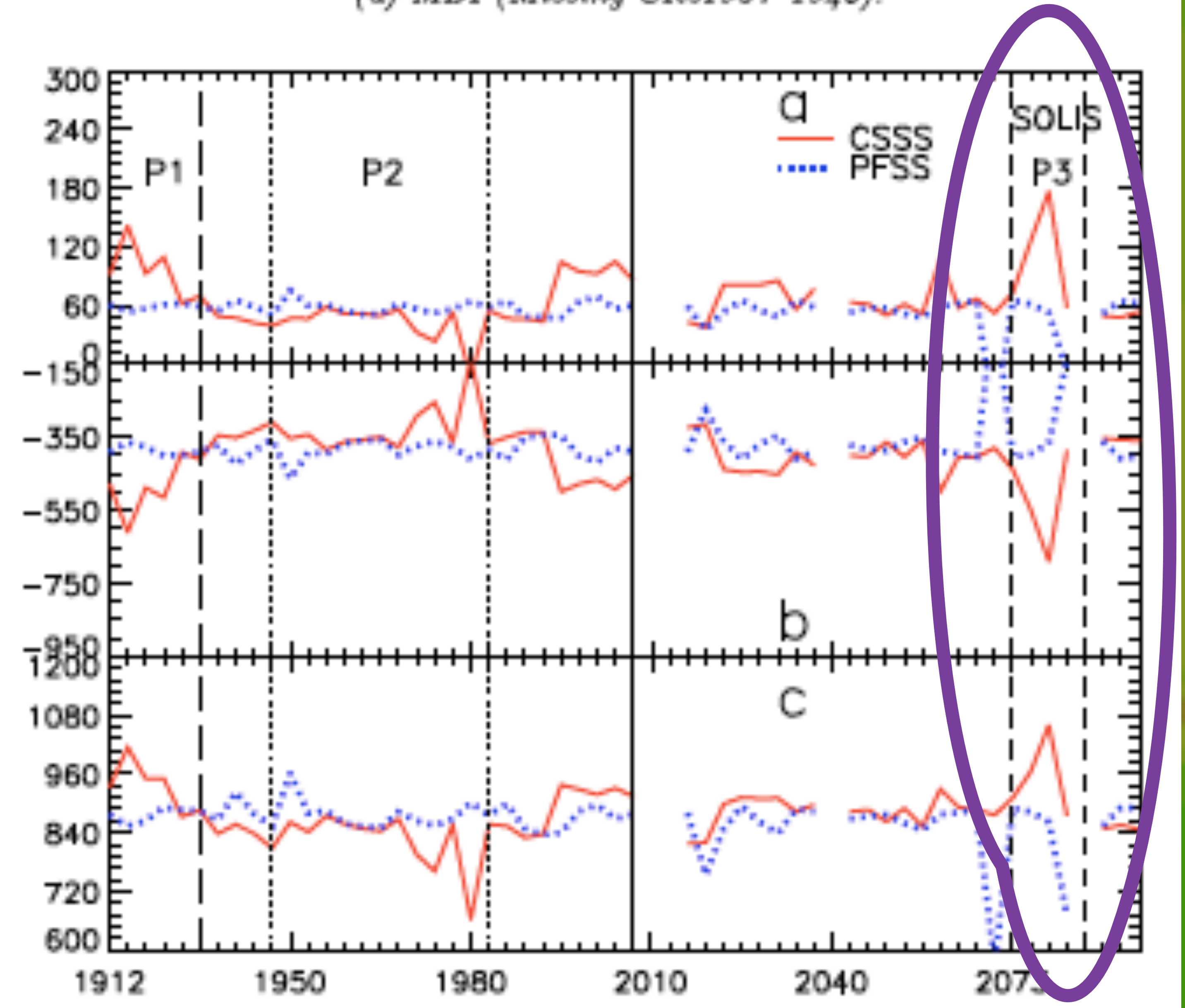


(a) *CRs 1947–1985 (1999–2002).* (b) *CRs 2073–2092 (2008–2010).*

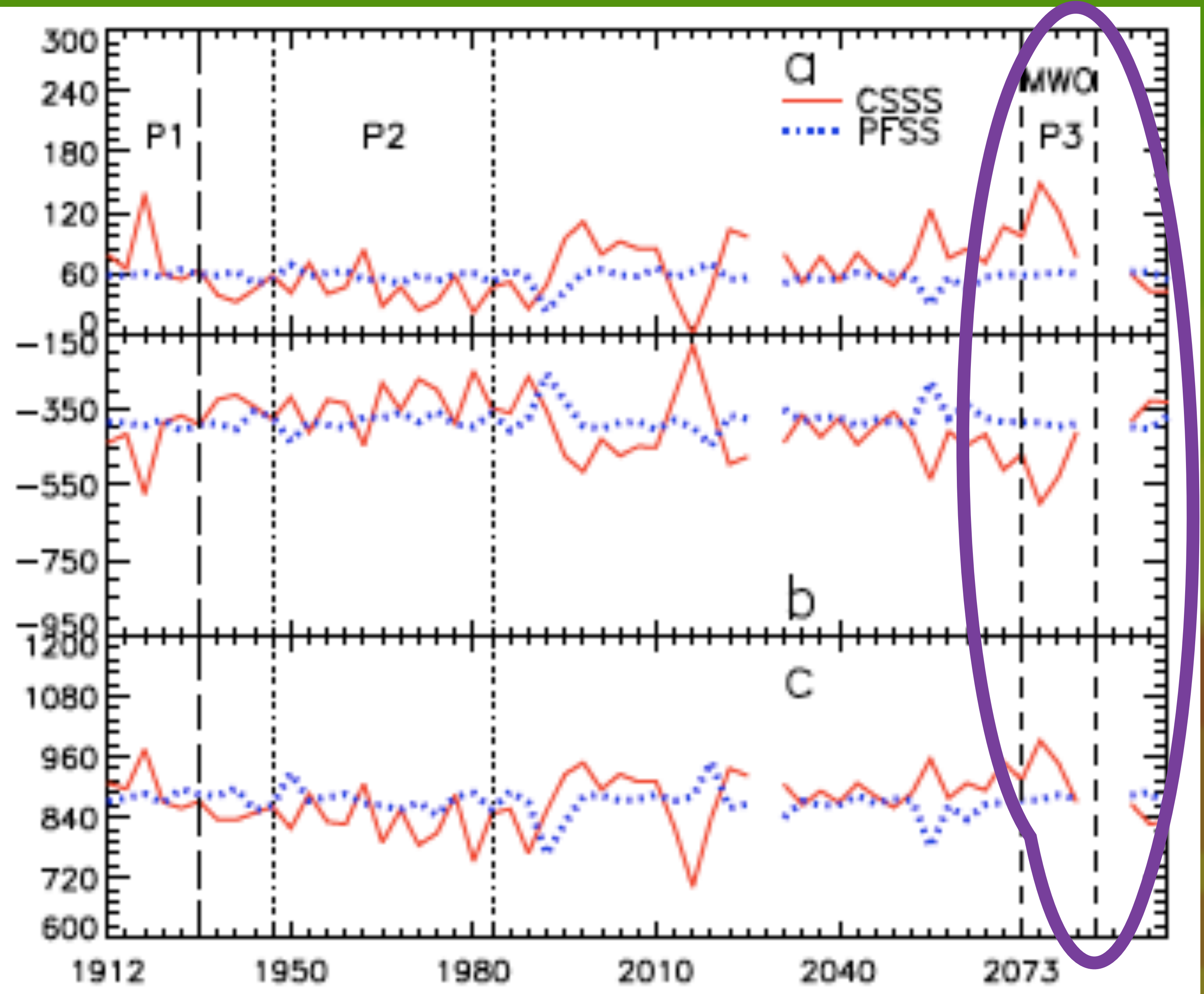
Poduval, 2016: ApJ, 827, L6



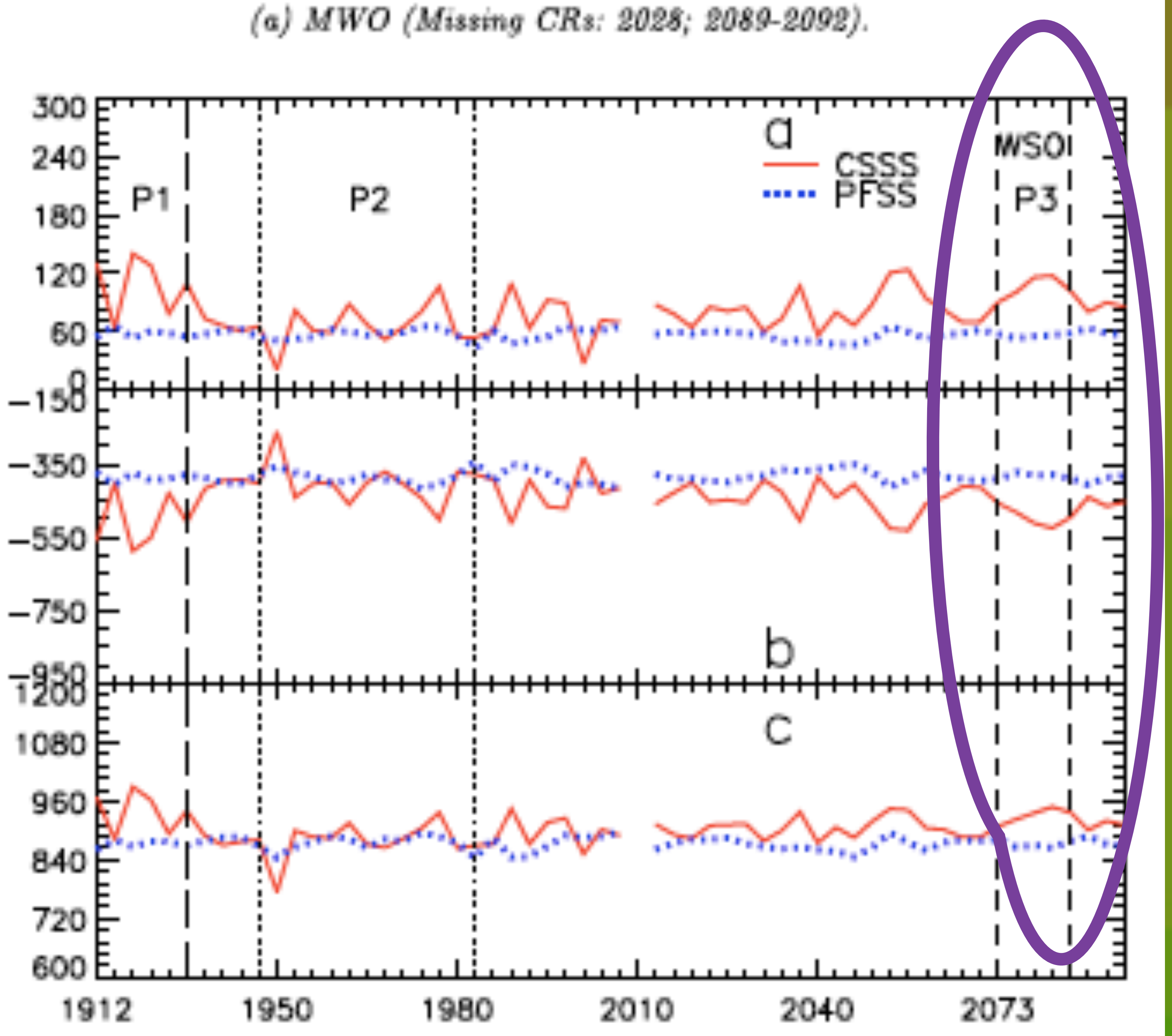
(a) MDI (Missing CRs1937–1946).



(b) KPK/SOLIS (Missing CRs2006;2010;2013;2089–2092).



(a) MWO (Missing CRs: 2028; 2089–2092).

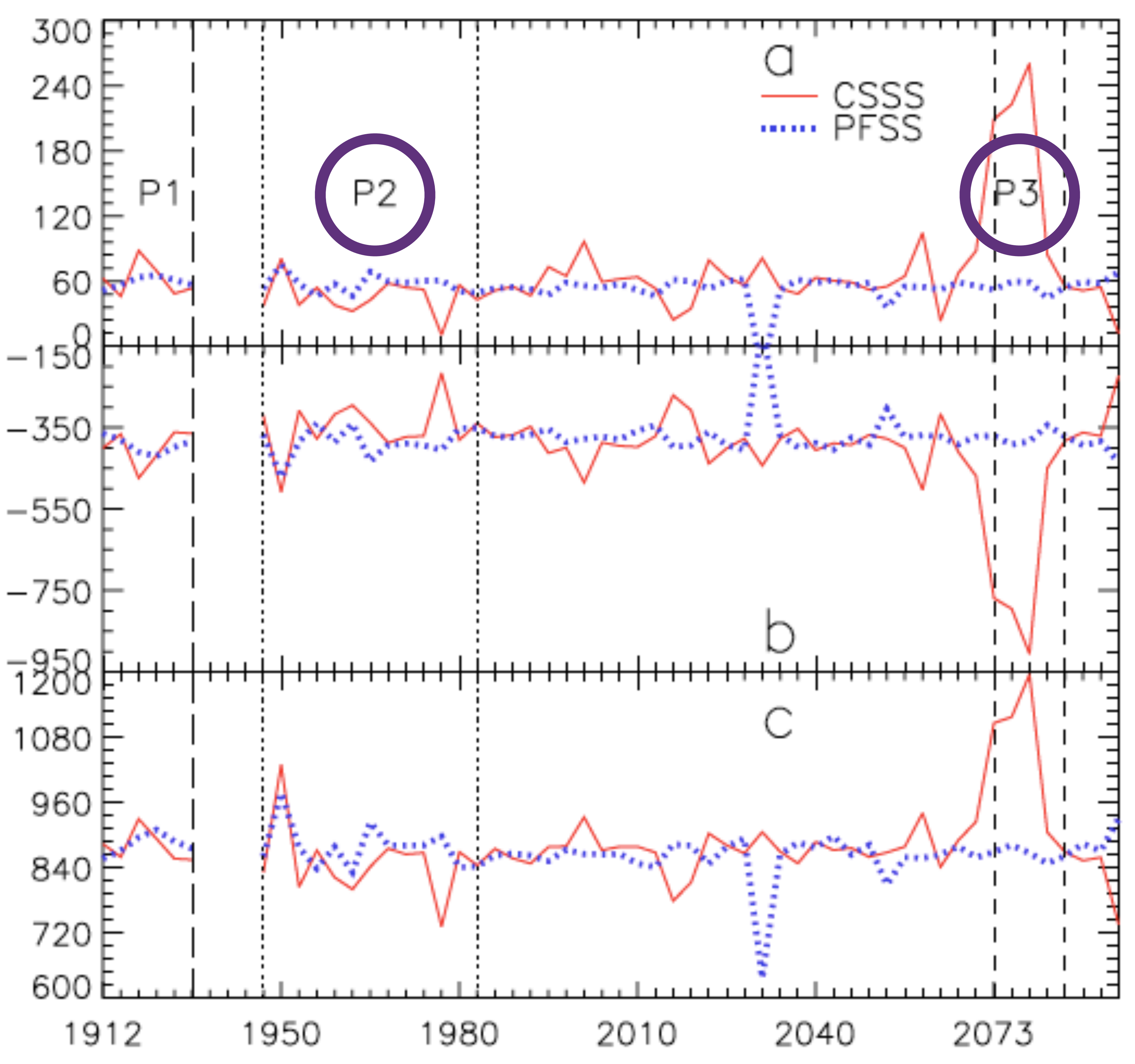


(b) WSO (Missing CR: 2011).

Anomaly

variation of the fitted coefficients a, b, c during CRs 1912 - 2104

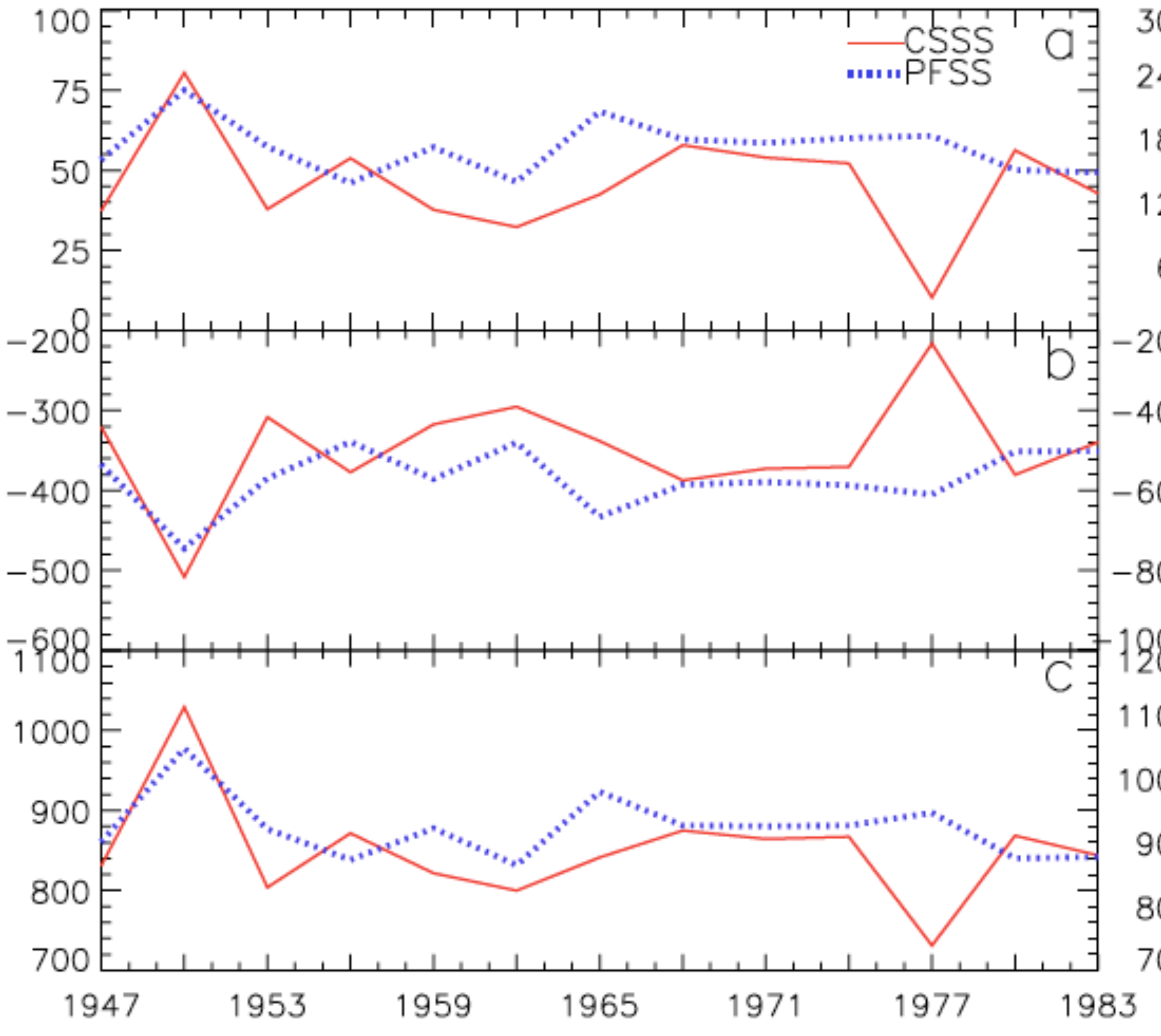
Poduval, 2016: ApJ, 827, L6

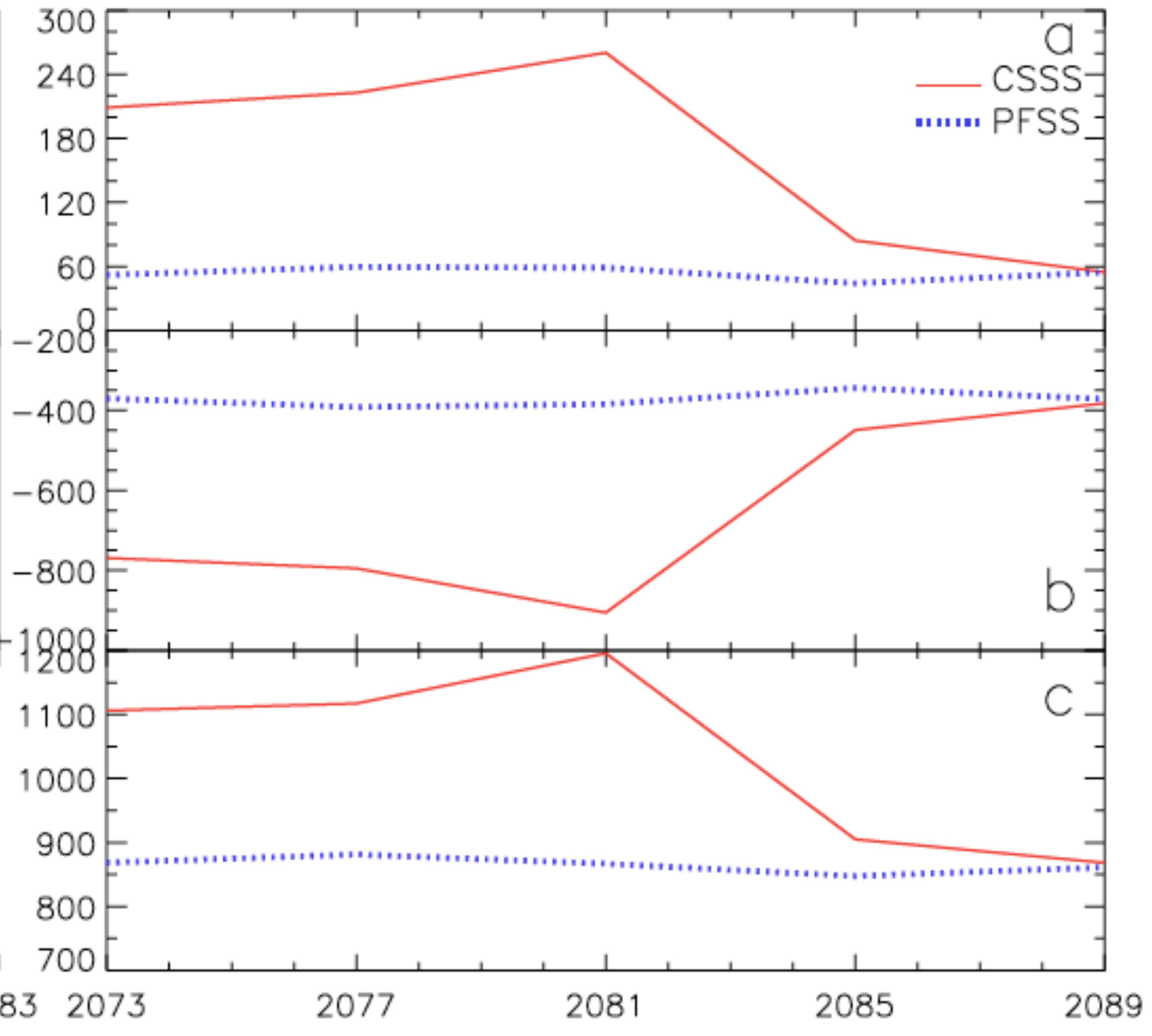


(a) *CRs 1912–2104 (1996–2010)*

P2

P3

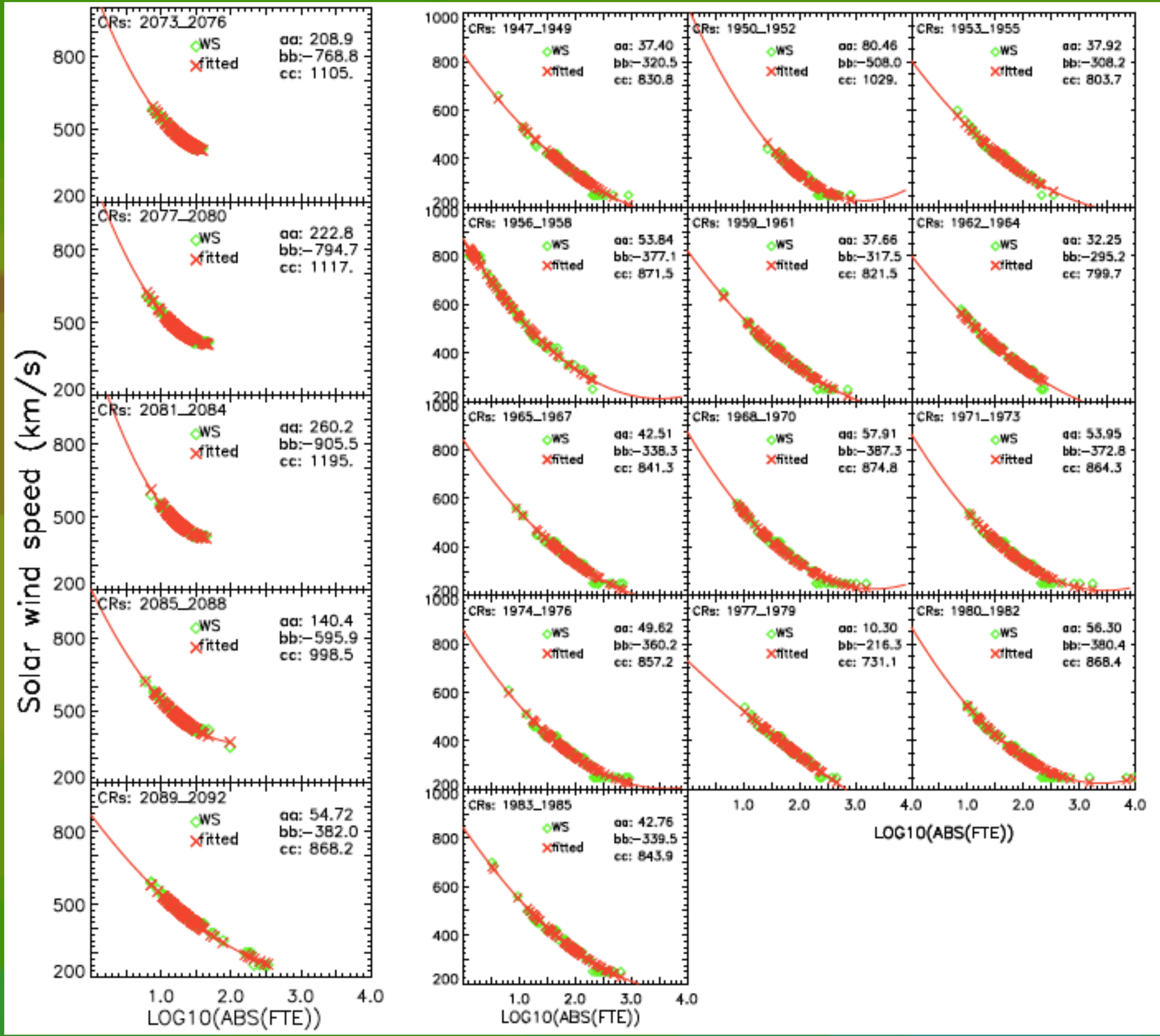




(b) *CRs 1947–1985 (1999-2002).*

(c) *CRs 2073–2092 (2008–2010).*

Poduval, 2016: ApJ, 827: L6

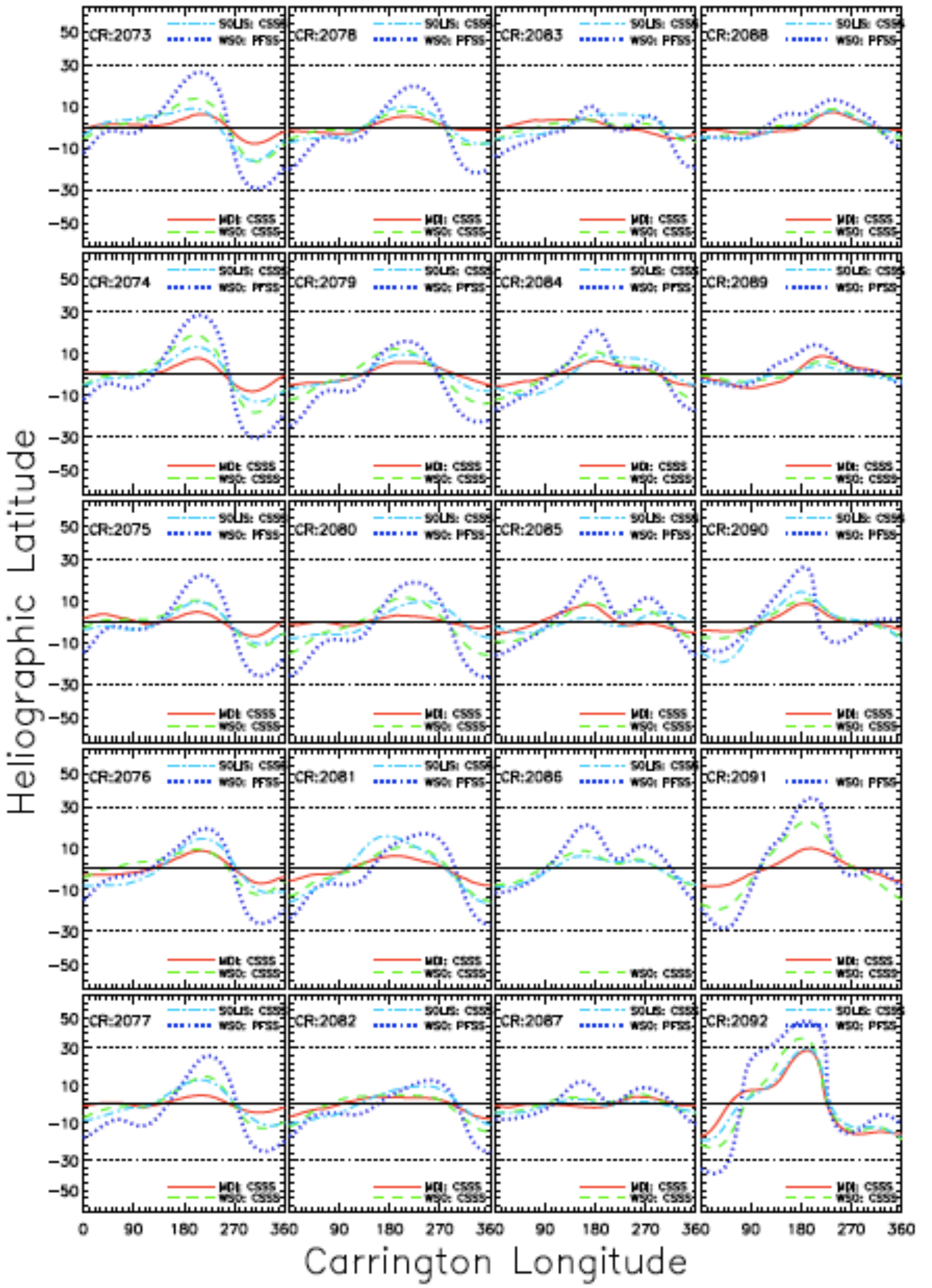


variation of the coefficients of the fitted quadratic function during a solar cycle

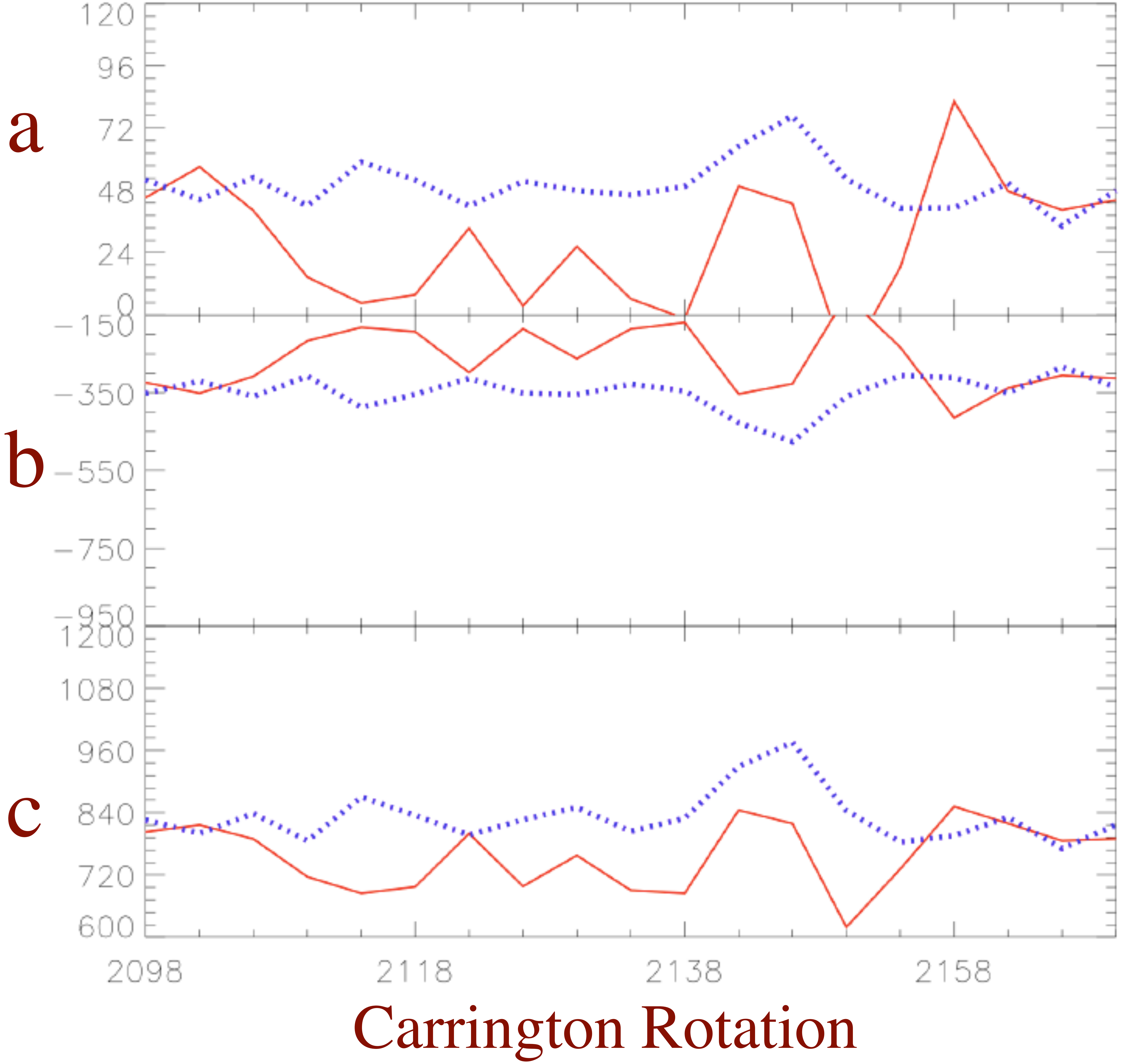
sws = a * (fte)^2 + b * fte + c

almost linear fit

Poduval, 2016: ApJ, 827, L6


CR:2073
CR:2078
CR:2083
CR:2088
CR:2074
CR:2079
CR:2084
CR:2089
CR:2075
CR:2080
CR:2085
CR:2090
CR:2076
CR:2081
CR:2086
CR:2091
CR:2077
CR:2082
CR:2087
CR:2092
SOLIS: CSSS
WSO: PFSS
MDI: CSSS
WSO: CSSS
Heliographic Latitude
Carrington Longitude

HMI DATA

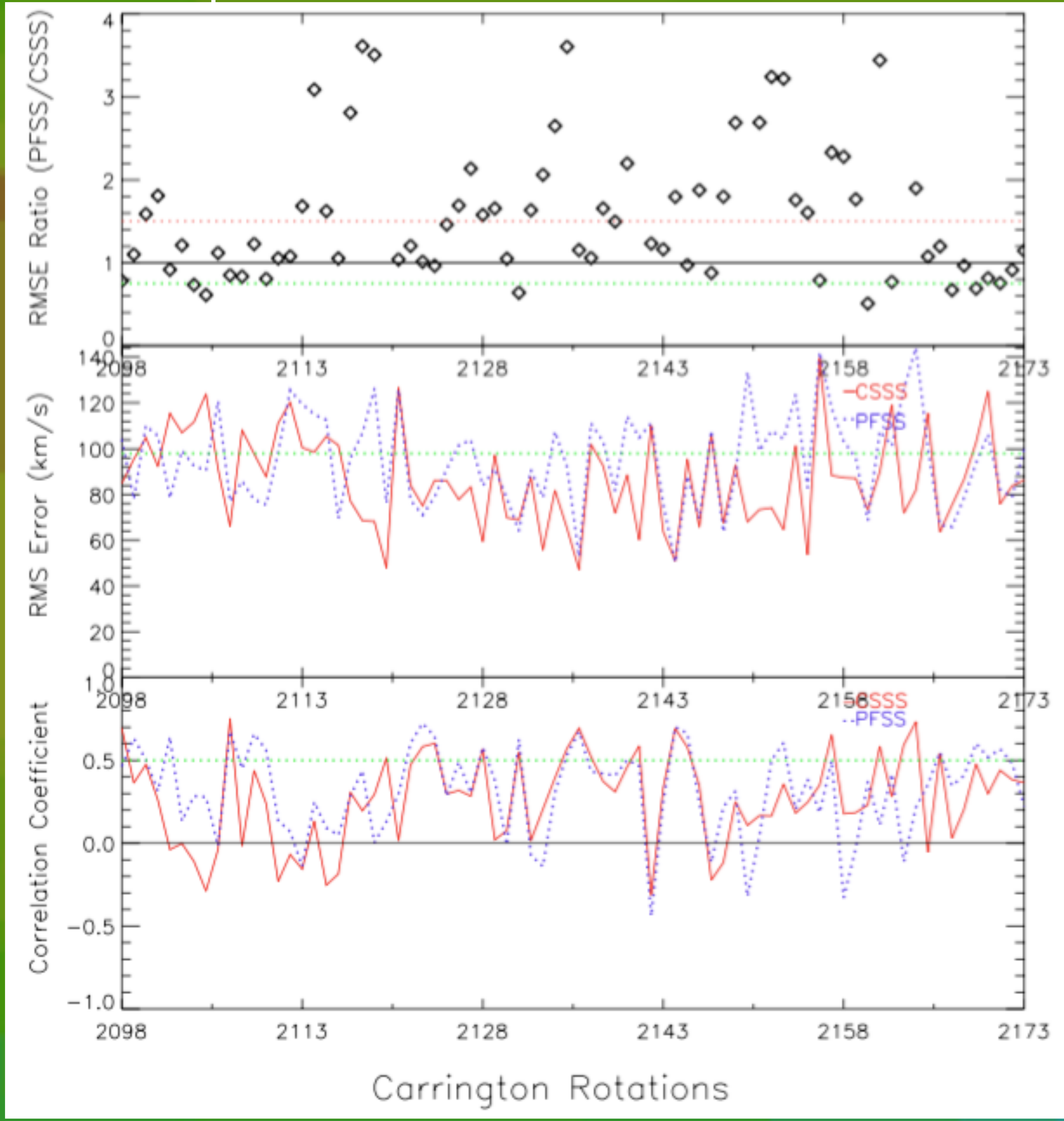


HMI data   2010–2016

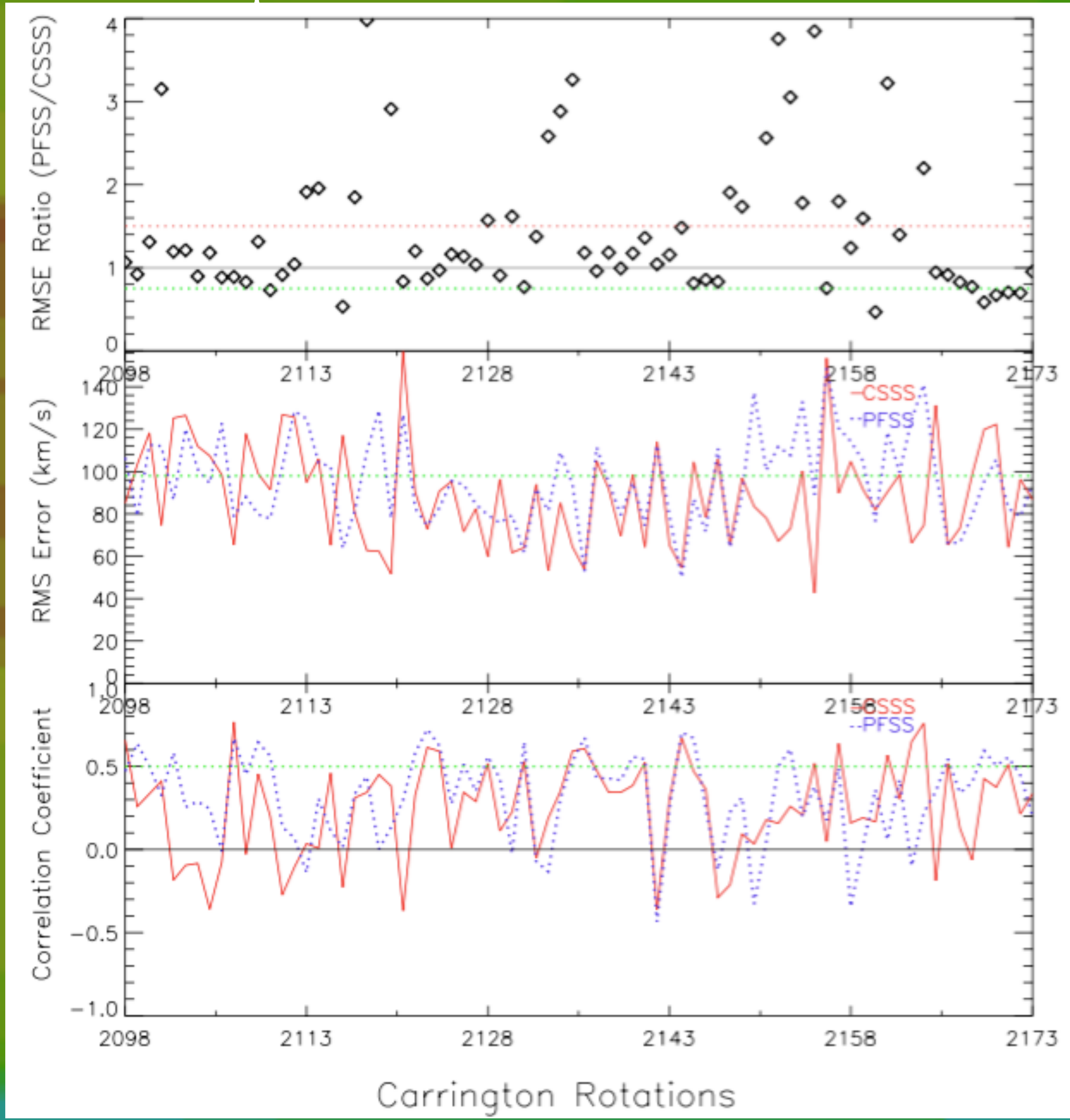
sws = a * (fte)^2 + b * fte + c

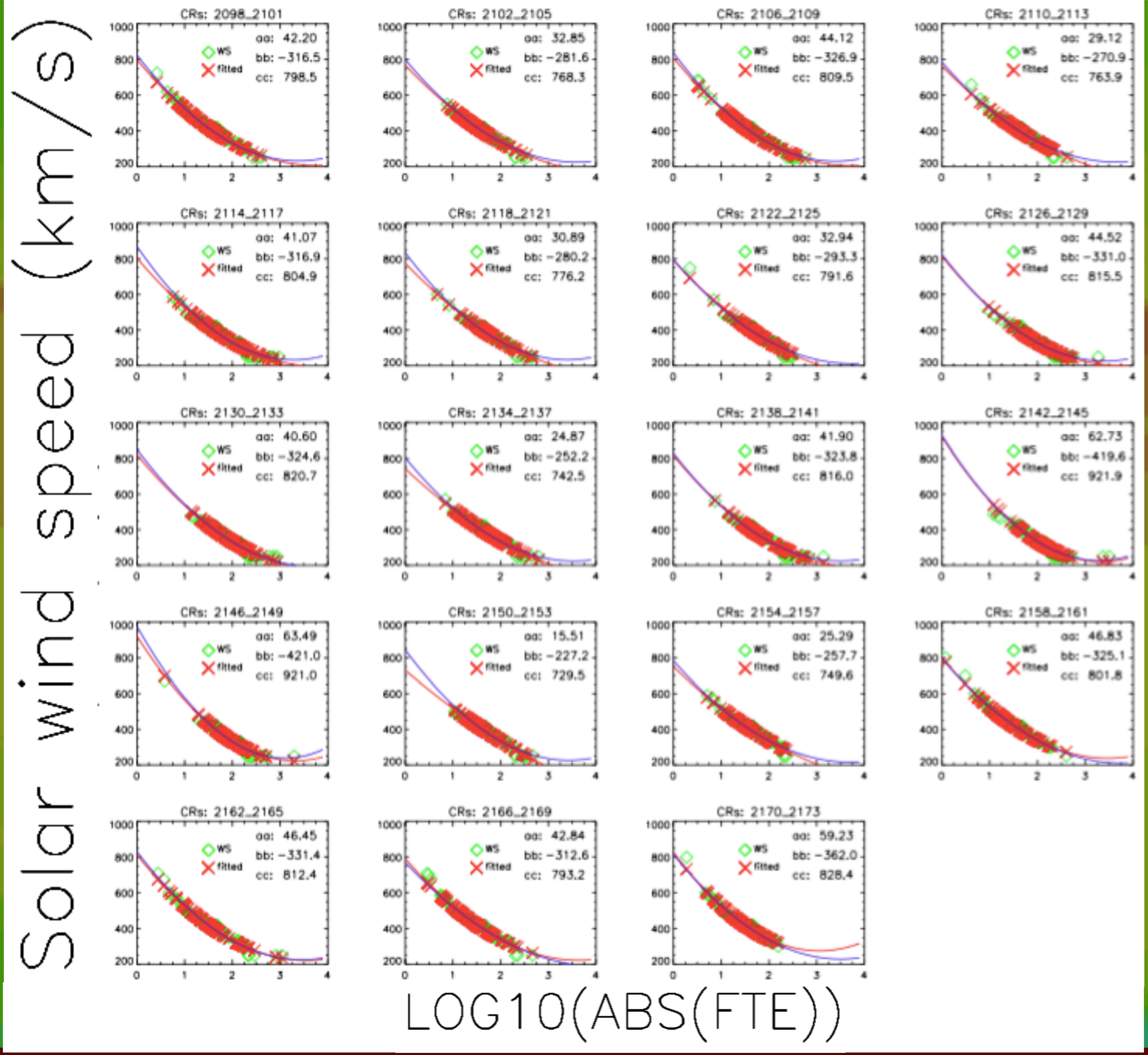
HMI data 2010-2016

$r_{cp} = 2.5R_s$ t_test: > 90%



$r_{cp} = 2.25R_s$ t_test: > 95%





HMI data 2010-2016
CRs 2098-2173

variation of the coefficients of the fitted quadratic function during a solar cycle

CONCLUDING REMARKS

Investigation of the controlling influence of magnetic field on solar wind outflow

$$FTE = B_r/B_r(ss) * (R/R_{ss})^2$$

B_r; R : photospheric magnetic field & radius

$B_r(ss)$; R_{ss}: source surface magnetic field & radius

CONCLUDING REMARKS

temporal variation of FTE-SW speed relationship

— quadratic term in the best fit to SWS-FTE

— nearly disappearing during certain solar rotations, giving rise to an almost linear fit

— **significant in CSSS model**

— nearly negligible in PFSS model

CONCLUDING REMARKS

to establish Sun—Solar wind connectivity:

mapped observed solar wind back to corona & predicted speed using magnetic field properties at the foot points, represented by FTE

PFSS: solar wind mapped back to 2.5 R_{sun}

CSSS: 15R_{sun} — avoids the region below Alfven critical point, where SW is still accelerating

CONCLUDING REMARKS

PFSS: magnetic field constrained to be radial at 2.5 R_{sun} —> larger uncertainties in the photospheric footpoints

CSSS: magnetic fields allowed to be nonradial between 2.5 & 15R_{sun}

better performance of CSSS model indicates —>

solar wind sources are traced more accurately

— nearly twice better than PFSS & WSA/ENLIL

CONCLUDING REMARKS

CSSS: source surface location free to vary

— great advantage —

can be placed outside Alfven critical point

coronal and heliospheric magnetic

field strengths can be computed/predicted and compared

with in situ measurements

CONCLUDING REMARKS

For a given synoptic map (WSO; NSO/KittPeak):

CSSS model performs 1.5 – 2 times better than PFSS & WSA/ENLIL models,

taking RMS error as the metric of accuracy

CONCLUDING REMARKS

Solar Orbiter & Solar Probe Plus

obtain information on coronal conditions within 40 R_{sun}

CSSS predictions will be useful in interpreting the results …