Dynamic Time-Lag Regression: Predicting Time Lagged Effects of Solar Activity

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From the Ph.D. Thesis - Mandar Chandorkar,

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Dynamic Time-Lag Regression, DTLR, is a novel method for modeling the temporal dependency between two spatio-temporal phenomena where one is caused by the other with a non-stationary time delay.

There exists significant discrepancy between the background solar wind observed near the Earth's orbit and that predicgted by the current space weather prediction model, WSA-Enlil and there has been ongoing efforts to improve the prediction accuracy.

We introduced DTLR as an attempt to improve the solar wind prediction in the context of space weather forecast.

Problem Definition

To forecast solar wind speed at L1 from solar & heliospheric observations

a complex regression problem

because

- badly conditioned input-output problem — large dimension of input signal (d = 512 x 512 x #channels —> scaled output solar wind speed) — dilution of the "cause" in the input signal due to bad SNR.
- stochastic non-constant time lag — range 1 to 5 days:

Dynamic Time-Lag Regression

Two time series:

- the cause series x(t) [$x(t) \in \chi \subset \mathbb{R}^d$]
- the observed effect series y(t) [a scalar]

to establish a connection between x(t) and y(t), we seek a mapping

f(.) that maps x(t) to y(t) and

g(.) that determines the time delay between x(t) and y(t)

Dynamic Time-Lag Regression

Deterministic formulation of the problem

$$y(\phi(t)) = f[x(t)]$$
$$\phi(t) = t + g[x(t)]$$

where, $f : \chi \to \mathbb{R}$; and $g : \chi \to \mathbb{R}^+$;

$x(t) \in \mathbb{R}^d, d \gg 1$, input data containing the hidden cause

$y(t)$, scalar, the effect

$\phi(t)$, the time-lag between cause and effect

Dynamic Time-Lag Regression

the time lag, g[x(t)]

- is non-stationary since it depend on x(t)

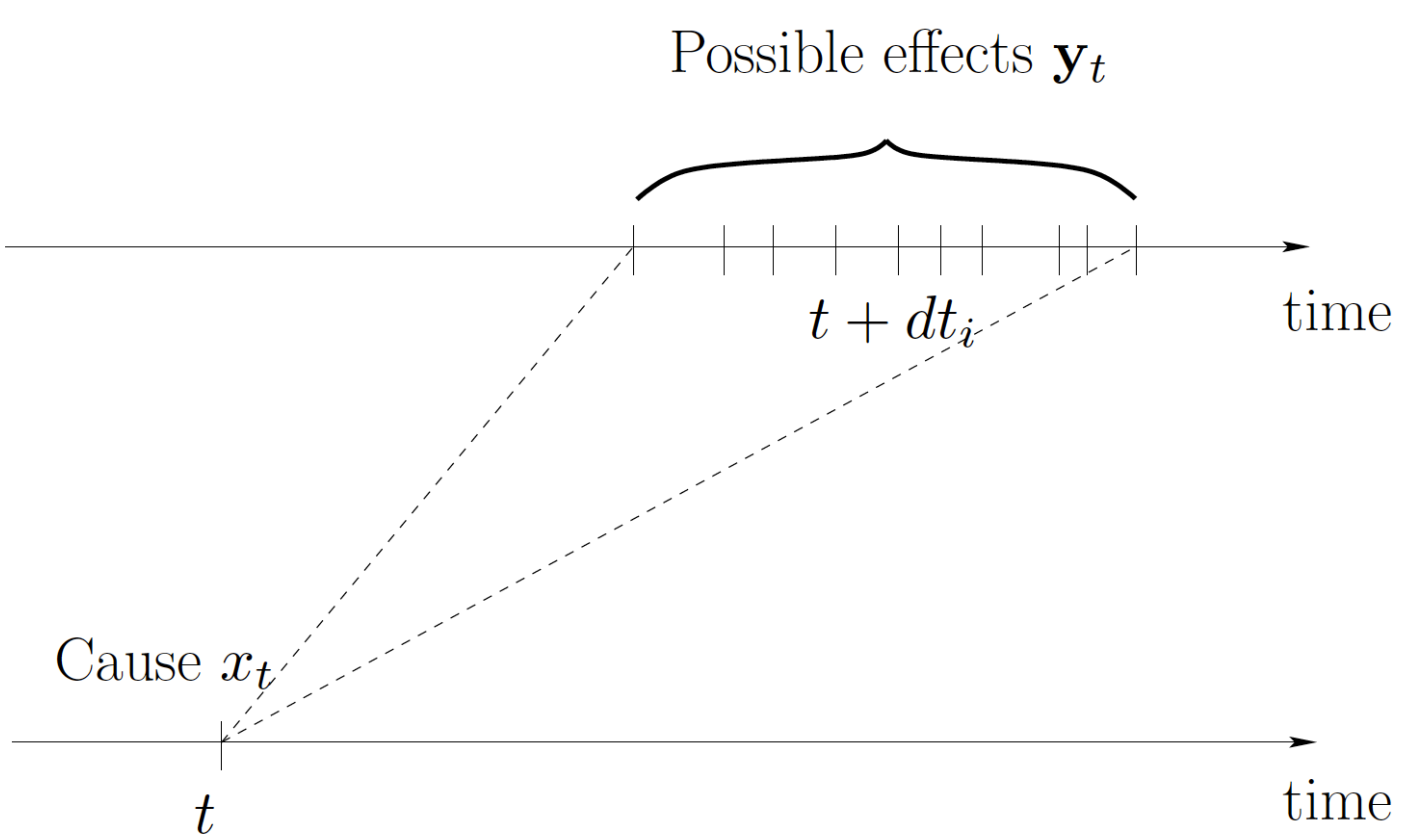
- is unknown (not explicitly recorded in the training data)

Dynamic Time-Lag Regression

the cause and effect series sampled at constant rates, x_t & y_t $(t \in \mathbb{N})$

mapping 'g' maps x_t onto finite set τ of possible time-lags

$\tau = \{\Delta t_{min} ... \Delta t_{max}\}$



Dynamic Time-Lag Regression

Baysian combination of experts:

$$P[\mathrm{y_t}|\mathrm{x_t} = \mathrm{x}] = \sum_{\{\tau_\mathrm{i} \in \{0,1\}, \mathrm{i} \in \mathcal{T}\}} \hat{\mathrm{p}}(\tau_1, ..., \tau_\mathrm{n})\, \mathbb{N}(\hat{\mathrm{y}}(\mathrm{x}),\ \sigma(\tau))$$

where,

$\sigma(\tau)$: the diagonal matrix of variance parameters of each time-lag $i \in \mathcal{T}$

$\hat{p}(\tau_1, ..., \tau_n|\mathrm{x})$: the joint probability measure of time-lagged effects caused by x.

Dynamic Time-Lag Regression

DTLR solution is obtained as a probability distribution conditioned on cause x, mixture of Gaussians centered on the predictors $\hat{y}(\mathrm{x})$, where the mixture weights are defined from $\hat{p}(\mathrm{x})$

Dynamic Time-Lag Regression

simplifying assumptions:

— the stochastic time lag is modeled as binary latent variables:

- $\tau_i = 1$ indicates if x_t drives y_{t+i}
- every cause has a single effect: $\sum_{i \in \mathcal{T}} \tau_i = 1$

— the variance of predictor $\hat{y}_i$ does not depend on x:

$$\sigma_i(\tau)^2 = \frac{\sigma^2}{1 + \alpha \tau_i}$$

here, $\alpha_{ij} \geq 0$

default variance: σ^2

Dynamic Time-Lag Regression

Model parameters:

- The predictors,
$$\hat{y}(x) = \{\hat{y}_i(x), i \in \mathcal{T}\}$$
- The probability weights,
$$\hat{p}(x) = \{\hat{p}_i(x), i \in \mathcal{T}\}$$
- σ^2
- α

- Learning Creterion: The loss function is the Log Likelihood of the data (x, y) -
$$\mathcal{L}[x, y|\hat{y}, \hat{p}, \sigma, \alpha]$$
- Learning strategy:

$\hat{y}$ & $\hat{p}$ are modeled using coupled neural nets

σ & α are optimized in an outer loop based on saddle point equations

$$\left(\frac{\partial \mathcal{L}}{\partial \hat{y}(x)}, \frac{\partial \mathcal{L}}{\partial \hat{p}(x)}, \frac{\partial \mathcal{L}}{\partial \sigma}, \frac{\partial \mathcal{L}}{\partial \alpha}\right) = 0$$

Dynamic Time-Lag Regression

Implemetation

Initialization of α and σ

$it \longleftarrow 0$;

while *it* $<$ *max* **do**

  while *epoch* **do**

    $\theta \longleftarrow Optimize(\mathcal{L}(\theta, \alpha, \sigma^2))$;

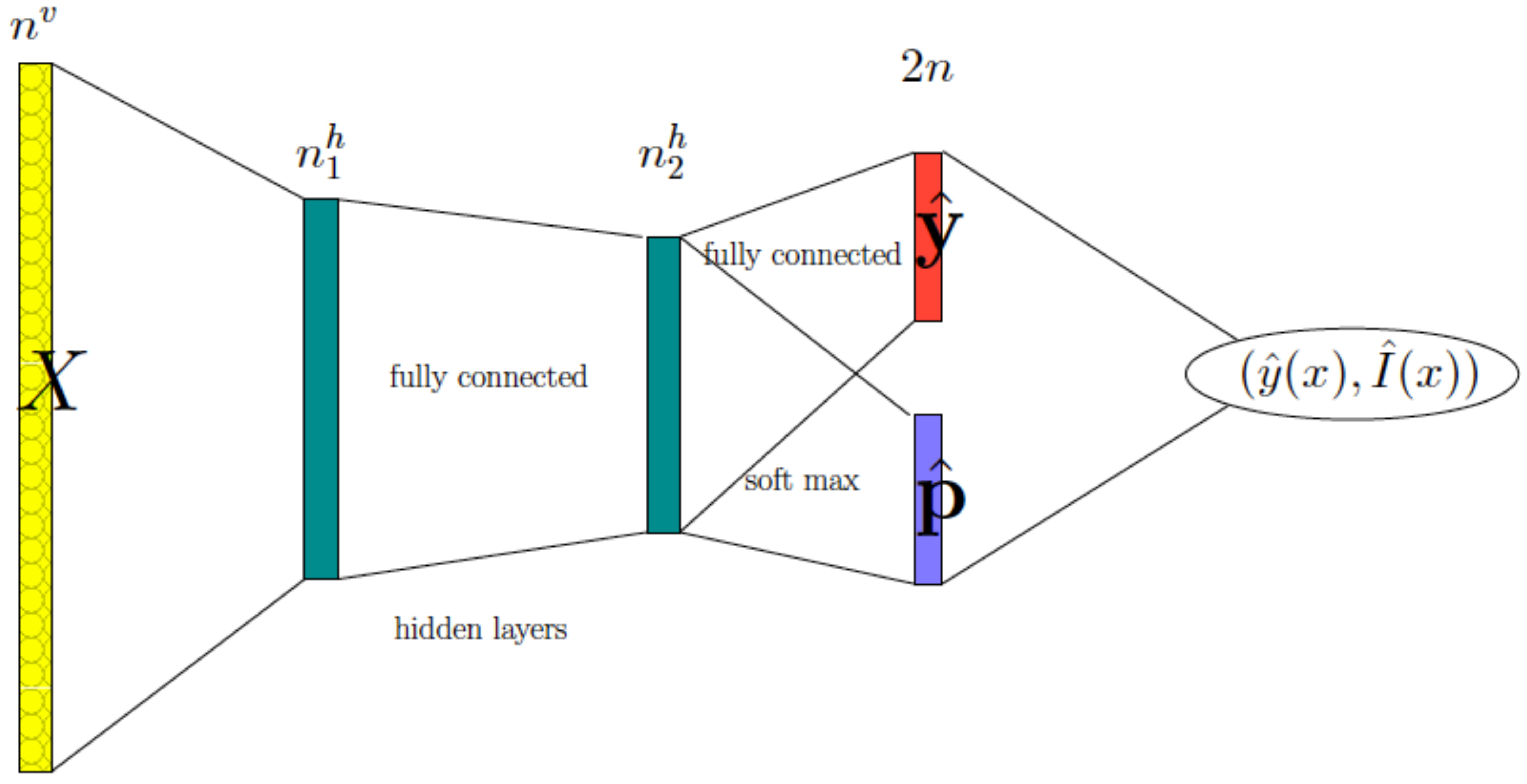
  end

  $\sigma^2 \longleftarrow \sigma_0^2 \frac{|T| - C_1[\mathbf{q}]}{|T| - 1}$;

  $\alpha \longleftarrow \frac{|T|}{|T| - 1} \frac{1 - C_1[\mathbf{q}]}{C_1[\mathbf{q}]}$;

end

Result: Model parameters $\theta = (\hat{\mathbf{y}}, \hat{\mathbf{p}})$,
hyper-parameters α, σ^2



Predicted time-lag index :

$$\hat{I}(x) = \underset{i}{\text{argmax}}(\hat{p}_i(x))$$

DTLR - Solar Wind Prediction at L1

$$FTE = \left(\frac{R_{phot}}{R_{ss}}\right)^2 \frac{B_{r(phot)}}{B_{r(ss)}}$$

Input data at t_0:

$x(t) = (\log(FTE); B_{cp}; v_{27}; SSN; F_{10.7}$

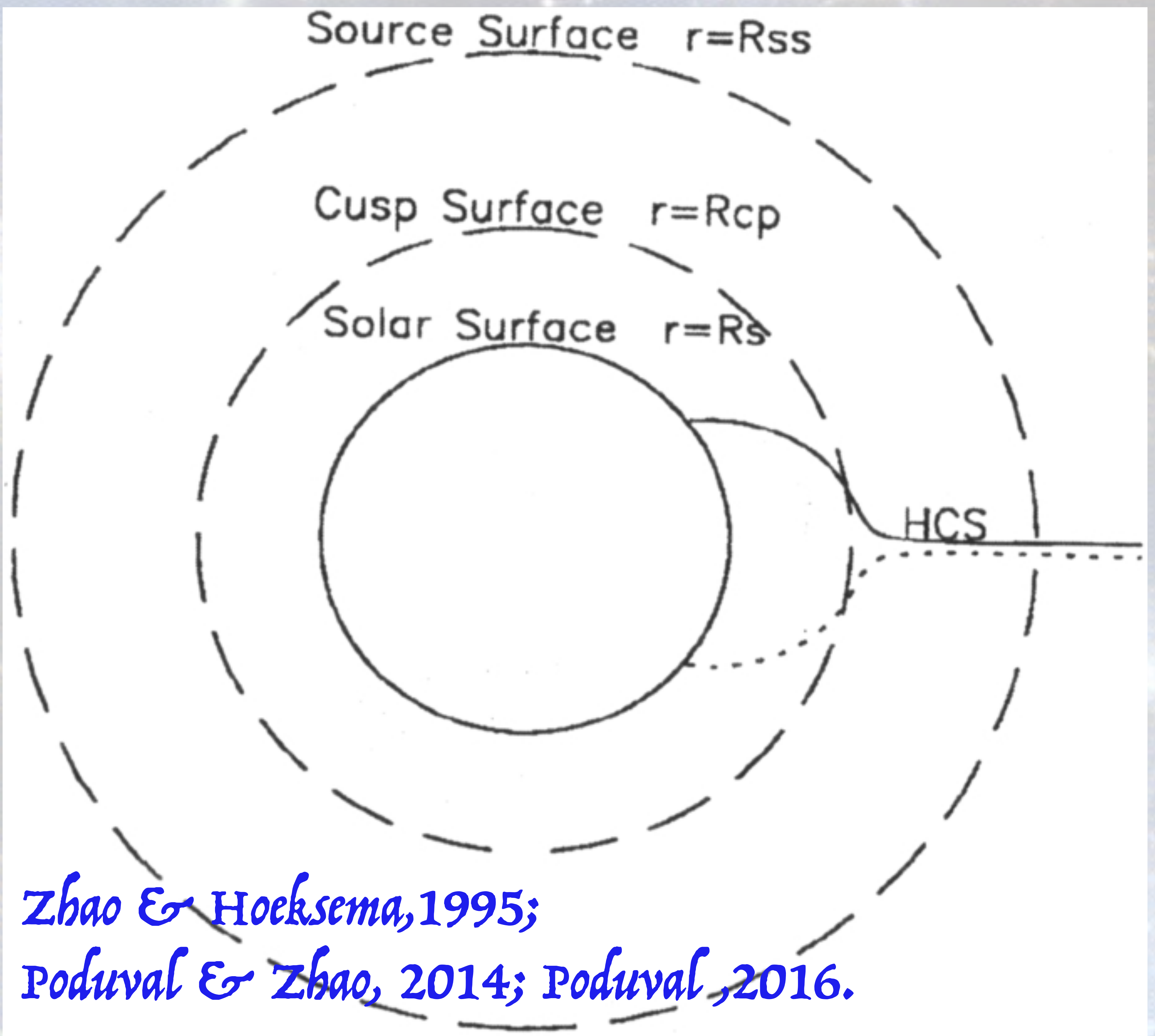
Here, FTE: the magnetic flux tube expansion factor, computed using the Current Sheet Source Surface (CSSS) model,

B_{cp}: the radial magnetic field at 2.5 Rsun

v_{27}: the 27-day prior solar wind speed

SSN: the Sunspot number

$F_{10.7}$: the Solar Radio Flux at 10.7 cm



Zhao & Hoeksema,1995;
Poduval & Zhao, 2014; Poduval ,2016.

DTLR - Solar Wind Prediction at L1

The FTEs were computed using the GONG synoptic maps and the CSSS model.

Output data $y(t)$: solar wind speed (t_0+2days; t_0+5days) with time-lag discretization, $|T| = 12$.

Speed	FTE
> 750	< 4.5
650 - 750	4.5 - 8
550 - 650	8 - 10
450 - 550	10 - 20
< 450	> 20

Wang & Sheeley empirical relationship between solar wind speed and FTE (1990;1993;1995)

DTLR - Solar Wind Prediction at L1

DTLR Performance

	9-fold	baseline
Mean Absolute Error	56.35	66.45
Root Mean Square Error	74.20	84.53
Pearson Correlation Coeft.	0.6	0.41

Note: 9-fold is the cross validation adopted in this work.

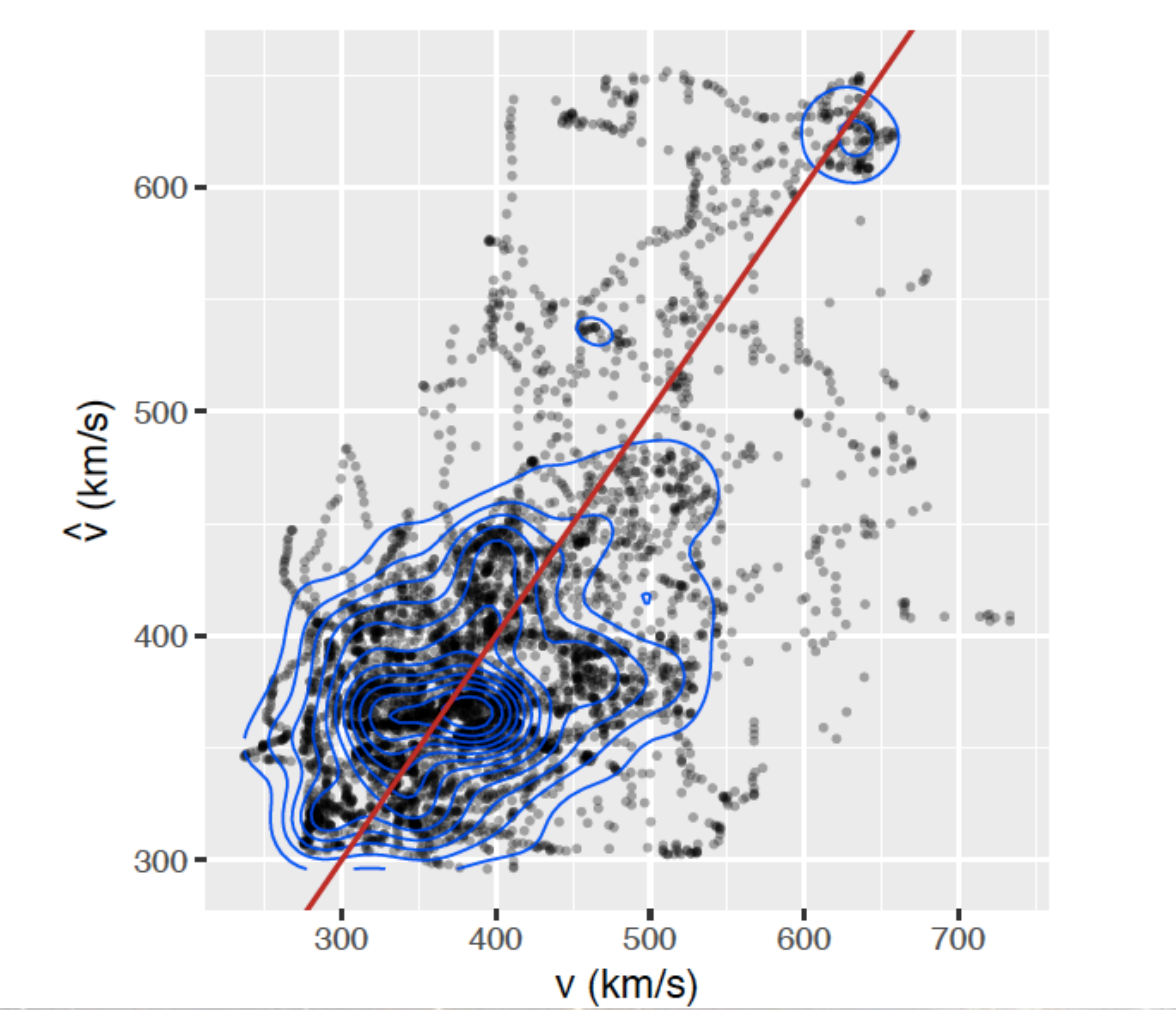
DTLR - Solar Wind Prediction at L1

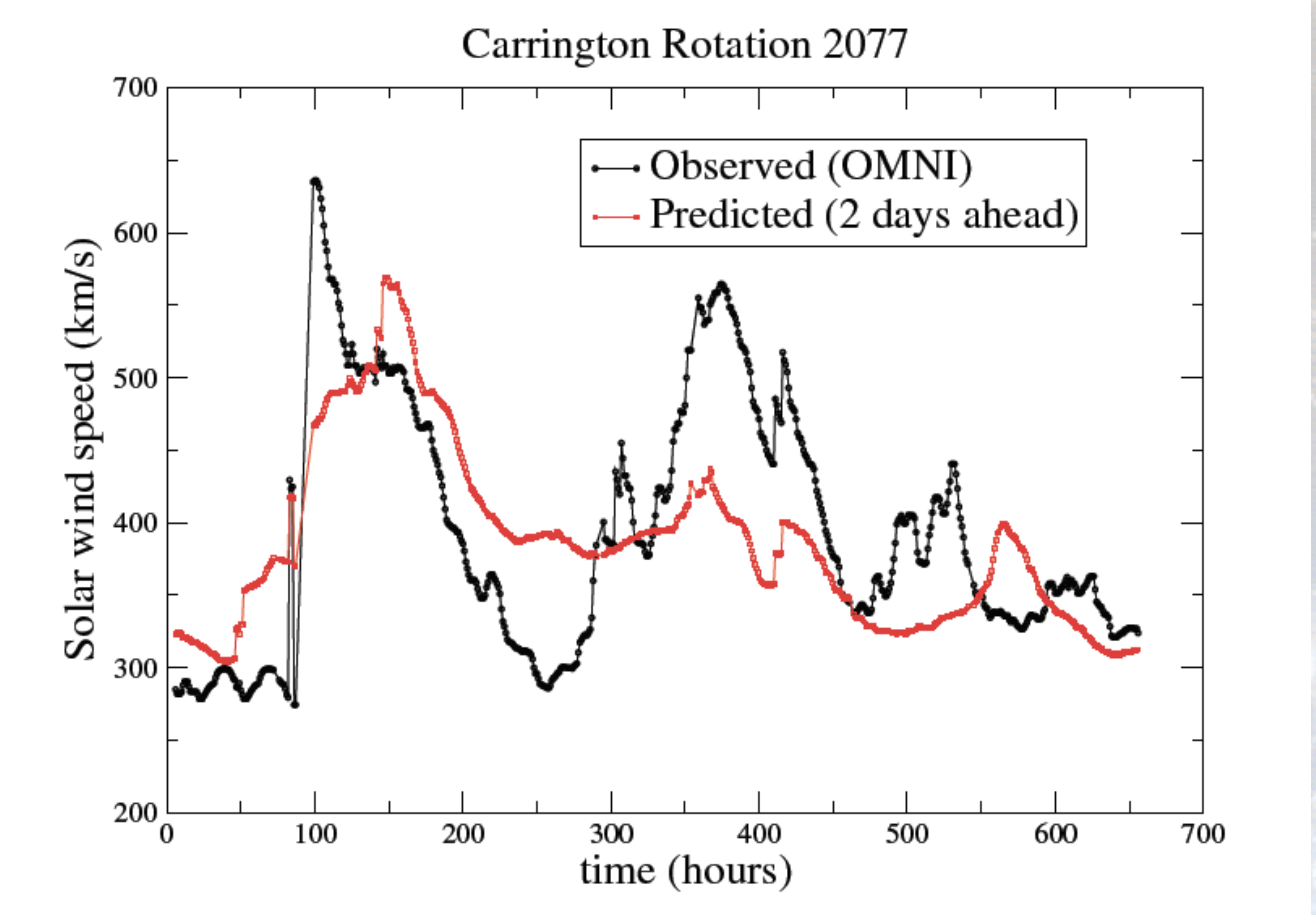
Comparison with the state of the art .

Reiss et al (ApJ Suppl. 2019)

Model	M.A.E km/s	R.M.S.E
WS	74.09	85.27
DCHB	83.83	103.43
WSA	68.54	82.62
Ensemble Median (WS)	71.52	83.36
Ensemble Median (DCHB)	78.27	100.04
Ensemble Median (WSA)	62.24	74.86
Persistence (4 days)	130.48	161.99
Persistence (27 days)	66.54	78.86
DTLR	**54.41**	**64.18**

DTLR - Solar Wind Prediction at L1





Concluding Remarks

- DTLR : motivated by space weather forecasting but is more general.
- Our Bayesian approach is based on a minimal model : possible refinements planned.
- The neural net architecture is also minimal : should combine and pre-train with autoencoder.
- More experiments needed, to extend and select the relevant input information

Thank you!