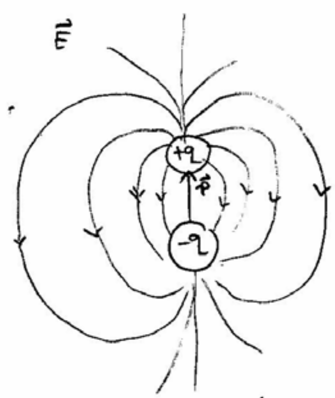
Chapter 32: Magnetism of Matter & Maxwell's Equations

see Sections 1-4, 5-11

Dipoles & Monopoles

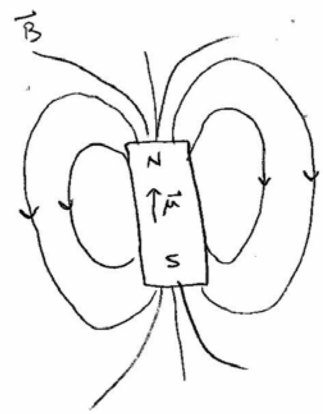
electric dipole $\vec{p}$



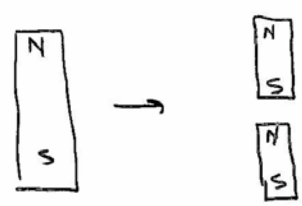
Break an electric dipole in half, and you get a bare charge = <u>a monopole</u>

+q ← electric monopole

magnetic dipole $\vec{\mu}$



but break a magnet in half, and you get <u>two</u> magnets:

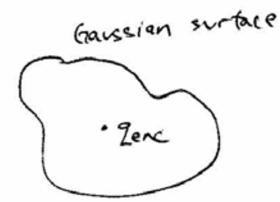


You <u>don't</u> get a magnetic monopole...

The simplest magnetic structure is a dipole . . .
No one has ever detected a magnetic monopole (but high-E physicists are looking for them)

Recall Gauss' Law for $\vec{E}$-fields:

electric flux $\quad \Phi_E = \oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0} \leftarrow$ charge on enclosed monopoles



so what is Gauss' law for $\vec{B}$-fields?

magnetic flux $\quad \Phi_B = \oint \vec{B} \cdot dA = ?$

$= 0$ since there are no magnetic monopoles.

These are the first two of Maxwell's 4 eqn's that describe all electro-magnetic phenomena.

The other two are simply Faraday's and Ampere's Laws, which we have already seen.

The magnetic properties of matter, & induced $\vec{B}$ fields

are a consequence of the quantum-mechanical properties of electrons & atoms... pricipally due to an electron's spin and its orbital angular momentum when in an atom

These details will be left to your quantum mechanics class, and not treated here (but you might have a look at section 32-4)

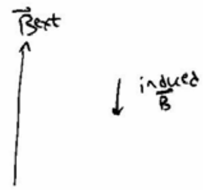
But we will discuss the consequences of magnetization...

- There are 3 types of magnetism:

diamagnetism, paramagnetism } magnetic fields that are induced by an external field $\vec{B}_{ext}$

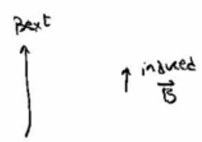
ferromagnetism - persistent magnetism that can occur w/out $\vec{B}_{ext}$

diamagnetism: results when diamagnetic material is placed in external field $\vec{B}_{ext}$.
This results in an induced field that points <u>opposite</u> to $\vec{B}_{ext}$.



If $\vec{B}_{ext}$ is nonuniform, then the diamagnet is <u>repelled</u> to where $\vec{B}_{ext}$ gets weak.

paramagnetism: The induced field is parallel to $\vec{B}_{ext}$.
The paramagnet is attracted to where $\vec{B}_{ext}$ gets stronger, ie, opposite to paramagnetism



ferromagnetism: 'always on' magnetism, like a fridge magnet
iron, nickel, & other metals

Induced Magnetic Fields

Recall Faraday's Law: $\oint \vec{E} \cdot ds = -\frac{d\Phi_B}{dt}$

where $\Phi_B = \oint \vec{B} \cdot d\vec{A}$ = magnetic flux

This law tells us that a time-changing B field will induce an $\vec{E}$ field.

Can the reverse be true? Can a time-changing $\vec{E}$-field induce a $\vec{B}$ field? Yes:

electric flux

$$\underbrace{\underbrace{\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}}_{\text{This is Ampere's law, which is valid when } \vec{E} = \text{constant in time}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}_{\text{but this is the more general formula, known as the Ampere-Maxwell law.}}$$

$\Rightarrow$ a time-changing $\vec{E}$-field can induce a $\vec{B}$ field, even when all currents $i_{enc} = 0$.

Recap: Maxwells 4 Fundamental Eqn's of E&M:

integrals over a Gaussian surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$ Gauss law for $\vec{E}$

$$\oint \vec{B} \cdot d\vec{A} = 0$$ Gauss' law for $\vec{B}$

integral over an Amperian loop

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$ Faraday's law

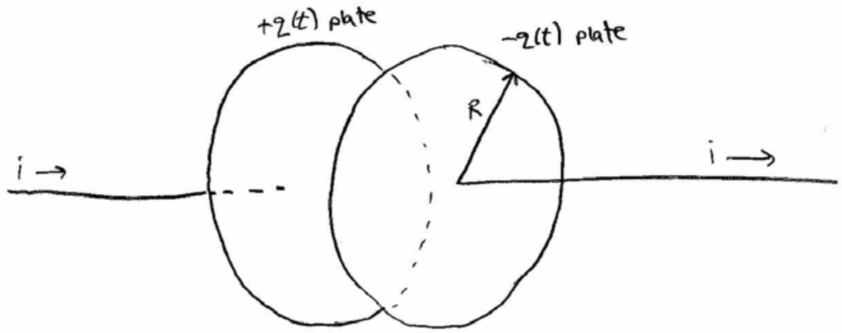
flux thru Amperian loop

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$ Ampere-Maxwell law

Note: these eqn's assume there is no dialectric present

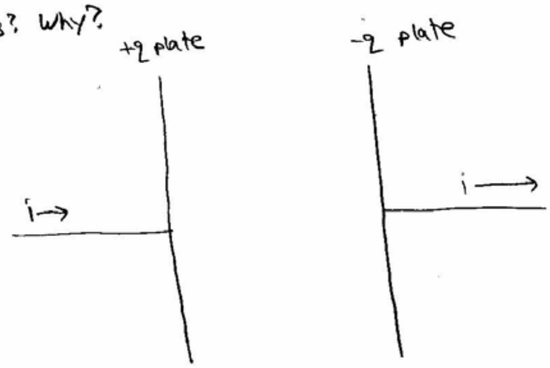
Sample Problem 32-3:

a current i charges up a circular, parallel-plate capacitor.



Calculate $\vec{B}$ between the plates.

- should we expect $\vec{B} \neq 0$ between plates? Why?
- Which of Maxwell's eqn's should we use?



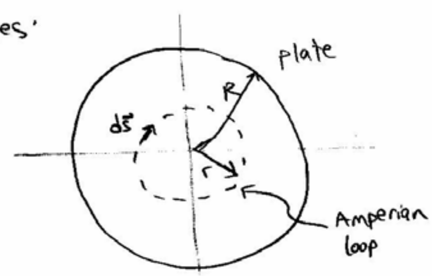
Try the Amp'-Max' eqn'

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

↑
what Amperian loop should I use?

use a circle concentric with the plates.

what is $\vec{B} \cdot d\vec{s}$?



First, note that $\vec{B}$ must be tangential to the loop.

(If it were not, ie, if $\vec{B}$ pointed radially, then $\oint \vec{B} \cdot d\vec{s} = 0$ would imply that $d\Phi_E/dt = 0$, ie, $\vec{E}$ = constant, which is NOT the case as you charge up a capacitor)

Thus $\vec{B} \cdot d\vec{s} = B\,ds$.

Also, B = constant due to the azimuthal symmetry.

$\Rightarrow \oint \vec{B} \cdot d\vec{s} = B \int ds = B \cdot 2\pi r$ = LHS of the eqn.

Now lets deal with the RHS.

What is i_{enc} between the plates? $i_{enc} = 0$

So the Amp-Max Eqn is

$$B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

where $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ = flux thru loop

Draw the $\vec{E}$ field between the plates.

what is $\Phi_E = ?$

$$= EA = E \cdot \pi r^2$$

so $$B \cdot 2\pi r = \mu_0 \varepsilon_0 \cdot \pi r^2 \frac{dE}{dt}$$

or $$B = \frac{1}{2} \mu_0 \varepsilon_0 r \frac{dE}{dt}$$

Now calculate $\frac{dE}{dt}$

way back in Chapter 26 we showed that

$q = \varepsilon_0 E A$ = charge on a capacitor plate

so $$\frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}$$

so $$\frac{dE}{dt} = \frac{i}{\varepsilon_0 A}$$

what is A ?

$A = \pi R^2$ = area of plate

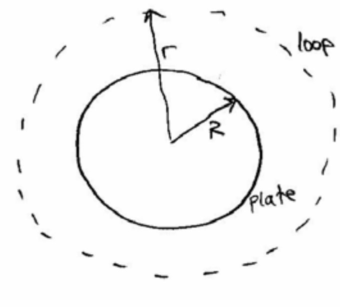
$$= \frac{i}{\varepsilon_0 \pi R^2}$$

$\Rightarrow B = \frac{\mu_0 i r}{2\pi R^2}$ = B-field that induced as you charge up the capacitor.

what is B at $r > R$, <u>outside</u> the capacitor.

Note that the LHS of the Amp-Max eqn is unchanged:

$$\oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi r = LHS$$



while the $RHS = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

where $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ = flux thru the loop

$= ?$

Recall that $\vec{E}$ is zero (or nearly so) outside the capacitor, so

area of disk

$$\Phi_E = EA = E \cdot \pi R^2$$

and $\frac{d\Phi_E}{dt} = \pi R^2 \frac{dE}{dt} = \frac{\pi R^2 i}{\varepsilon_0 \pi R^2} = \frac{i}{\varepsilon_0}$

so $B \cdot 2\pi r = \mu_0 i$

so $B = \frac{\mu_0 i}{2\pi r}$ = field outside capacitor

same as the field due to a long, uninterrupted wire!