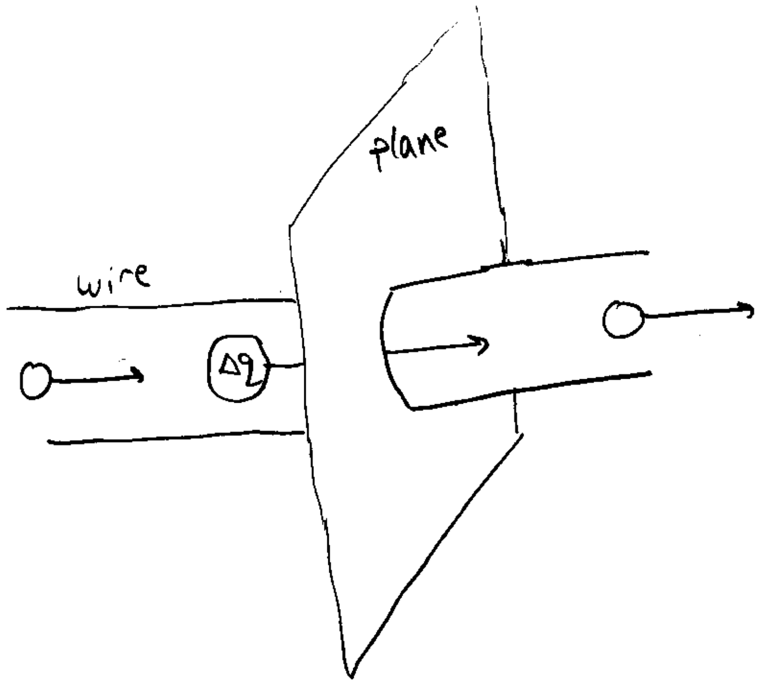
Chapter 27: Currents & Resistance

see sections 1-5, 7

An <u>electric current</u> = rate at which charge passes across some boundary

ex: charge flowing down a wire. Charge Δq crosses plane in time Δt, so

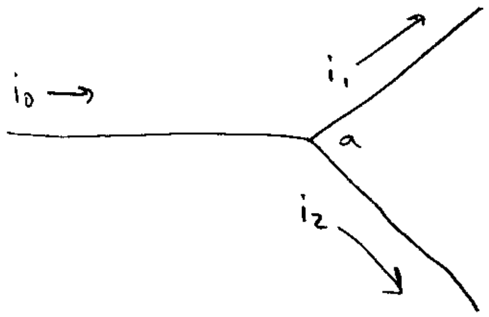


$$i = \frac{\Delta q}{\Delta t} \rightarrow \frac{dq}{dt} = \text{current}$$

i has units of charge/time, or

C/s = 1 ampere (or amp) in SI units.

Think of wires as hoses or pipes in which a charged fluid flows:



Charge conservation requires

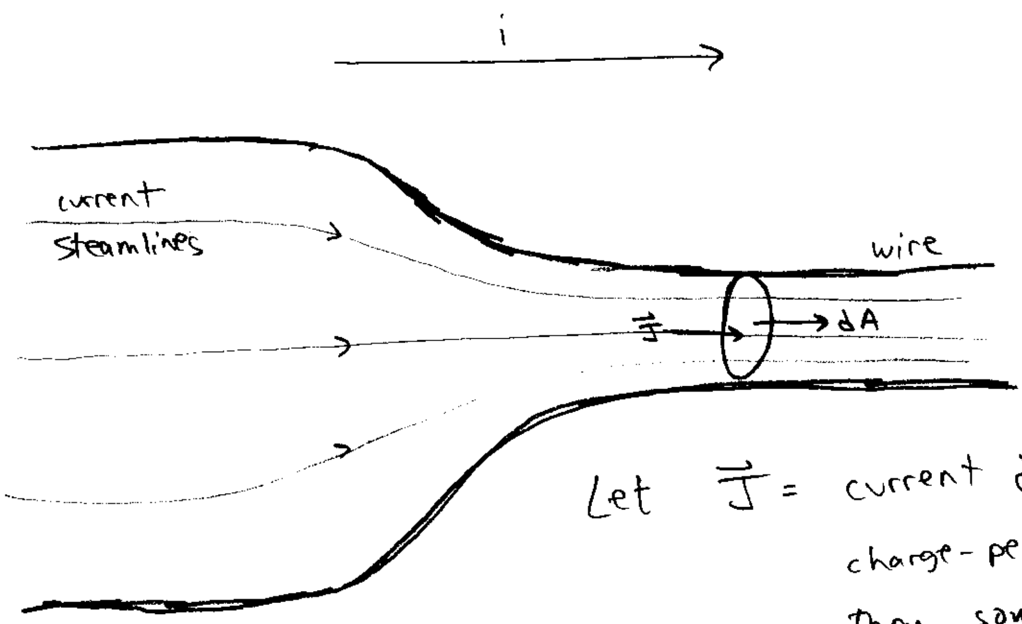
$i_0 = i_1 + i_2$ at junction a.

By convention, the arrow indicates the direction in which positive charge carriers would be traveling ... if present

If the above wires were instead carrying conduction e's, which way would they be travelling?

Current Density

Some times current flows along rather fat wires:



Let $\vec{J}$ = current density, ie, charge-per-area passing thru some surface

so $\quad i = \int \vec{J} \cdot d\vec{A}$ = total current passing thru surface

Note: if $\vec{J}$ is uniform, and parallel to surface element $d\vec{A}$,

then $\quad i = J \int dA = JA$

$$\Rightarrow J = \frac{i}{A}$$

Suppose the above wire's radius decrease by $\times 2$.

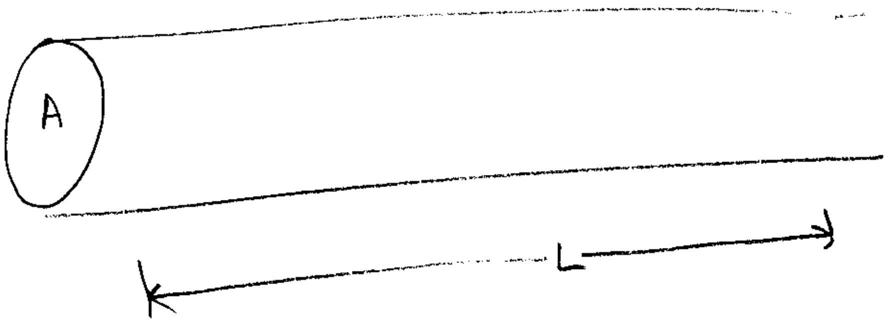
By what factor does J change?

Electron Drift Speed

You can think of conduction electrons as a hot gas whose 'molecules', ie the e's, travel in random directions at high speeds $\sim 10^6$ m/sec

Now put a potential difference across the ends of the wire due to, say, a battery.

The conduction electrons in the wire start to <u>drift</u> down it.



if n = number density of e's in wire with
A = cross sectional area,

Then $q = n \cdot ALe$ = total |charge| on wire of length L

The time for the e's to travel a distance L down the wire is

$$t = \frac{L}{v_d} \quad \text{where } v_d = \text{e's drift speed}$$

so $\quad i = \dfrac{q}{t} = \dfrac{nALe}{L/v_d} = nAev_d =$ current

Note that $\quad J = \dfrac{i}{A} = nev_d =$ current density

or $\quad \vec{J} = ne\vec{v}_d \quad$ in vector form.

Typical drift speeds are $v_d \sim 10^{-5 \text{ to } -4}$ m/sec in household wiring

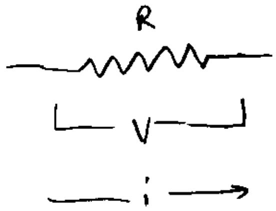
~ 4 to 40 cm/hour!

So why does the light go out immediately after flicking a swith?

Think about opening a valve to an already full hose.

Resistance – tendancy of matter to oppose a current flow.

Which has a larger resistance – copper or plastic?

Let 

represent a resistor.

If V = potential across resistor,
i = current flowing thru resistor

Then $R = \dfrac{V}{i}$ ← Note: The lower the current, the higher the resistance.

↑ sometimes called Ohm's law

Note that R has units of volts/amps ≡ 1 ohm $= 1\Omega$

A related definition is a materials resistivity:

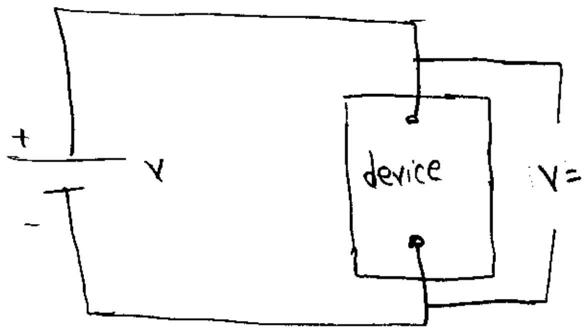
$\rho = \dfrac{E}{J}$ ← E-field, ← current density, which is tabulated for copper, glass, etc on Table 27-1

so $\vec{E} = \rho \vec{J}$ = electric field in a conductor of resistivity ρ carrying a current density $\vec{J}$

Also, conductivity $\sigma = \frac{1}{\rho}$ so $\vec{J} = \sigma \vec{E}$

Power in Electrical Circuits

Connect a battery to some electrical device,
which might be a <u>motor</u> (which does work)
or a resistor (which simply dissipates electrical energy)



V = potential diff' across device

Recall that $U = qV$ = potential energy that you 'store' when you push charge q across potential difference V

so $dU = V\,dq$ = change in small charge dq's potential energy as it travels thru the 'device'

but $i = \frac{dq}{dt}$ so $dU = iV\,dt$

or $\frac{dU}{dt} = \underline{\underline{P = iV}}$ = rate at which electrical energy is transferred from battery to the device

If The device were say a motor with an attached load (such as a winch That is attached to a weight), then

$$P = iV =$$ rate at which the motor does work on the load.

If the device were instead a resistor That obeys $V = iR$,

then $P = i^2 R = \frac{V^2}{R}$ = rate at which the resistor dissipates electrical energy