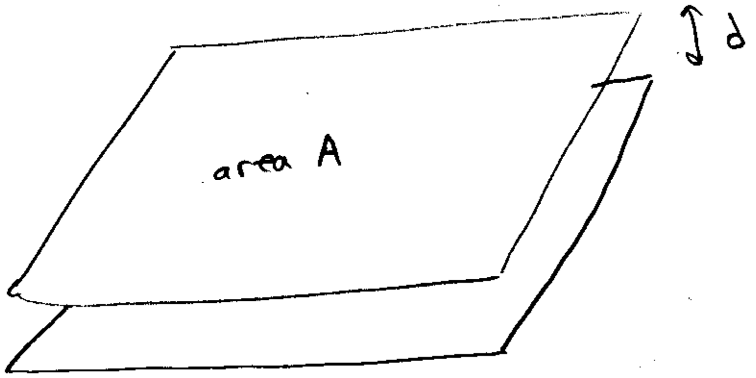
Chapter 26: Capacitance

See Sections 1-8

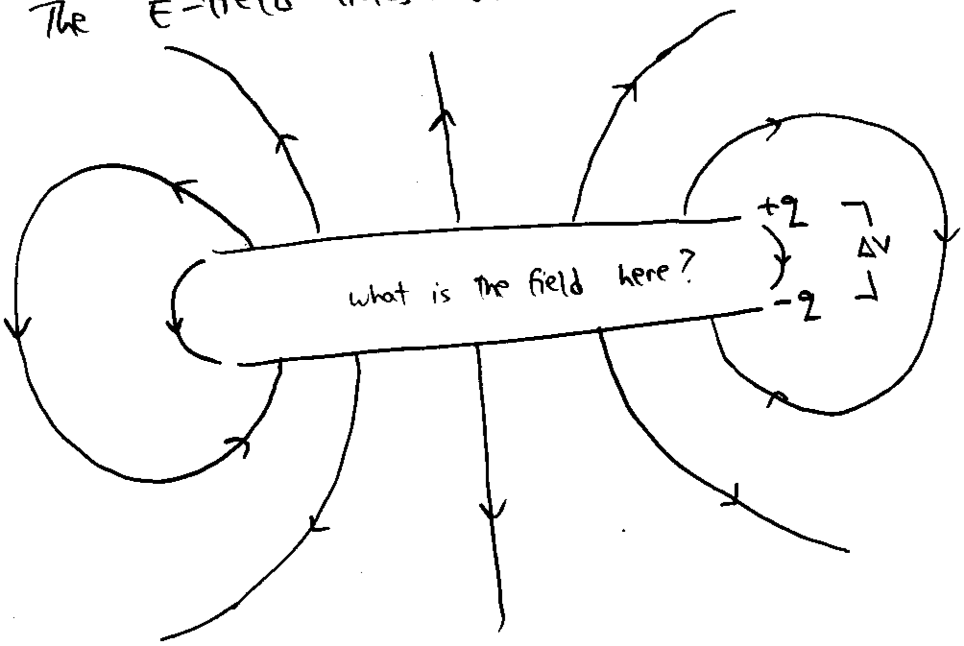
A capacitor stores potential energy.

The typical capacitor is comprised of 2 parallel conductors of area A & seperation d:



You store energy in a capacitor by placing charge q on one plate, and charge $-q$ on the other.

The $\vec{E}$-field lines then look like:



Because the plates are charged, there is a potential difference ΔV between the plates

The capacitance C of a device is obtained from

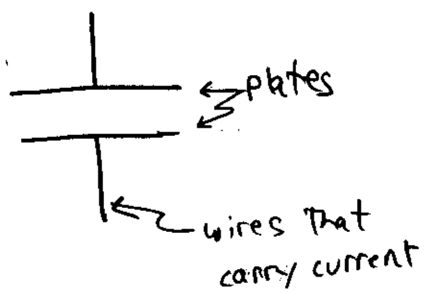
$$q = C\,\Delta V$$

absolute value of charge on one plate

Historically, we use the symbol V to represent the potential difference ΔV... try not to get confused by this!

So $q = CV$ is the equation for a capacitor.

In electrical circuits, we represent a capacitor with the symbol

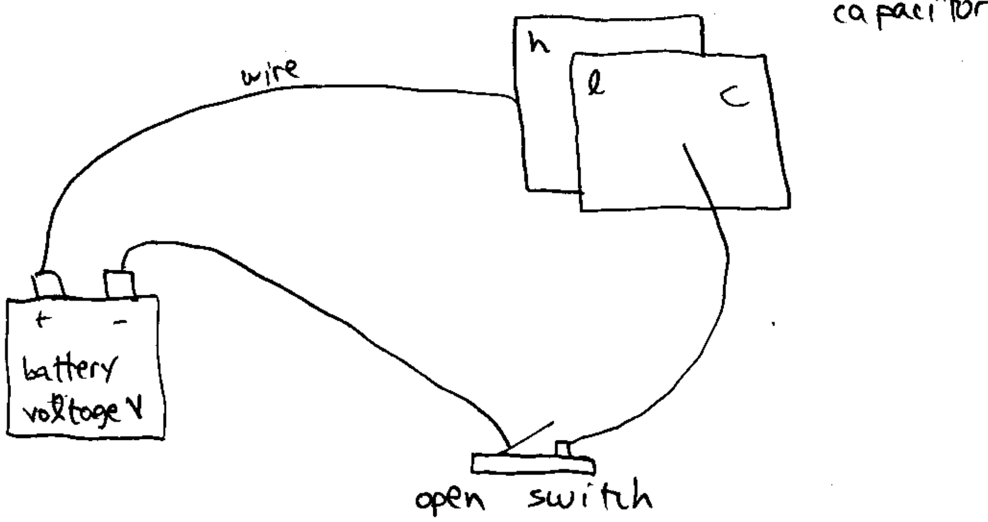


What are the units of C?

coulomb/volt $\equiv$ 1 farad

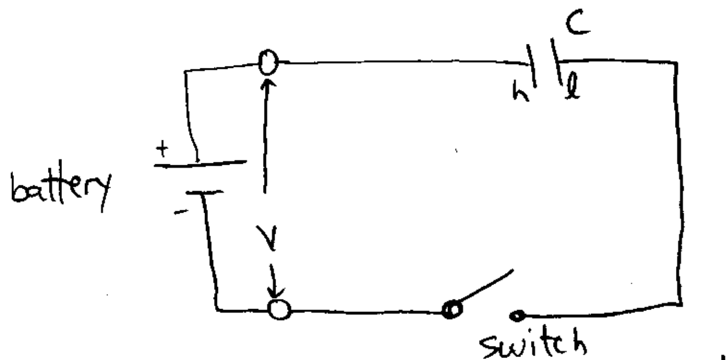
Note that C depends only on the capacitors geometric properties (A and d), and NOT on q, V, etc.

Our first circuit: hook a battery up to a capacitor & a switch:



What happens when switch S is closed?

equivalent circuit diagram:



Closing the switch allows conducting e's to flow along the wires.

conduction e's in plate h are attracted to the batteries positive terminal,

while e's flow from - terminal to plate l

What is the PD across the capacitors plates? V

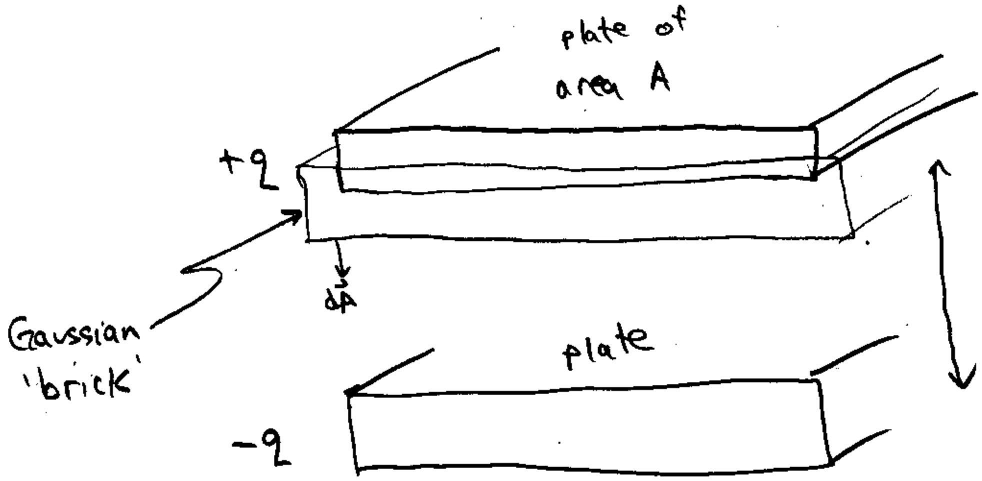
What is the charge on plate h? $q = +CV$

plate l? $q = -CV$

The capacitor is fully charged.

Parallel - Plate Capacitor:

Calculate C for two plates having charges q and a potential difference (pd) V:



what do the $\vec{E}$-field lines look like?

Step 1: Use Gauss' law to calculate $\vec{E}$ between the plates:

$$\varepsilon_0 \oint \underbrace{\vec{E} \cdot d\vec{A}}_{\text{flux}} = q$$

← charge enclosed by Gaussian brick

Note that $\vec{E} \cdot d\vec{A} = E\,dA$ so $q = \varepsilon_0 E A$

Note that, $\sigma = \frac{q}{A}$ = surface charge density

so $E = \frac{\sigma}{\varepsilon_0}$ = 2x that for a single plate.

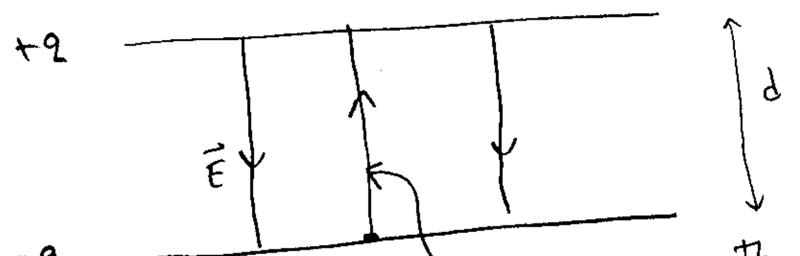
why?

step 2:

Calculate V (formerly $\Delta V = V_f - V_i$):

recall $\quad V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

choose a path that runs from − plate to + plate:



integration path runs from − plate to + plate, by convention

so $\vec{E} \cdot d\vec{s} = -E\,ds$

and $\quad \Delta V = V_f - V_i \longrightarrow V = -\int_{-\text{plate}}^{+\text{plate}} (-E\,ds)$

$$= +E \underbrace{\int_-^+ ds}_{\text{what is this?}}$$

$\Rightarrow \quad V = Ed$

<u>Step 3</u>: calculate $C = \frac{q}{V}$

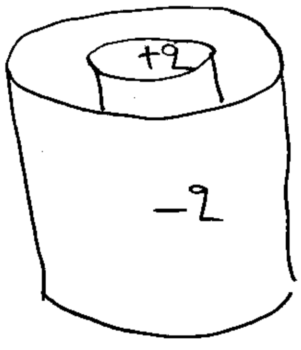
$$= \frac{\varepsilon_0 E A}{E d}$$

so $C = \frac{\varepsilon_0 A}{d}$ for a parallel-plate capacitor.

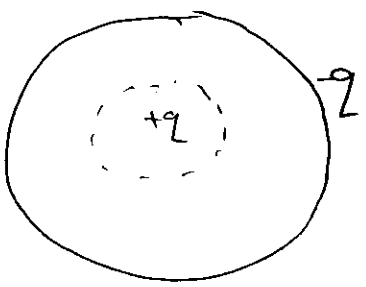
Note that C depends only on the capacitors' <u>physical</u> properties (A & d), and NOT on any <u>electrical</u> properties (like E, V, q)

Your text shows how to derive C for:

cylindrical capacitor:



and a spherical capacitor

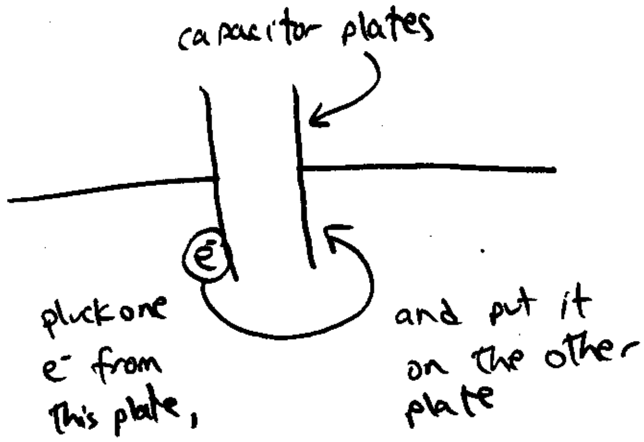


Please read & confirm!

Also read Section 26-4

Energy stored in a capacitor:

Suppose you want to charge up a capacitor, 1 conduction electron at a time...



Recall from Chapter 25: $W_{app} = q' \Delta V$ (Eqn 25-14)

work done by an applied force (ie, by you) — that moves charge q' — across the PD ΔV (which we now call V...)

If you want to move a tiny charge dq', then

$$dW = V \cdot dq' = \text{small differential work you do}$$

but $V = \frac{q'}{C}$, so $dW = \frac{q' dq'}{C}$

and $W = \int dW = \int_0^q \frac{q' dq'}{C} = \frac{q^2}{2C}$

= work you must do to charge up this capacitor

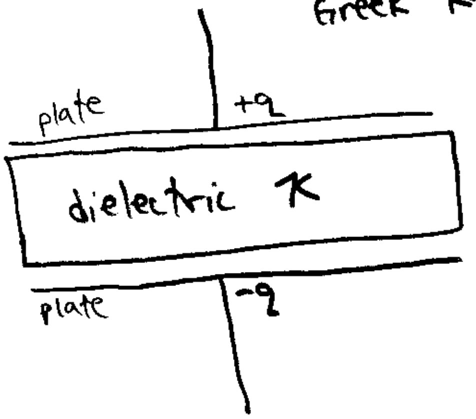
$= $ energy U stored in capacitor

$$\Rightarrow \quad U = \frac{q^2}{2C} = \frac{1}{2} C V^2 \quad \text{since} \quad q = CV$$

<u>Dielectrics</u> = insulating material, often mineral oil or plastic

Michael Faraday (1837) showed that adding a dielectric between the plates in a capacitor can <u>increase</u> its capacitance by a factor κ = dielectric constant

Greek Kappa



$$\underset{\text{capacitance with dielectric}}{C} = \kappa \underbrace{C_{air}}_{\text{capacitance with air between the plates}}$$

<u>See table 26-1 for κ:</u>

κ (air) = 1.00054

κ (pyrex) = 4.5

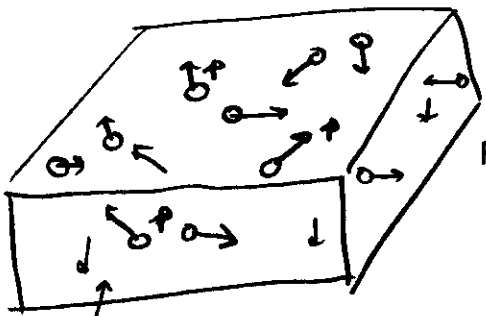
κ (water) = 80

Consider what happens when we put a <u>polar</u> dietectric in a capacitor.

What is a polar substance? a substance whose molecules are little dipole moments. Water, for example:

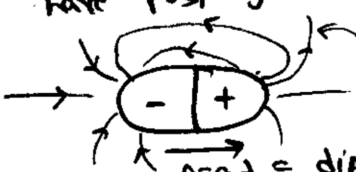
Recall our sketch of a water molecule: (Section 23-9)

'slab' of water



each dipole contributes a small $\vec{E}$-field

water molecules are <u>polar</u>, ie have pos/negative ends,



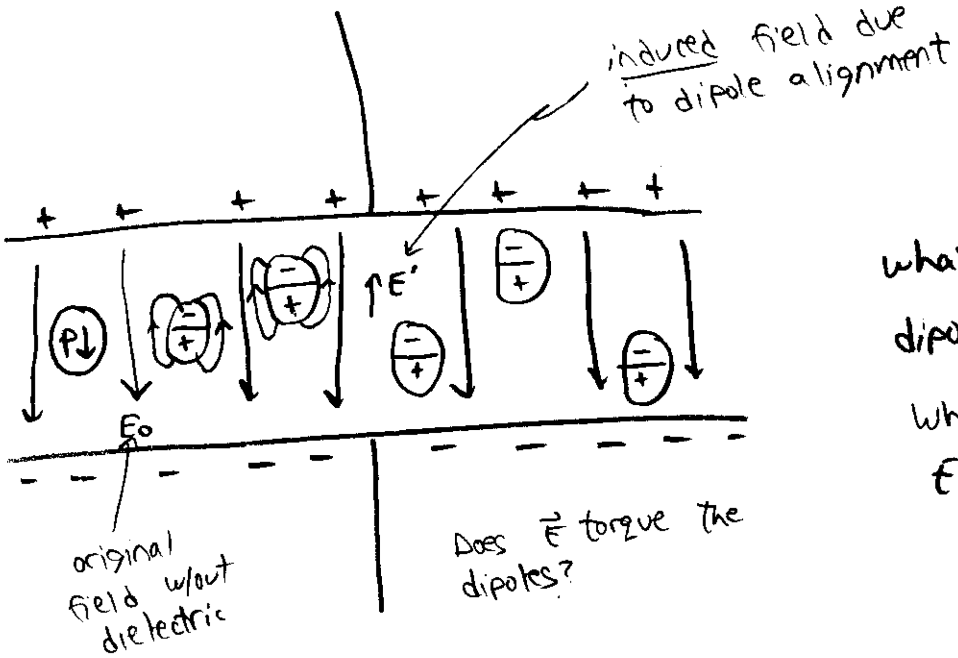
dipole $\vec{E}$-field

$p=qd$ = dipole moment

What is the <u>net</u> $\vec{E}$-field in this water due to all the dipole moments (prior to inserting this slab inside the capacitor)?

What happens when you slide that water-slab inside a <u>charged</u> capacitor?

Remember, $E_0 = \frac{q}{\varepsilon_0 A}$ between plates (initially...)



what will the dipoles do?

what will their E-fields look like?

Has the net E-field increased or <u>decreased</u>?

$$E = \underbrace{E_0}_{\text{original field}} + \underbrace{E'}_{\text{field due to polar water}} \equiv \frac{E_0}{K} = \text{E-field with dielectric}$$

$$= \frac{q}{K\varepsilon_0 A}$$

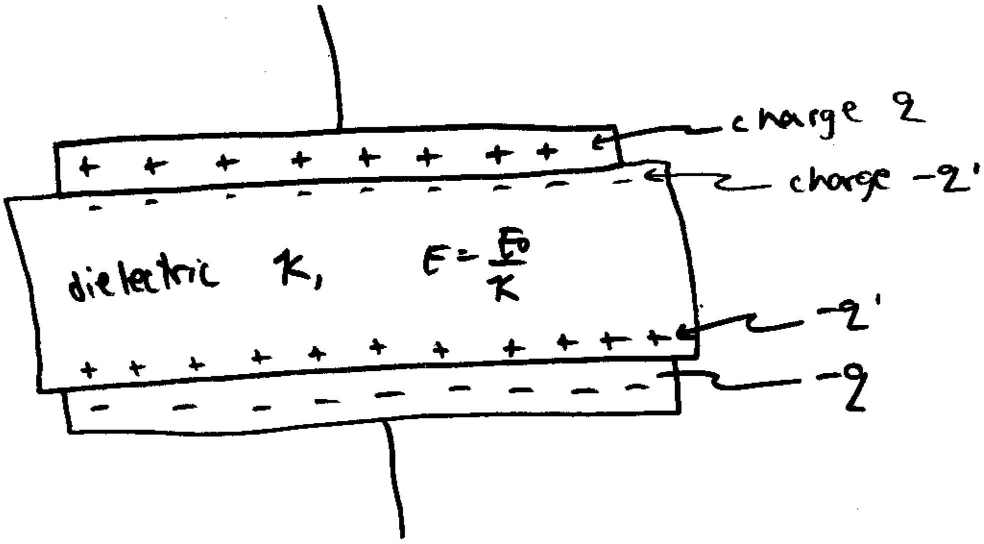
Useful rule of thumb: to account for the effects of a dielectric, replace $\varepsilon_0 \rightarrow K\varepsilon_0$ in all equations

Example: parallel-plate capacitor, has $C = \frac{\varepsilon_0 A}{d}$ (no dielectric)

$$\rightarrow \frac{K\varepsilon_0 A}{d} = K\, C_{air} \quad \text{(as expected)}$$

Induced Charges :

adding a dielectric to a charged capacitor induces charge at the surface of the dielectric :



Section 26-8 shows that the net charge at one plate is

$$q - q' = \frac{q}{\kappa}$$

free charge due to mobile electrons on plate

charge induced in dielectric

$\Rightarrow$ the dielectric : reduces the net charge at a capacitors plates by a factor κ,

reduces E between plates by factor κ

increases C by factor κ

this is effected by replacing $\varepsilon_0 \to K\varepsilon_0$ in all equations

A dielectric forces us to revise Gauss' Law: $\varepsilon_0 \oint K\vec{E}\cdot d\vec{A} = q$ when there is <u>no</u> <u>dielectric</u>

$$\varepsilon_0 \oint K\vec{E}\cdot d\vec{A} = q$$ with dielectric

<u>free</u> charge on surface,
NOT the net charge $q - q'$

keep K inside the integral
since it might vary with
position on your Gaussian surface.