SMALL SHEPHERD SATELLITES IN SATURN'S ENCKE GAP?. J. M. Hahn, *Institute for Computational Astrophysics, Saint Mary's University, Halifax, NS, B3H 3C3, Canada (jhahn@ap.stmarys.ca).*

The Cassini spacecraft has imaged a number of interesting features in Saturn's rings, some of which can be interpreted as disturbances due to small unseen moonlets that might also inhabit the rings (see Porco *et al.* 2005; henceforth P05). Of particular interest here is the Encke gap, which is the narrow gap in Saturn's main A ring maintained by the small satellite Pan. P05 identify four ringlets also residing in the Encke gap (Fig. 1). One ringlet is coincident with Pan's orbit and is likely due to particles in horseshoe orbits. However the other ringlets are puzzling, since Pan's perturbations tends to shepherd particles away. P05 suggest these ringlets could indicate the presence of other small moonlets, since such moonlets might be able to counterbalance Pan's perturbations and confine particles in rings. The following will assess this possibility, and will also estimate the masses and orbits of these hypothetical moonlets.

Pan has mass $\mu_p = 8.7 \times 10^{-12}$ in Saturn masses, semimajor axis $a_p = 133,584$ km, and orbits near the center of the Encke Gap whose edges lie $\Delta_{pe} = 160$ km to either side of Pan's orbit (P05). Pan also interacts with the A ring at its many Lindblad resonances (LRs) in the ring. Resonant interactions cause Pan to exert a negative torque on the ring material at its inner LRs (ILRs), and a positive torque at its outer LRs (OLRs). These Lindblad torques maintain the gap, despite the ring's viscosity which tries to close it. However if a small moonlet also inhabited the gap, these same Lindblad torques exerted by the ring would tend to drive the moonlet towards the gap center. But a stable orbital configuration is possible if this moonlet instead gets trapped at one of Pan's LRs.

To consider this, assume the ring exerts an acceleration on the moonlet of the form $\mathbf{a}_r = 2\beta\dot{r}\hat{\mathbf{r}} + (T_r/r)\hat{\theta}$, where r and $\dot{r}$ are the moonlet's radial position and velocity, and β, T_r are constants. Gauss' planetary equations shows this alters the moonlet's eccentricity at the rate $\dot{e} \simeq \beta e$, and drives it radially at the rate $\dot{r} = 2T_r/r\Omega$, where Ω is the moonlet's angular velocity. Newton's second law of motion then provides the evolution of the moonlet's position vector: $\ddot{\mathbf{r}} = -\nabla(\phi_S + \phi_p) + \mathbf{a}_r$, where ϕ_S and ϕ_p are Saturn's and Pan's gravitational potentials. Note that a similar problem is also considered in Hahn *et al.* (1995), which examined the delivery of planetesimals to a protoplanet's Lindblad resonance via solar nebula drag. Aerodynamic drag has a form similar to that considered above, so that problem's solution may also be applied here.

Begin by assuming that the moonlet's radial and tangential coordinates vary as $r(t) = r_0 + r_1(t)$ and $\theta(t) = \theta_0 + \Omega_0 t + \theta_1(t)$, where r_0 and $\theta_0 + \Omega_0 t$ is the moonlet's unperturbed motion, and r_1, θ_1 are small disturbances satisfying the linearized equation of motion:

$$\ddot{r}_1 + \left(r\frac{\partial\Omega^2}{\partial r}\right)_{r_0} r_1 - 2r_0\dot{\theta}_1 - 2\beta\dot{r}_1 \simeq -\left(\frac{\partial\phi_p}{\partial r}\right)_{r_0}$$
$$\text{and} \quad r_0\ddot{\theta}_1 + 2\Omega_0\dot{r} \simeq -\left(\frac{1}{r}\frac{\partial\phi_p}{\partial\theta}\right)_{r_0} + T_r/r_0 \qquad (1)$$

where $\Omega^2(r) = r^{-1}\partial\phi_s/\partial r$ and $\Omega_0 \equiv \Omega(r_0)$ is the moonlet's angular velocity. We can treat's Pan's orbit as circular, so a Fourier expansion of its potential has the form $\phi_p = \text{Re}\sum\Phi_m(r)e^{i(m\theta-\omega_m t)}$ where the sum ranges over all integral values of m, the $\Phi_m(r)$ are real functions of r, and the disturbing frequency $\omega_m = m\Omega_p$ is the m^{th} harmonic of Pan's angular velocity Ω_p; see Goldreich & Tremaine (1982, henceforth GT82) for details.

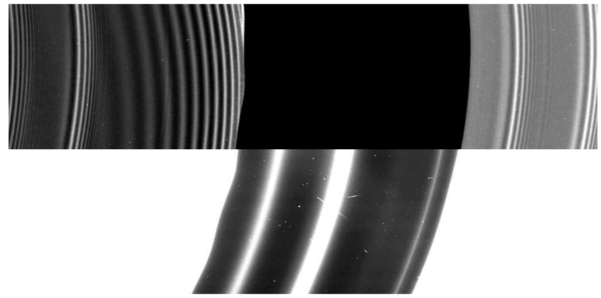

Figure 1: Cassini ISS image N1467351325_2 from the Planetary Rings Node. Upper image shows the Encke gap (black) and nearby ring material with brightness $0.07 < I/F < 0.1$. Lower image is of fainter material having $0.002 < I/F < 0.007$, revealing four ringlets in the gap. Leftmost is the "Encke I" ringlet of P05, a distance $\Delta_I = 65$ km from the gap edge. The central ringlet is at Pan's orbit, the fainter "Encke O1" ringlet lies $\Delta_{O1} = 83$ km from the gap's far side, and the narrow "Encke O2" ringlet is $\Delta_{O2} = 20$ km from the edge.

There are two types of driving forces on the right hand side of Eqn. (1): oscillatory terms due to Pan's periodic perturbations, and the secular acceleration exerted by the ring. The solution to the above equation thus has three parts:

$$r_1(t) = r_e(t) + r_r(t) + \text{Re}\sum_m R_m e^{i(m\theta-\omega_m t)}$$
$$\theta_1(t) = \theta_e(t) + \theta_r(t) + \text{Re}\sum_m \Theta_m e^{i(m\theta-\omega_m t)} \qquad (2)$$

where r_e and θ_e are the moonlet's epicylic motions that satisfy the homogeneous form of Eqn. (1), r_r and θ_r are the secular rates at which the ring causes the moonlet's orbit to drift, and

$$R_m = \frac{\psi_m}{r(D - 2i\beta\omega'_m)}, \qquad \Theta_m = \frac{2i\Omega}{\omega'_m}\frac{R_m}{r} + \frac{im\Phi_m}{(r\omega'_m)^2} \qquad (3)$$

are the complex amplitude's of the moonlet's response to Pan's m^{th} perturbation. In the above, $\omega'_m = m(\Omega - \Omega_p)$ is the Doppler–shifted forcing frequency, $\psi_m = -r\partial\Phi_m/\partial r - 2m\Omega\Phi_m/\omega'_m$ is Pan's forcing function, $D(r) = \kappa^2 - \omega'^2_m$ is the moonlet's distance from the m^{th} Lindblad resonance, and all quantities are evaluated at $r = r_0$. Note that when the ring perturbation is absent, the above solution reduces to that given in GT82. We also make the usual assumption that the moonlet is dominated by perturbations from a single m^{th} resonance, so $r_1 \simeq \text{Re}(R_m e^{i\omega'_m t})$ and $\theta_1 \simeq \text{Re}(\Theta_m e^{i\omega'_m t})$. However this assumption is dicey in much of the Encke gap

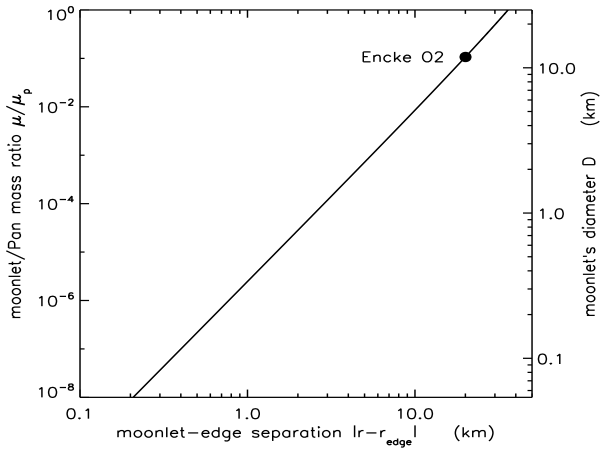

Figure 2: Moonlet mass μ (in units of Pan's, left axis) and diameter D (right axis) versus distance from the gap edge, assuming the nearby ring has $\sigma \sim 50$ gm/cm^2 and $\mu_d = \pi\sigma r^2/M_S \sim 5\times10^{-8}$. The dot is the distance of the O2 ringlet from the gap edge. Note the curve is unreliable right of the dot, since the calculation does not account for the torque that a large, nearly Pan–sized moonlet exerts on Pan.

since orbits within $\Delta_c \simeq 1.3\mu_p^{2/7}a_p \simeq 120$ km of Pan's are chaotic due to overlapping resonances (Wisdom 1980).

The specific torque Pan exerts on the moonlet is $T_p' = -|\mathbf{r}\times\nabla\phi_p| = -\partial\phi_p/\partial\theta = m\Phi_m(r)\sin(m\theta-\omega_m t)$. Inserting $r(t)$ and $\theta(t)$ into T_p', Taylor expanding to first order, and time averaging yields $T_p' \simeq m\psi_m \mathrm{Im}(R_m)/2r = m\beta\omega_m'(\psi_m/r)^2/[D^2+(2\beta\omega_m')^2]$; see Hahn *et al.* (1995) for details. The Lindblad resonance is where $D(r_m)=0$, so $\omega_m'(r_m) = s\kappa(r_m)$ where $s=+1$ (-1) at an ILR (OLR); this lies a distance $\Delta_m = r_m - a_p \simeq -2sr/3m$ away from the moonlet. The forcing function is $\psi_m \simeq sfm\mu_p(r\Omega)^2$ where $f \simeq 1.6$ when evaluated at the m^{th} resonance (GT82), so the maximum torque Pan exerts on the moonlet as it drifts into the m^{th} resonance is

$$T_p = T_p'\mu M_S \simeq -\frac{2f^2\mu_p^2\mu}{27}\left(\frac{\Omega}{\beta}\right)\left(\frac{r}{r-a_p}\right)^3 M_S r^2\Omega^2 \quad (4)$$

where μ is the moonlet's mass in units of Saturn's mass M_S. Since the ring torque is driving the moonlet towards Pan, T_p must be positive (negative) at an OLR (ILR) if the moonlet is to get trapped via a balance of torques. This in turn requires $\beta < 0$, *i.e.*, the moonlet's eccentricity must also be damped.

To evaluate β we need the rate at which the moonlet's eccentricity e evolves. GT82 show that a narrow ring of mass $\delta m = 2\pi\sigma r\delta r$, mass surface density σ, and radial width δr orbiting a distance Δ away will damp the moonlet's eccentricity at the rate $\delta\dot{e} = -(g/2)\mu(\delta m/M_S)|r/\Delta|^5\Omega e$, where $g=0.148$, and Δ is the ring–moonlet separation. Provided motions at the corotation resonances are not saturated, the moonlet's total e–damping rate due a broad ring is

$$\frac{de}{dt} = \int\delta\dot{e} \simeq -\frac{1}{4}g\mu\mu_d\left|\frac{r}{r-r_{\rm edge}}\right|^4\Omega e \quad (5)$$

where $\mu_d = \pi\sigma r^2/M_S$ is the normalized disk mass, and $|r-r_{\rm edge}|$ is the moonlet's distance from the nearest gap edge. Since $de/dt = \beta e$, β is the coefficient multiplying e in Eqn. (5). Inserting β into (4) yields the torque Pan exerts on the moonlet:

$$T_p \simeq \mathrm{sgn}(r_0-a_p)\frac{8f^2\mu_p^2}{27g\mu_d}\left|\frac{r_0}{r_0-a_p}\right|^3\left|\frac{r_0-r_{\rm edge}}{r_0}\right|^4 M_S r_0^2\Omega_0^2, \quad (6)$$

which is repulsive—this torque drives the moonlet away from Pan.

However the ring also exerts a torque that drives the moonlet towards Pan. The ring's radial torque density is $\mathcal{T}(r) = \mathrm{sgn}(r_0-r)(8\pi f^2/81)\mu^2\mu_d|r_0/(r_0-r)|^4 M_S r_0\Omega_0^2$ (GT82), so the total torque integrated across the nearby ring is

$$T_r = \int\mathcal{T}dr \simeq -\mathrm{sgn}(r_0-a_p)\frac{8\pi f^2}{243}\mu^2\mu_d\left|\frac{r_0}{r_0-r_{\rm edge}}\right|^3 M_S r_0^2\Omega_0^2. \quad (7)$$

The moonlet is trapped at Pan's m^{th} LR when these torques sum to zero, which results in a relationship between the moonlet's mass μ and its position in the gap:

$$\frac{\mu}{\mu_p} \simeq \frac{3}{\sqrt{\pi g\mu_d}}\left|\frac{r_0}{r_0-a_p}\right|^{3/2}\left|\frac{r_0-r_{\rm edge}}{r_0}\right|^{7/2} \quad (8)$$

which is plotted in Fig. 2. This figure indicates that resonant effects tends to sort ring particles near a gap edge according to size, with larger particles being driven further away from the edge. Large ring particles are in fact *attracted* to gaps. We also wonder whether this size–sorting might play a role in the development of the ropy features that P05 spot at the edges of the Encke gap. Lastly, we note that this phenomena might also play a role during the early stages of planet formation, when protoplanets begin to carve open gaps in the planetesimal disk.

Unfortunately, it is unlikely that the shepherding mechanism envisioned here can account for all three Encke ringlets I, O1, and O2. For instance, if one assumes the O2 ringlet is confined by torques from Pan and a moonlet, then the net torque on the ringlet, $-\mathcal{T} \propto [\mu^2/\Delta_{mr}^4 - \mu_p^2/\Delta_{pr}^4]$, must be zero, so the moonlet–Pan mass ratio obeys $\mu/\mu_p = (\Delta_{mr}/\Delta_{pr})^2$ where Δ_{mr} and Δ_{pr} are the moonlet–ringlet and Pan–ringlet separations. Combining this result with Eqn. (8) then yields a unique mass and orbit for the shepherd satellite: $\mu \simeq 0.0061\mu_p$ orbiting a distance $\Delta_{me} \simeq 9.2$ km inwards of the gap's edge. Such a body would have a diameter $(\mu/\mu_p)^{1/3} \sim 1/5^{\rm th}$ of Pan's, and its brightness would be $1/25^{\rm th}$ that of Pan's, causing us to wonder if this $D \sim 5$ km–sized body would already have been detected in Cassini images. Invoking this scenario for the other ringlets is even more problematic, since this would require moonlets with masses $\sim 20\%$ to $\sim 50\%$ of Pan's.

If small unseen moonlets are in fact shepherding the Encke ringlets, then they must also be susceptible to additional eccentricity damping that is more vigorous than that considered here (*i.e.*, the β in Eqn. 4 must be larger). Radiation forces like the Yarkovsky effect come to mind, and this possibility is being explored.

References: Goldreich & Tremaine, 1982, *ARA&A*, 20, 249. Hahn *et al*, 1995, *Icarus*, 117, 25. Porco *et al*, 2005, *Science*, 307, 1226. Wisdom, 1980, *AJ*, 85, 1122.