DISRUPTION OF A SMALL ICY SATELLITE AND THE EVOLUTION OF THE RESULTING DEBRIS RING. K. Peek, *Mount Holyoke College, South Hadley MA 01075, USA, (kmpeek@mtholyoke.edu)*, J. M. Hahn, *Lunar and Planetary Institute, Houston TX 77058, USA, (hahn@lpi.usra.edu)*.

It has been known for some time that the small icy satellites of the giant planets are susceptible to disruption by cometary impacts [1]. The disruption of a satellite will produce a very narrow and dense debris ring that is likely to suffer gravitational instabilities that is manifested as density wakes and/or waves. Since vigorous angular momentum transport is also associated with wakes and waves, it is likely that a newly–formed ring would spread quite rapidly due to the density contrasts within the ring. Indeed, our initial interest was to determine whether the disruption of a Mimas–class satellite orbiting exterior to Saturn's Roche limit could deliver additional mass to Saturn's rings that largely orbit interior to the Roche limit. However it is shown below that the short answer to this question is no. A self–gravitating debris ring initially spreads quite rapidly due to the formation of wakes, but the process soon stalls due to the concurrent decrease in the ring's surface density. Although the disruption of a major satellite like Mimas is likely to have occurred several times over the age of the solar system, debris–ring spreading still proceeds too slowly to substantially contribute to the mass–budget of Saturn's rings. Nonetheless, the frequency of satellite disruption and the subsequent evolution of the debris is of some interest, which we detail below.

The size of the smallest impactor that is able to disrupt a target satellite is obtained via energy considerations. The most likely impactor is an ecliptic comet that has diffused inwards from the Kuiper Belt, and such bodies approach Saturn with a typical velocity–at–infinity $v_\infty \sim 3$ km/sec [2]. The impactor also acquires the additional velocity $\Delta v = \sqrt{2}v_t$ as it falls down Saturn's gravitational well, where v_t is the target satellite's orbital velocity about Saturn. Thus the impact velocity will be about $v_i \sim \sqrt{v_\infty^2 + \Delta v^2 + v_t^2} \sim \sqrt{3GM_s/a_t}$ where G is the gravitation constant, M_s is the mass of Saturn, and a_t is the target satellite's semimajor axis. Disrupting a satellite requires a kinetic energy $m_i v_i^2/2$ in excess of $fE_b + E_m$ where m_i is the impactor mass, $E_b = 3GM_t^2/5R_t$ is the gravitational binding energy of the target satellite that has mass M_t and radius R_t, the adjustment factor $f \simeq 1 - a_{roche}/2a_t$ accounts for the fact that the resulting debris need only be launched beyond the target's Hill sphere with a_{roche} being Saturn's Roche limit, and $E_m = 4\pi S_t R_t^3/3$ is the mechanical energy required to fragment a target of strength S_t. Disruption thus requires an impactor of radius

$$R_i \gtrsim \gamma[1 + (R_t/R_\star)^2]^{1/3} R_t \qquad (1)$$

where $R_\star \equiv \sqrt{5S_t/4\pi fG\rho_t^2}$ is the target radius that has equal mechanical and gravitational energies, and the dimensionless coefficient $\gamma \equiv (2S_t a_t/3GM_s\rho_i)^{1/3}$ where ρ_i is the impactor's density. For example, disrupting a satellite such as Mimas requires an impactor of radius $R_i \gtrsim 5$ km assuming an impactor density $\rho_i = 1$ gm/cm^3 and a target strength of $S_t = 10^7$ dynes/cm^2 that is typical of icy satellites [3].

The frequency of this impact event at Saturn can be scaled from impact rates recently reported for the Galilean satellites at Jupiter. For an order–of–magnitude estimate of the impact rate r_x onto object x, take $r_x \sim N_x P_x$ where N_x is the number of impactors within striking distance of planet or satellite x and P_x is the impactors' characteristic impact probability per unit time. The ratio of impact rates at Jupiter (J) and Saturn (S) is $r_J/r_S \sim (N_J/N_S)(P_J/P_S) \simeq 2.4$ according to Table I of [2]. The impact probability per orbital period is $P_x = (R_x/a_x)^2 G_x f_x$ where R_x is the target's radius, a_x is its semimajor axis, $G_x = 1 + (v_{esc,x}/v_\infty)^2$ is the target's gravitational focusing factor that depends upon the target's surface escape velocity $v_{esc,x}$, and the function $f_x(a_i/a_x, e_i, i_i)$ depends upon the impactor's scaled semimajor axis a_i/a_x, eccentricity e_i, and inclination i_i [4]. Ecliptic comets approach Jupiter at a typical velocity of $v_\infty \sim 4$ km/sec [2] so Jupiter has a focusing factor $G_J \simeq 220$ while Saturn has $G_S \simeq 150$. If it is also assumed that each comet swarm in the vicinity of Jupiter and Saturn have similar orbital distributions, then the functions $f_J \sim f_S$ and the relative impact probabilities are $P_J/P_S \sim (G_J/G_S)(R_J/a_J)^2/(R_S/a_S)^2 \sim 7.4$. Since $r_J/r_S \simeq 2.4 \sim (N_J/N_S)(P_J/P_S)$, this implies that the ratio of ecliptic comets within striking distance of each planet is $N_J/N_S \sim 0.3$; this is also the ratio of potential impactors within reach of a Galilean satellite at Jupiter and our target satellite at Saturn.

The orbits of the swarms of comets at Jupiter and Saturn should also be distributed similarly since these impactors' orbits are nearly parabolic with isotropic inclinations in a planet–centered coordinate system. Thus the impactors at the target satellite as well as at the Galilean satellite g have $f_t \sim f_g$. The gravitational focusing factors of these small satellites are $G_t \simeq G_g \simeq 1$, so their relative impact probabilities are $P_t/P_g \sim (R_t/a_t)^2/(R_g/a_g)^2$ and the relative impact rates are $r_t/r_g \sim (N_S/N_J)(P_t/P_g)$. Impacts at Io will be used to calibrate impact rates in the Saturnian system, so $g = Io$ henceforth. The frequency of impacts at Io relative to Jupiter is $r_{Io}/r_J = 1.4\times10^{-4}$ [5] where $r_J(d_i > 1 \text{ km}) \simeq 3.3\times10^{-3}$ impacts/year is the rate of impacts at Jupiter by ecliptic comets having diameters $d_i > 1$ km [2], so $r_{Io}(d_i > 1 \text{ km}) \simeq 5\times10^{-7}$ impacts/year. Assuming impactors at Jupiter and Saturn have the same cumulative power–law size distribution $N(d_i) \propto d_i^{-Q}$, the impact rate at the target satellite is $r_t \sim (N_S/N_J)(R_t/R_{Io})^2(a_{Io}/a_t)^2(d_i/1 \text{ km})^{-Q}\times r_{Io}(d_i > 1 \text{ km})$. Combining this result with Eqn. (1) yields the satellite's disruption timescale $\tau = r_t^{-1}$,

$$\tau \sim \frac{N_J}{N_S}\left(\frac{a_t}{a_{Io}}\right)^2\left(\frac{R_{Io}}{1 \text{ km}}\right)^Q\left(\frac{R_t}{R_{Io}}\right)^{Q-2}\times \left[1+\left(\frac{R_t}{R_\star}\right)^2\right]^{Q/3}\frac{(2\gamma)^Q}{r_{Io}(d_i > 1 \text{ km})}. \qquad (2)$$

If we adopt the $Q = 2$ size distribution that is in some agreement with the Galilean crater record [5], then Eqn. (2) gives a

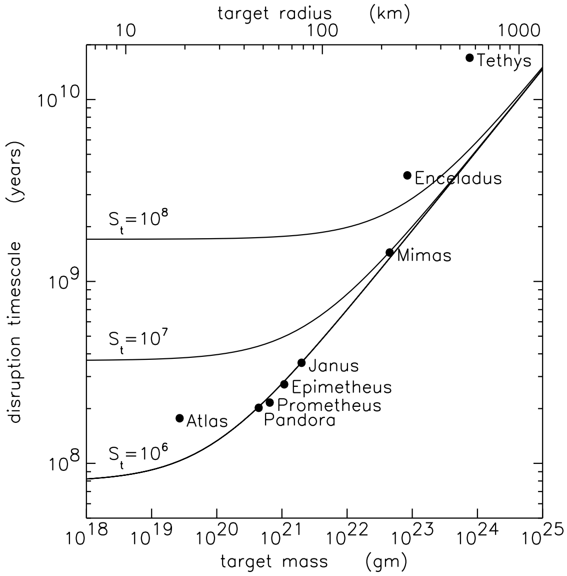

Figure 1: The solid curves give the disruption timescale τ versus satellite mass M_t for the indicated values of satellite strength S_t in dynes/cm^2; these curves are evaluated at Mimas' distance from Saturn. The dots indicate various satellite lifetimes assuming $S_t = 10^7$ dynes/cm^2.

disruption timescale of $\tau \sim 10^9$ years for Mimas. The disruption timescale versus impactor mass is shown in Fig. 1 with the solid curves demonstrating how these results depend on the strength S_t that is known only to an order of magnitude [3]. Figure 1 also shows the disruption timescale for various Saturnian satellites assuming $S_t = 10^7$ dynes/cm^2. But keep in mind that Fig. 1 adopts a relatively shallow $Q = 2$ size distribution. If impactors at Saturn instead have a steeper $Q = 3.2$, as is observed among Kuiper Belt Objects [6], then disruption timescales can be considerably longer.

Figure 1 suggests that a satellite like Mimas has been disrupted ~ 4 times over the age of the solar system and that the smaller satellites have been disrupted many times over. Using Mimas as an example, its complete disruption requires tossing debris beyond the satellite's Hill radius $R_h = (M_t/3M_s)^{1/3}a_t$. Since the most probable disruption scenario is the minimum–energy event, this would result in a debris ring of radial width $\Delta r_o \sim 2R_h \sim 1000$ km for Mimas. Azimuthal variations in the ring are smoothed out after a few synodic periods or in a few years. This dense debris ring would have an initial surface density of $\sigma_o = M_t/2\pi a_t \Delta r_o \sim 3000$ gm/cm^2 and a normal optical depth of $\tau = 3\sigma_o/4\rho_t R \sim 20(R/1\text{ m})^{-1}$ assuming the debris particles have a characteristic radius R. These particles will have initial eccentricities of $e_0 \sim R_h/a_t \sim 0.003$, so the debris will recollide at velocities $c_0 \sim e_0 v_K \sim 40$ m/sec. Laboratory experiments and impact scaling laws indicate that collisions at these velocities will grind the ring particles down to sizes $\ll 1$ m [7], so the ring's high optical depth results in very frequent collisions that steadily damps the particles' random velocities. This velocity damping continues until the ring approaches gravitationally instability.

The threshold for gravitational instability is when $Q_T = c\Omega/\pi G\sigma = 1$. Rings with $Q_T > 1$ are stable whereas rings with $Q_T < 1$ can break up into concentric ringlets having characteristic radial separations $\lambda_c = 4\pi^2 G\sigma/\Omega^2$ [8]. However a marginally stable ring with $1 \lesssim Q_T \lesssim 2$ exhibits transient clumps of particles known as wakes [9, 10]. The gravitational scattering of particles by these wakes counterbalances collisional velocity damping and tends to keep the ring near the $Q_T = 1$ threshold. Consequently, collisions in the debris ring damps particle velocities down to $c_1 \simeq \pi G\sigma Q_T/\Omega \sim 8$ cm/sec which corresponds to eccentricities $e_1 \simeq c_1/v_K \sim 6 \times 10^{-6}$. Collisional damping also flattens the ring until the particles have inclinations of $\sin i \sim e_1/2$. If the ring particles have a coefficient of restitution ϵ, then the number of collisions needed to drive the ring eccentricities from $e_0 \rightarrow e_1$ is $N \sim \ln(e_1/e_0)/\ln\epsilon$. Although ϵ for water ice is a sensitive function of particle size and impact velocities [7, 11], it is likely smaller than 0.9 and perhaps closer to 0.1. For $\epsilon < 0.9$, $N < 60$ collisions. Since each particle suffers $\sim 2\tau > 40$ collisions per orbit, collisional velocity damping proceeds very quickly indeed.

The azimuthal separation between wakes is about $\sim 2\lambda_c$ [9], so a narrow annulus in the debris ring should contain about $m \sim 2\pi a_t/2\lambda_c \sim M_s/4\pi\sigma a_t^2 \sim 4 \times 10^4$ wakes. Since the wakes impose a non–axisymmetric density pattern upon the ring, the ring's self–attraction results in a torque that causes the ring to spread radially. The viscosity associated with this gravitational torque is $\nu_g \simeq CG^2\sigma^2/\Omega^3$ [12] where the factor $C \simeq 18(r/a_{roche})^5 \simeq 160$ in this application. Wakes also appear to be genetically related to the spiral density waves that are common in marginally stable $Q_T \simeq 1$ particle disks (c.f. Takeda and Ida 2001): both have the same pitch angle of about 27°, the same radial spacings of $\sim \lambda_c$ between density crests, and both cause radial spreading due to the viscosity ν_g.

The timescale for the gravitational torque to cause the ring to spread a distance Δr is $\tau_g \simeq 2r\Delta r/3\nu_g \simeq 4(M_s/M_t)^2(\Delta r/r)^3 P_{orb}/C$ where P_{orb} is the orbital period. Although the ring–doubling timescale is initially quite short, *i.e.*, 2×10^3 years for $\Delta r \simeq \Delta r_o$, ring spreading soon stalls due to the associated decrease in the ring's surface density. Meanwhile, accretion within the debris ring will generate a subsequent generation of satellites. However these satellites will also be susceptible to disruption by cometary impacts, so this disruption–accretion cycle is destined to repeat again.

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