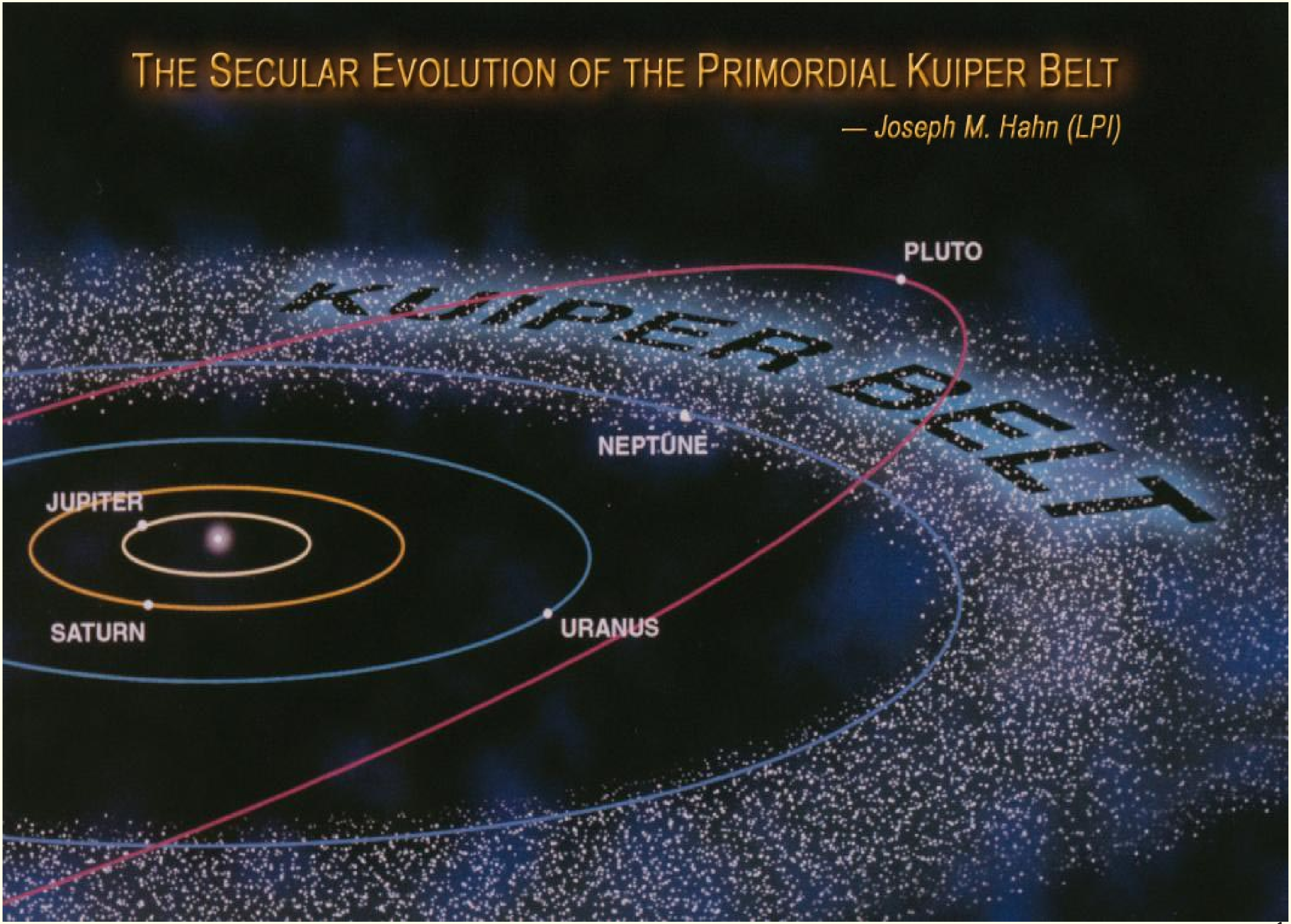
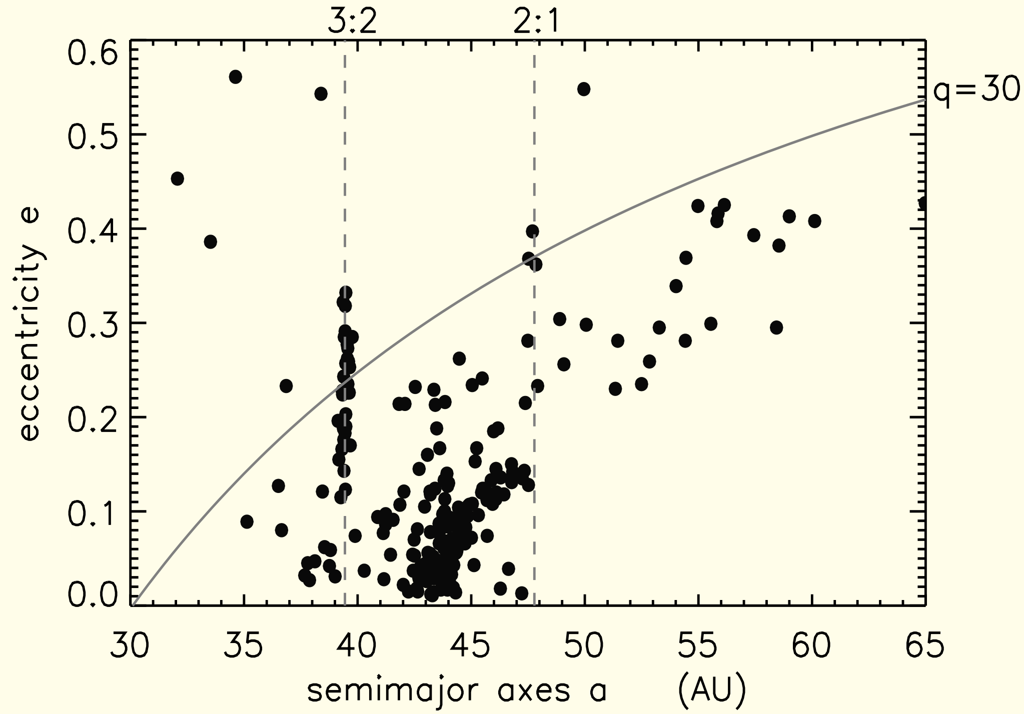
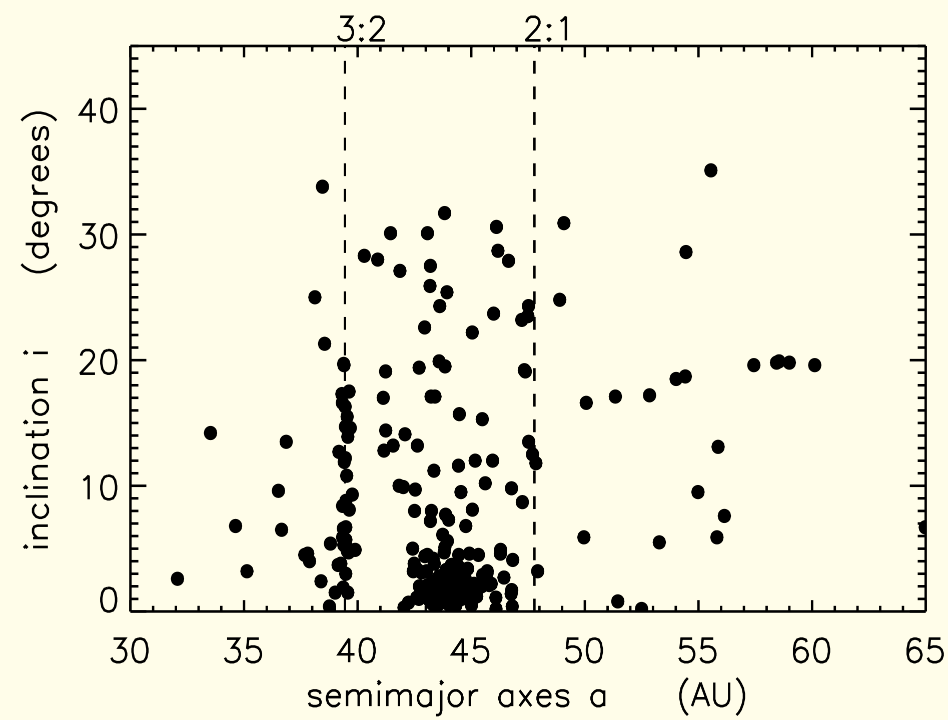

THE SECULAR EVOLUTION OF THE PRIMORDIAL KUIPER BELT
— Joseph M. Hahn (LPI)
PLUTO
KUIPER BELT
NEPTUNE
JUPITER
SATURN
URANUS

# What Happened to the Kuiper Belt?

- evidently some process has excited the KB's $e$'s and $i$'s
- Neptune's outward migration can explain the cluster of KBOs at the 3:2
  - but this does not account for the high $i \sim 15°+$
  - nor the low–$q$ KBOs in the Scattered Disk.
    - ∗ Gomez (2003) reports that high $e, i$ Scattered KBOs can 'invade' the Main Belt, but $\varepsilon \sim 0.001$
- I will consider a more efficient processes



# Secular Evolution of the Kuiper Belt

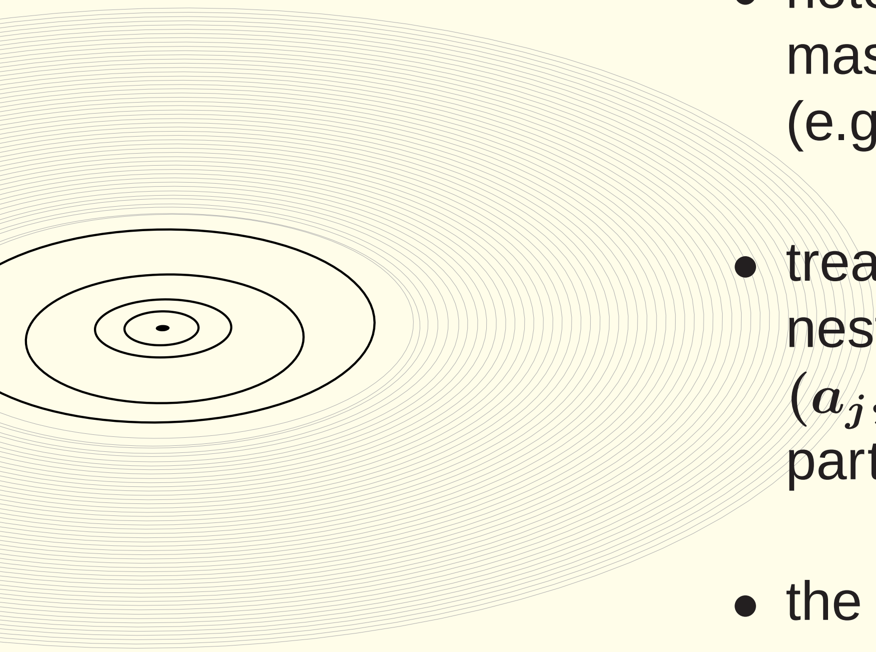
- secular perturbations are the low–frequency gravitational forces exerted by a perturber

- of particular interest are secular resonances, which are sites in a disk where a planet can excite large $e$'s and $i$'s

- in a gravitating disk, this $e$–disturbance can propagate away from resonance as a spiral density wave [aka, apsidal wave (Ward and Hahn 1998)].

- the $i$–disturbance can propagate away from resonance as a spiral bending (or nodal) wave (Ward and Hahn 2003).

# Simulate Waves Using a Rings Model



- note that the secular evolution of a system of point-masses is identical to that of gravitating rings (e.g., Murray and Dermott 1999).

- treat a disk of numerous small bodies as a nested set of interacting rings of mass $m_j$, orbits $(a_j, e_j, i_j, \tilde{\omega}_j, \Omega_j)$ and thickness $h_j$ due to their particles dispersion velocities $c_j$.

- the planets are thin $h_j = 0$ rings.

- evolve the system as per the Lagrange planetary equations
  - apply the Laplace–Lagrange solution to obtain the system's secular evolution
  - note that the rings' finite thickness $h$ softens their gravitational potential, which also softens the solution's Laplace coefficients over the scale $h/a$.

# Spiral Wave Theory

- Treating the disk as a set of rings also allows one to use the Lagrange planetary equations to re–derive spiral wave theory
  - this yields the waves' dispersion relation $\Omega_{\text{pattern}}(k)$ which provides the properties of the apsidal density waves:
    - long waves with wavelength $\lambda_L \propto \sigma \propto M_{KB}$ — $g$–mode of Tremaine (2001)
    - short waves with wavelength $\lambda_S \lesssim 10h$ — $p$–mode of Tremaine (2001)
    - apsidal density waves propagate between a resonance and the $Q$–barrier, which lies downstream where $h$ exceeds the threshold

$$h_Q \simeq 0.3 \frac{M_{KB}}{M_{\text{Sun}}} \left| \frac{n}{\Omega_{\text{pattern}}} \right| a \tag{1}$$

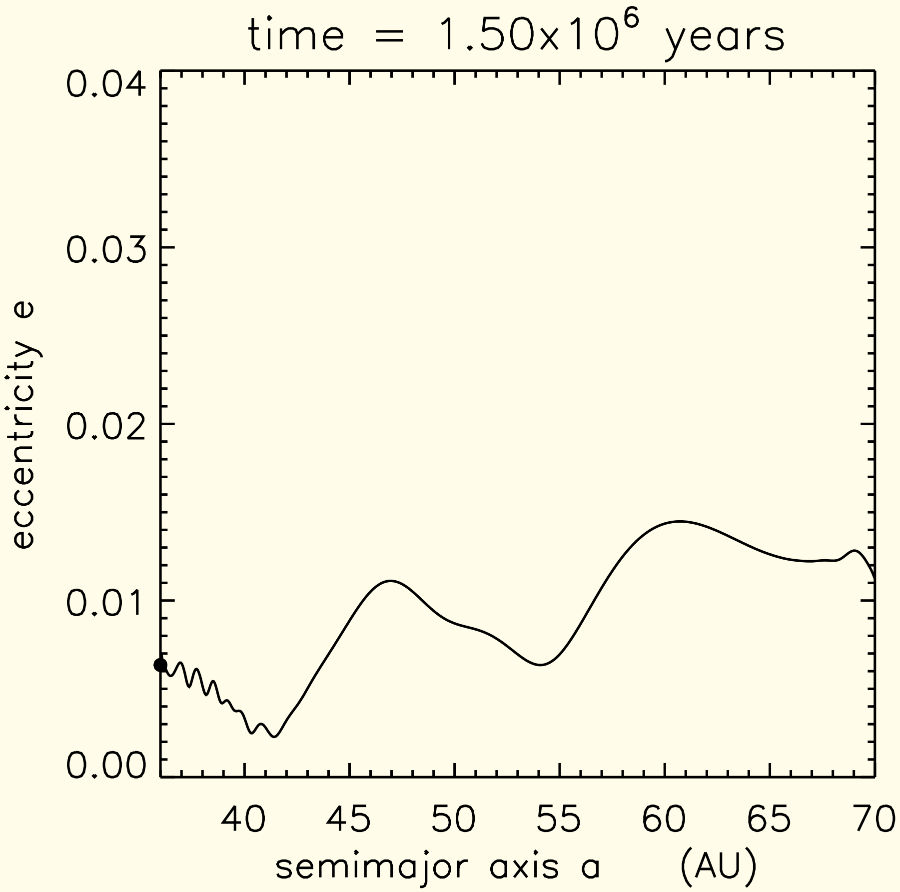
    - if long density waves encounter a disk edge or a $Q$–barrier, they reflect as short density waves
  - these results are not new—they may also be obtained from Toomre's (1969) dispersion relation in the limit $\Omega_{\text{pattern}} \ll n$

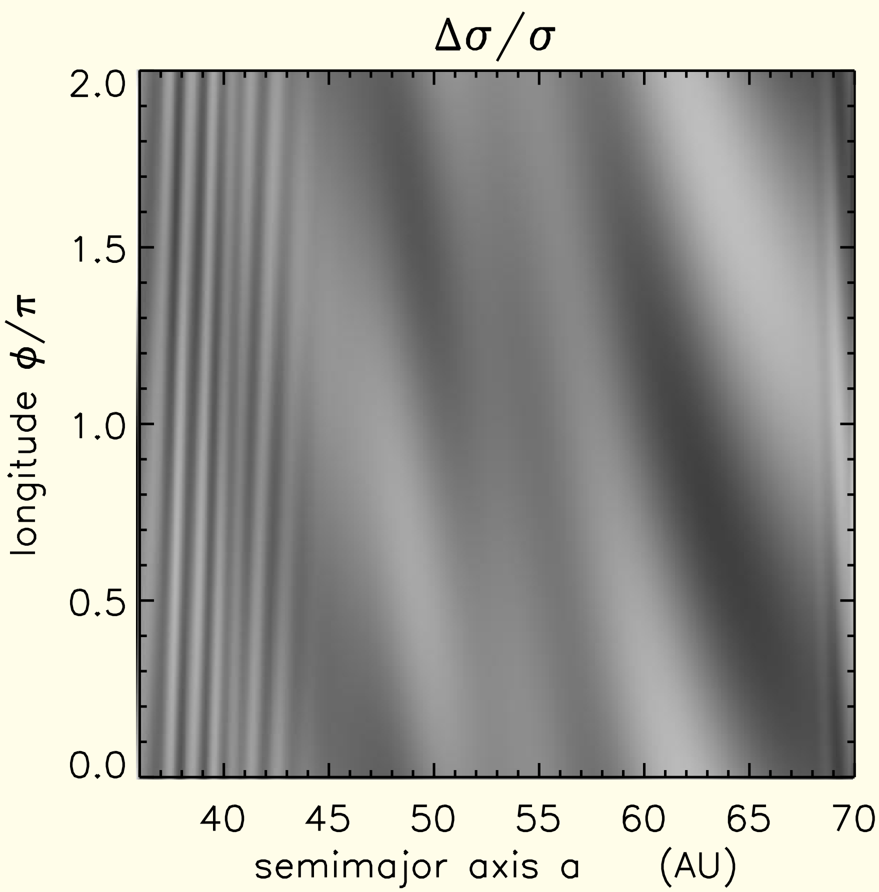
# Nodal Bending Waves

- the dispersion relation $\Omega_{\text{pattern}}(k)$ also provides the properties of nodal bending waves:
  - there are only a long wave solution having a wavelength $\lambda_L \propto \sigma \propto M_{KB}$
  - nodal bending waves propagate between the launch site and the disk edge
    * unless they encounter a zone downstream where $h \gtrsim 3h_Q$ where they *stall*, ie., $c_{\text{group}} \rightarrow 0$
  - New!
    * wave–stalling phenomenon disappears in a thin $h = 0$ disk

# Simulation of Apsidal Density Waves

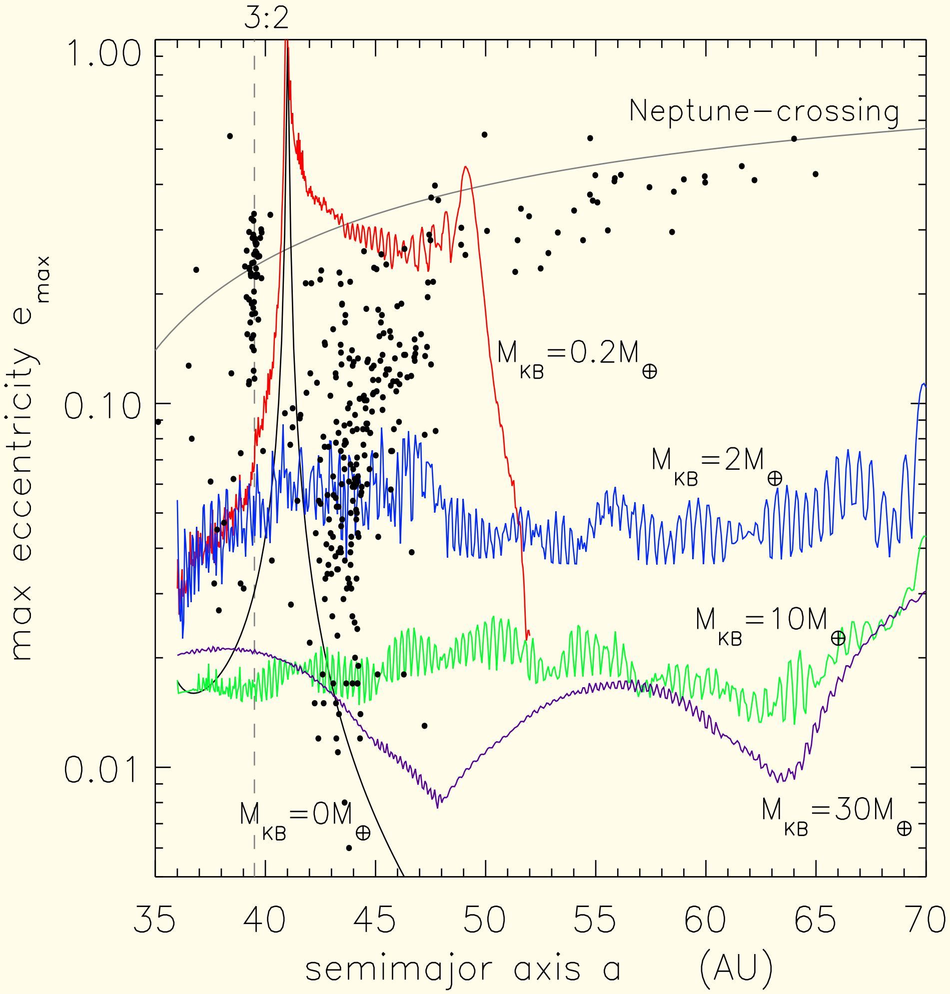
# in a $M_{KB} = 10$ M$_\oplus$ Kuiper Belt with $h = 0.01a$



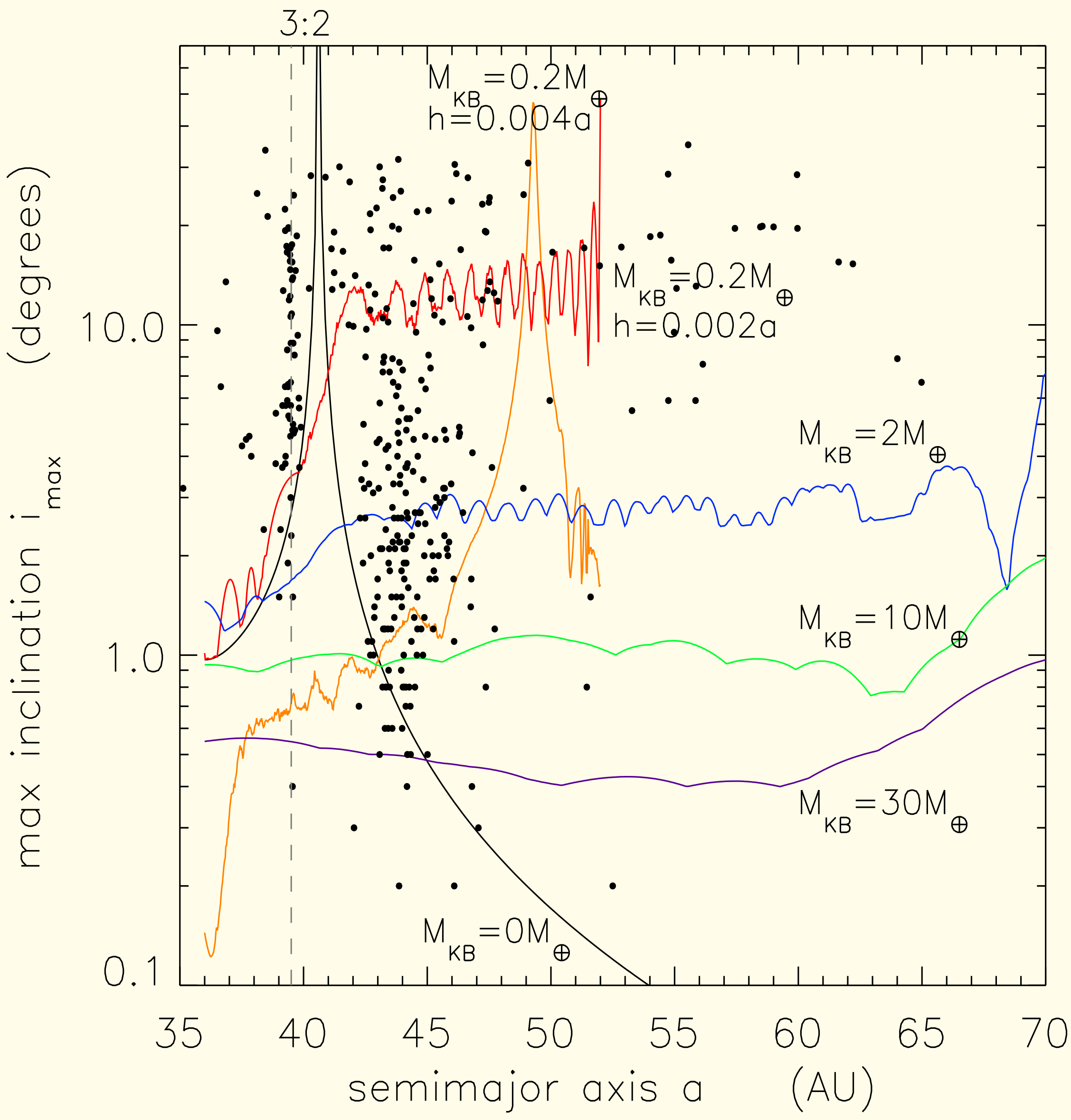
# Simulations of Apsidal Density Waves in the KB

- simulated Belt's have $h = 0.002a$
  - $M_{KB} = 30$ M$_\oplus$ (primordial mass)
  - $M_{KB} = 0.2$ M$_\oplus$ (current mass)
- density waves reflect at the disk's outer edge or at a $Q$–barrier.
  - reflected short waves are nonlinear, ie., $\Delta\sigma/\sigma \sim 1$
- the giant planets deposit $\sim 1\%$ of their $e$–AMD into the disk in the form of spiral density waves.
  - larger $e$'s are excited in lower–mass disks
  - exciting large $e \sim 0.3$ in the $M_{KB} = 0.2$ M$_\oplus$ Belt requires a very thin disk, $h \sim 0.002a$



# Summary of Nodal Bending Waves in the KB

- similarly, larger $i$'s get excited in lower–mass disks
- bending waves also reflect at the disk edge at 70 AU or else they stall where $h \gtrsim 3h_Q$



# Waves & Their Implications for the Primordial Kuiper Belt

- when the KB was still young and quite massive, $M_{KB} \sim 30$ M$_\oplus$, then low–amplitude apsidal density waves ($e_{\rm max} \sim 0.02$) and nodal bending waves ($i_{\rm max} \sim 0.5^\circ$) were sloshing about the KB.
  - wave propagation times were short,

$$T_{\rm prop} \sim 10^6 \left(\frac{\Delta a}{30 \text{ AU}}\right) \left(\frac{M_{KB}}{30 \text{ M}_\oplus}\right)^{-1} \text{ years}$$

  - the density waves eventually reflect and return as nonlinear short waves having $\Delta\sigma/\sigma \sim 1$ which dominate the Belt's surface density structure

# Implications for the Current Kuiper Belt

- KBO accretion models tell us that gravitational stirring by large, recently–formed KBOs increased the disk thickness $h$ while collisional erosion decreased $M_{KB} \rightarrow 0.2$ M$_\oplus$
  - stirring/erosion draws the $Q$-barrier and the stall–zone inwards to the secular resonances at $\sim 40$ AU which ultimately shuts off wave action

- this epoch of wave propagation in the Belt likely lasted for
  - at least $\tau_{\rm form} \sim 10$ million years when the large $R \sim 100$ km KBOs formed and started to stir up the Belt (Kenyon & Luu 1999)
  - but no more than $\tau_{\rm erode} \sim 500$ million years when collisions eroded $99\%$ of the KB's mass away (Kenyon & Bromley 2001)

- thus gravitational stirring and collisional erosion likely shut off apsidal and nodal waves, preventing them from exciting the Kuiper Belt.

# Other Applications of the Rings Model:

- apsidal & nodal waves like to propagate in a thin disk