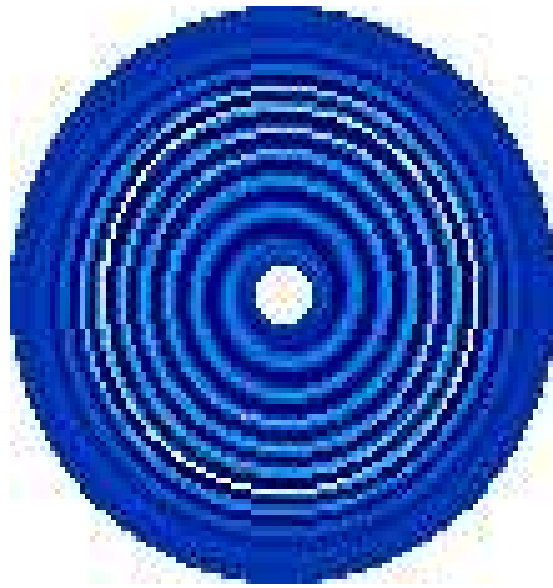


# Spiral Bending Waves Launched at a Vertical Secular Resonance



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- I. Examine secular forcing exerted between a perturber (e.g., planet or satellite) and a self-gravitating disk (e.g, planetesimal disk, solar nebula, planetary ring, stellar disk).

A. Secular resonances are sites in the disk where

1.

$$g_p \equiv \frac{d\tilde{\omega}_p}{dt} = g_{disk}$$

which excites eccentricities and can launch spiral *density* waves (Ward and Hahn 1998).

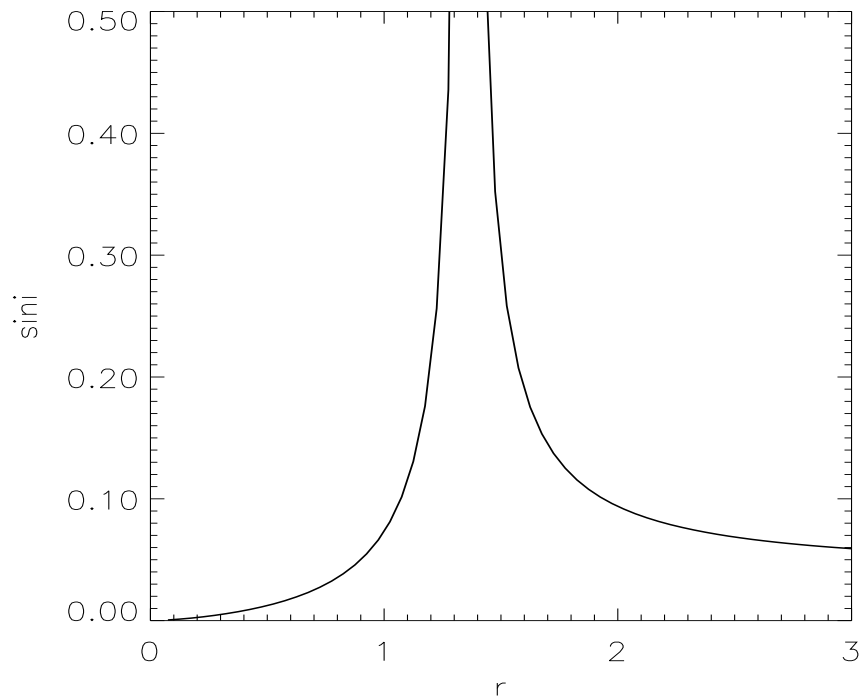
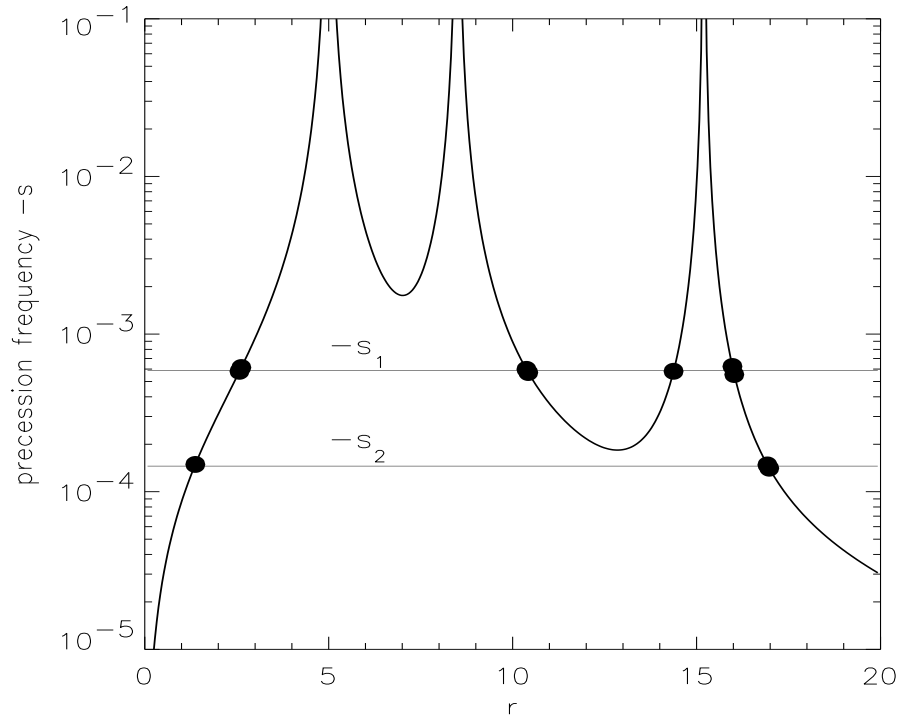
2.

$$s_p \equiv \frac{d\Omega_p}{dt} = s_{disk}$$

which excites inclinations and can launch spiral *bending* waves.

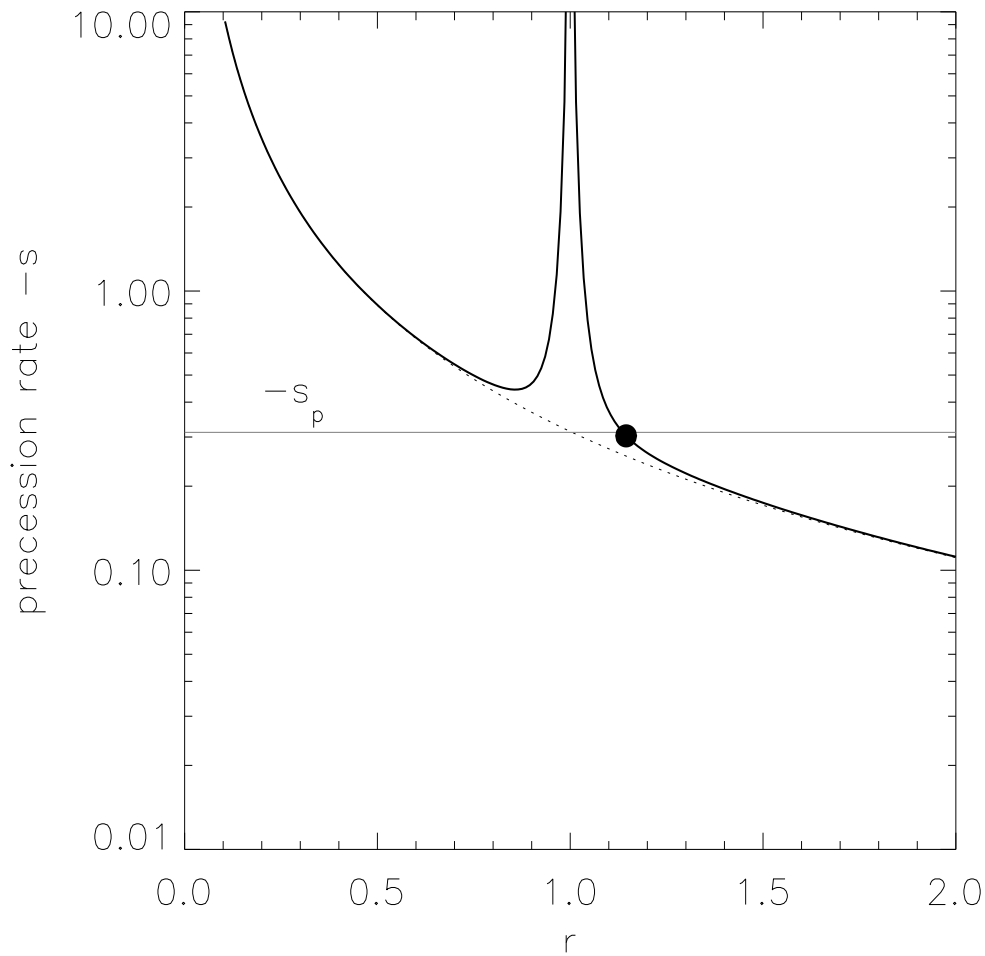
B. Review: Planets embedded in a zero-mass disk.

1. The planets' gravities drive the disk's nodal regression (*i.e.*,  $s < 0$ ).



- C. In a self-gravitating disk,  
the disk gravity drives the regression of the nodes:

$$s_{disk} = -\frac{1}{4}\mu_p\beta^2 b_{3/2}^{(1)}(\beta)\Omega - \frac{2\pi G\rho_{disk}}{\Omega}$$

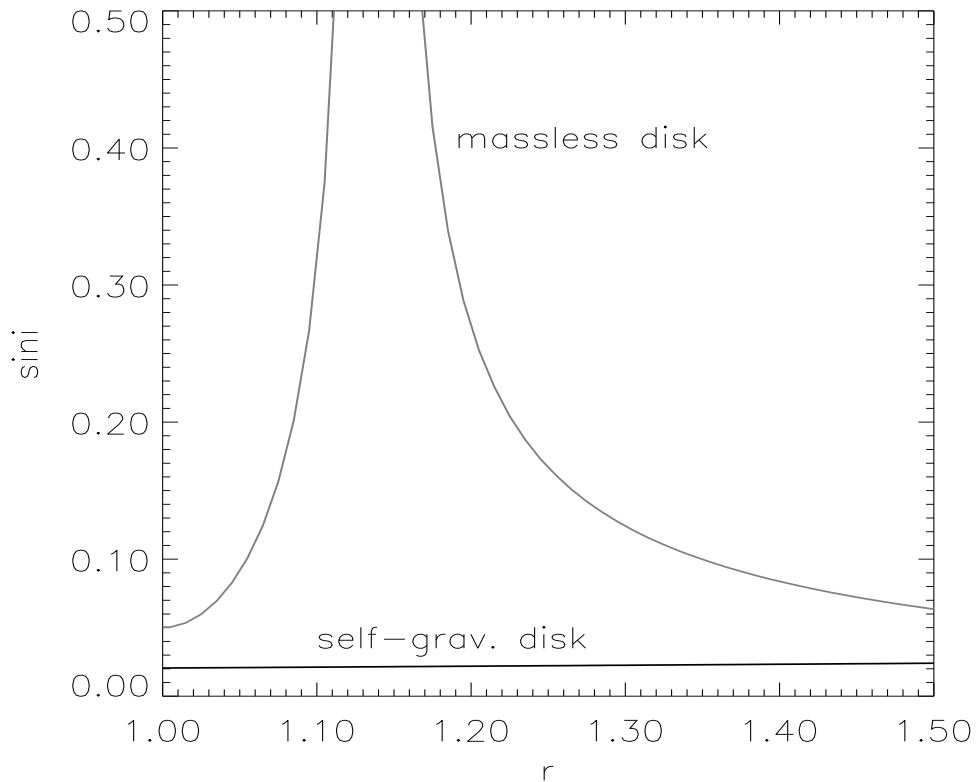


Massive disks having  $\pi\sigma_{disk}r^2 \sim M_p$  have secular resonance near the planet.

- D. A planet excites inclinations at the secular resonance. Disk self-gravity transports that disturbance away as a spiral bending wave with amplitude (Shu, Cuzzi, Lissauer 1983)

$$\sin i_d = \sqrt{\frac{\pi}{8\mu_d} \frac{\mu_p \beta^2 b_{3/2}^{(1)}(\beta)}{\left| r^2 \frac{d}{dr} \left( \frac{s}{r\Omega} \right) \right|^{1/2}}} \sin i_p$$

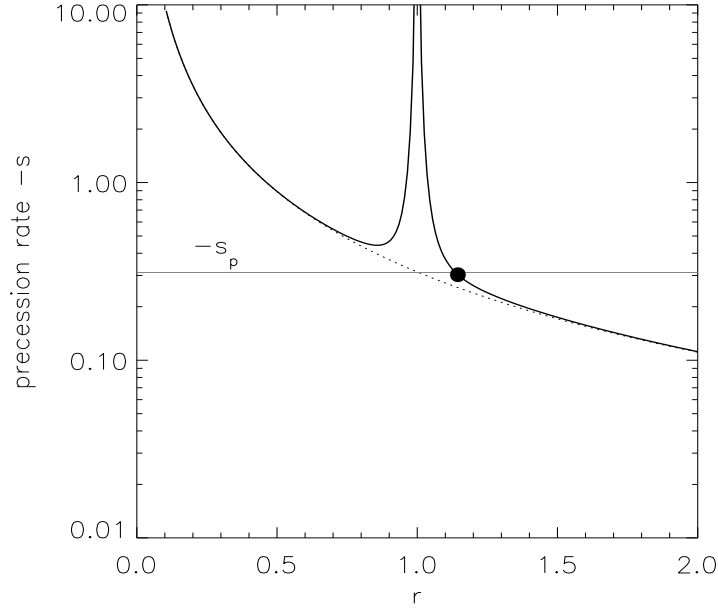
where  $\mu_p \equiv M_p/M_*$  and  $\mu_d = \pi\sigma r^2/M_*$ .



Note that  $i_{\text{self-grav.}} \ll i_{\text{test-part.}}$

## II. Resonance location & wave amplitude depends on the disk structure.

Scenario A. Consider a planet embedded in a continuous disk, **no gap**.



$$\Delta r \simeq \left( \frac{Q \mu_p}{3\pi} \right)^{1/3} r_p \simeq 0.007 Q^{1/3} \text{ AU}$$

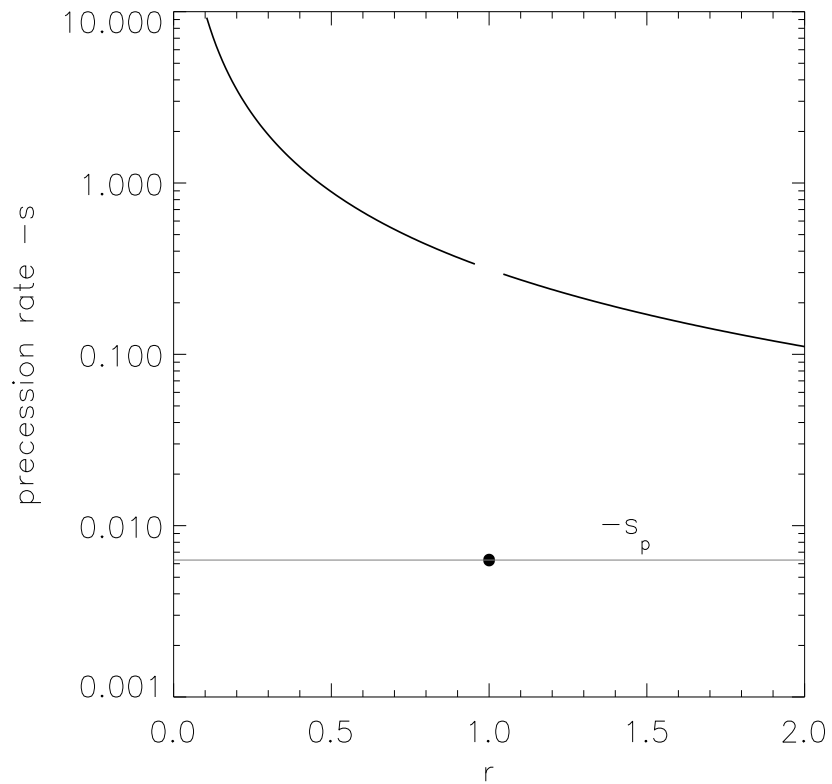
$$\lambda \simeq \sqrt{3Q \mu_d r_p} \simeq 0.01 Q^{1/2} \text{ AU}$$

$$\sin i_d \simeq \left( \sqrt{\frac{\pi}{Q}} \frac{\mu_p}{3} \right)^{1/3} \frac{\sin i_p}{\sqrt{\mu_d}} \simeq 2 Q^{-1/6} \sin i_p$$

where RHS is evaluated for an Earth-mass protoplanet at 1 AU in a minimum-mass *planetesimal disk*.

Scenario B. Planet opens gap in planetesimal disk  
 $w \sim 2\sqrt{3}R_H$  (width of feeding zone).

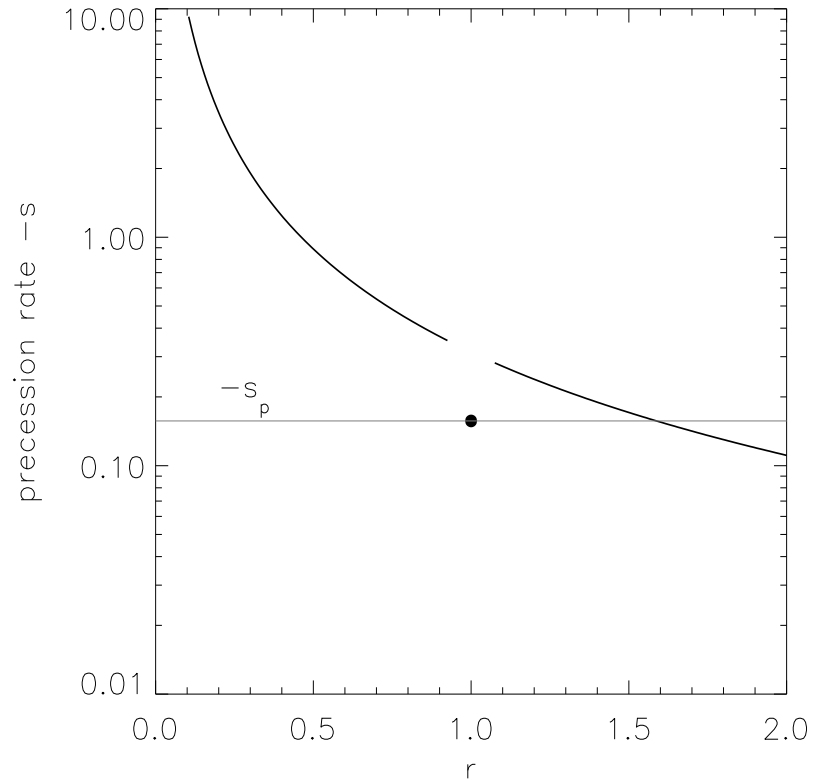
$$|s_p| \ll |s_{disk}|.$$



- planet's node *slowly* regresses,
- resonance lies far away,
- little (if any) wave excitation.

Scenario C. Giant planet opens gap in nebula gas disk  
 $w \sim h$  (width  $\sim$  disk scale height).

$$|s_p| \simeq \frac{1}{2}|s_{disk}|$$



- $r_{res} \simeq 1.6r_p$  (near  $m = 1$  OLR).
- $\lambda \simeq 3.6\sqrt{hr_p} \sim 5$  AU.
- $\sin i_d \simeq 2\sqrt{\frac{h\mu_p}{r\mu_d}} \sin i_p \sim 0.1 \sin i_p$ .

For Jupiter embedded in a minimum-mass nebula with  
 $h \sim 0.07r$ .

### III. The Planet's Response to the Spiral Wave.

- The mutual attraction of the disk and planet causes in-plane  $\vec{L}$  to flow from the planet  $\rightarrow$  resonance, which propagates away via the bending wave.
- Consequently the planet's inclination is damped at the rate (c.f., Borderies, Goldreich, & Tremaine 1984):

$$\frac{1}{i_p} \frac{di_p}{dt} = - \frac{\pi \mu_p \mu_d \beta^3 [b_{3/2}^{(1)}(\beta)]^2}{8 \left| r^2 \frac{d}{dr} \left( \frac{s}{r\Omega} \right) \right|} \Omega_p \equiv \tau_i^{-1}$$

which is similar to  $\dot{e}_p$  obtained by Ward & Hahn 1998 due to apsidal density waves.

Scenario A. For an Earth–mass planet embedded in a minimum–mass *planetesimal* disk, no gap

$$\tau_i \simeq \left( \frac{Q\mu_p}{3\pi} \right)^{1/3} \frac{3}{\mu_d\Omega_p} \sim 100Q^{1/3} \text{ orbits.}$$

For an Jupiter–mass planet embedded in a minimum–mass nebula *gas* disk,  
with  $Q \sim 20$  and no gap

$$\tau_i \sim 20 \text{ orbits.}$$

Scenario B. For a terrestrial planet embedded in a planetesimal disk with a gap width

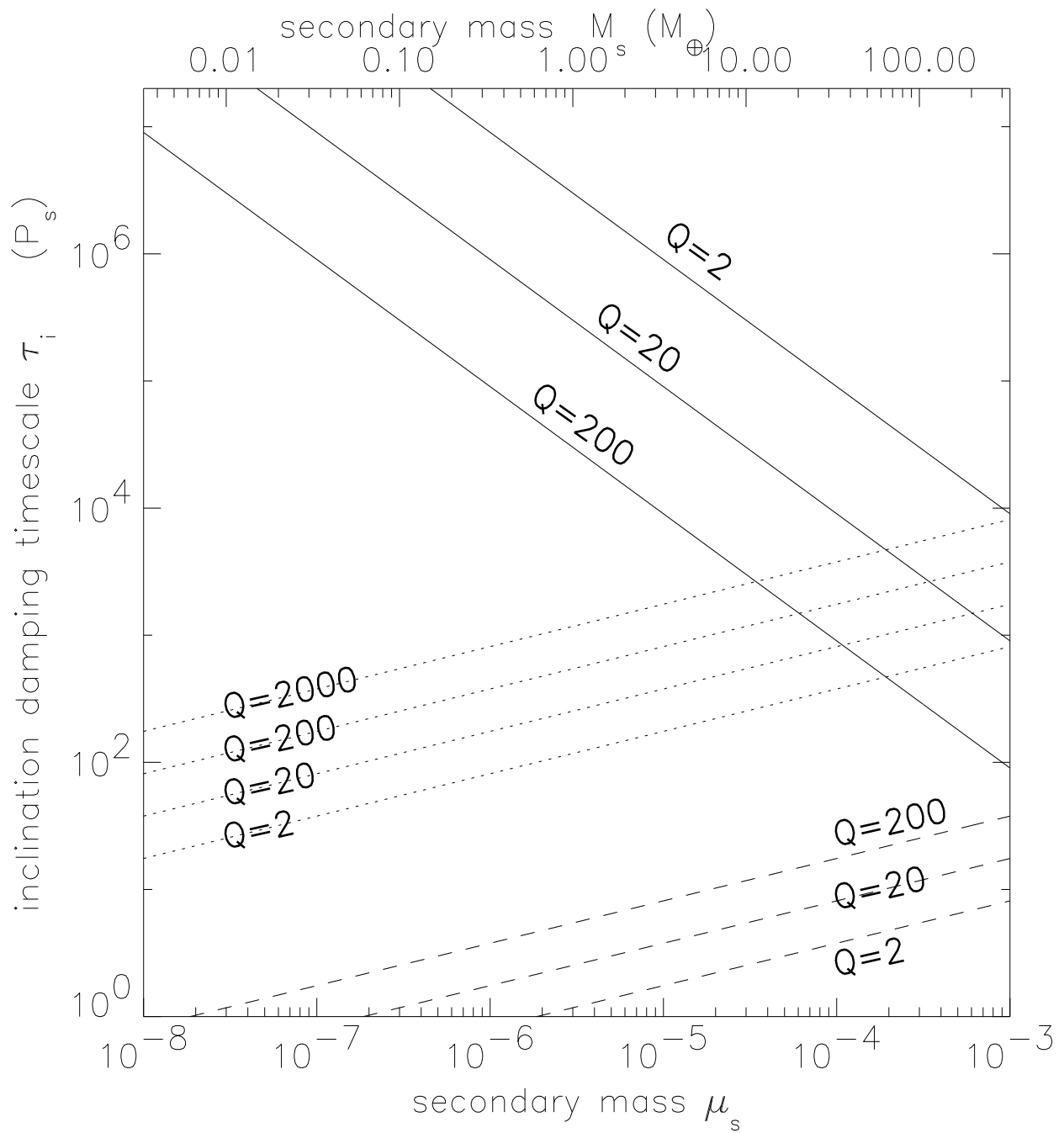
$$w \sim 2\sqrt{3}R_H,$$

- $\tau_i$  is very long since the resonance lies far from the planet.

Scenario C. For a Jupiter-mass planet embedded in a minimum-mass gas disk with a gap width

$$w \sim h$$

$$\tau_i \simeq \frac{0.4}{Q\mu_s\mu_d\Omega_p} \sim 10^3 \text{ orbits.}$$



## IV. Summary of Results.

- An inclined planet will excite  $i$  at a vertical secular resonance.
- Disk self-gravity transports that disturbance radially away as a spiral bending wave.
- In a typical planet-forming environment having  $\mu_p \sim \mu_{disk}$ :
  - The disk's gravity drives the planet's nodal regression  $s_p$  & determines the location of the resonance, which can lie quite near the planet.
  - However gap formation slows the planet's node and pushes the resonance away, weakening the disk-planet interaction.
  - $i_{self-grav.} \ll i_{non-grav.}$

- The excitation of spiral bending waves damps  $i_p$ :
  - For terrestrial protoplanets & no gap,  $\tau_i \sim 100$  orbits.
  - However  $\tau_i \rightarrow$  very long if a gap forms in the planetesimal disk.
  - For giant planets embedded in the solar nebula,  $\tau_i \sim 20$  orbits (no gap) and  $\tau_i \sim 1000$  orbits if gap width  $w \sim h$ .
  - this  $i_p$ -damping mechanism will have important implications for the planet's accretion history,
  - and may be responsible for the giant planets' low mutual inclination of  $\sim 1^\circ$ .

- Are there other applications for nodal bending waves?
  - launched by small moonlets embedded in Saturn's rings?
  - launched by satellite galaxies in stellar disks?
- Animations of N-ring simulations of spiral bending waves.

