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Diagnosing Circumstellar Debris Disks

Introduction

Debris disks are the dusty circumstellar disks that have been observed orbiting several nearby stars. These dust disks are transient and must also be replenished, since collisions tends to drain a disk of its dust. Consequently, these disks are believed to be replenished by unseen planetesimals whose mutual collisions generate the observed dust. The smaller dust formed in these collisions are immediately lofted outwards into wide orbits ($r \sim 100$–1000 AU) by radiation pressure (Strubbe & Chiang 2006). Radiation pressure also causes the dusty disk's surface brightness to break at the planetesimal disk's outer radius r_{out}. And when such a disk is viewed edge on, as is the case for AU Mic and β Pic, this causes the disk's surface brightness to fall of rapidly as $\propto r^{-3.5}$ at projected distances of $r \gg r_{out}$, and less rapidly at $r \ll r_{out}$ (Strubbe & Chiang 2006); see also Fig. 1.

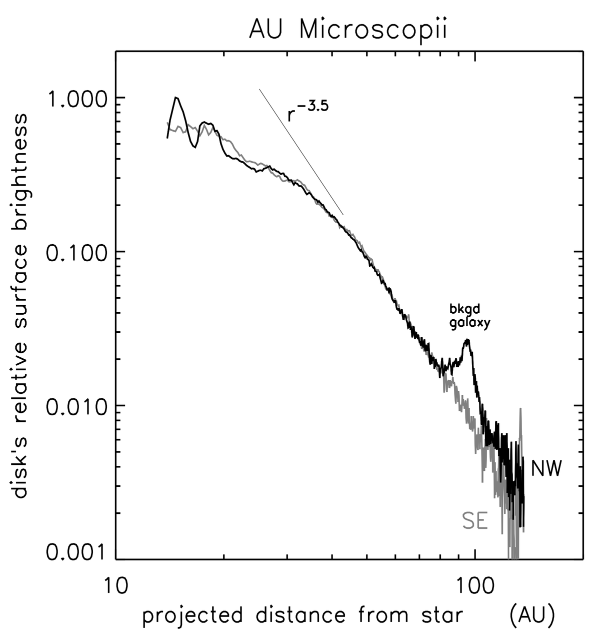

FIGURE 1: Surface brightness of edge-on debris disk orbiting AU Mic, plotted versus projected distance from the central star, along disk's northwest (NW) and southeast (SE) ansae. Profiles are extracted from an optical HST image described in Krist et al (2005). Note that $B(r) \propto r^{-3.5}$ at $r \gg 43$ AU, while the inner profile has a shallower slope. This is the signature of dust generated by a ring of colliding planetesimals orbiting at $r_{out} \simeq 43$ AU (Strubbe & Chiang 2006).

Dust orbits

An orbiting dust grain is characterized by its dimensionless size parameter $\beta = 3L_\star/16\pi GM_\star\rho Rc$, which is the ratio of radiation pressure to stellar gravity. All symbols have their usual meaning, so grains with smaller radii R have larger β. A dust grain is produced by a collision among planetesimals, and a small grain created this way is immediately injected into a wide eccentric orbit due to radiation pressure. The grain's semimajor axis a and eccentricity e are simple functions of the size parameter β (Burns et al 1979):

$$a(\beta) = \frac{1-\beta}{1-2\beta}r \quad \text{and} \quad e(\beta) = \frac{\beta}{1-\beta} \qquad (1)$$

where r is the collision's distance from the central star. Also note that a dust grain's longitude of periapse $\tilde{\omega}$ is the longitude where the grain formed.

Dust collisions

Collisions among dust grains also depletes the debris disk. To account for collisions in a simple and fast algorithm, we quantize the problem, and assume that the planetesimal disk is the source of numerous discreet dusty streamlines (or orbits) in the disk, each characterized by a grain size β and orbits a, e, and $\tilde{\omega}$ (see Eqn. 1). This quantization allows one to avoid slow Monte Carlo simulations and even slower Nbody simulations, and also replaces tedious 3D integrals over the disk's volume with simple sums. The code then inspects all sites where streamlines cross, and calculates a collision probability density α_{ij}, which is the probability per time that a dust grain in streamline i collides with one in in streamline j. The abundance $N_i(t)$ of dust in streamline i then evolves according to a rate equation $dN_i/dt =$ dust production - collisional destruction rates, or

$$\frac{dN_i}{dt} = p_i - \frac{N_i}{T_{out}}\sum_j \alpha_{ij}N_j \qquad (2)$$

where the dust production rate p_i is assumed to be a power law in the grain radiii R_i as $p_i(R_i) \propto R_i^{-q}$, and T_{out} is the orbital period at the planetesimal disk's outer edge r_{out}. This coupled system of equations is easily solved for the streamline populations $N_i(t)$, which in turn provides the debris disk's optical depth $\tau(r)$ versus distance r from the star; see Figs. 2–3.

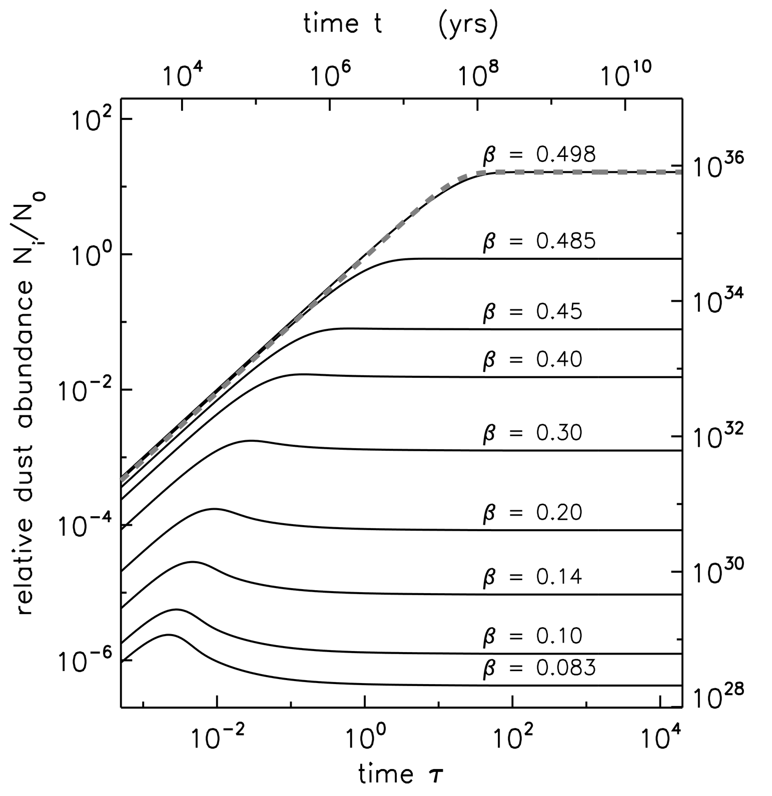

FIGURE 2: Solution to the dust evolution Eqns. (2) provides the dust abundances $N_i(t)$ versus time t for dust of various size parameters β_i. Shown are abundances for nine selected size bins, with 491 other curves not shown.

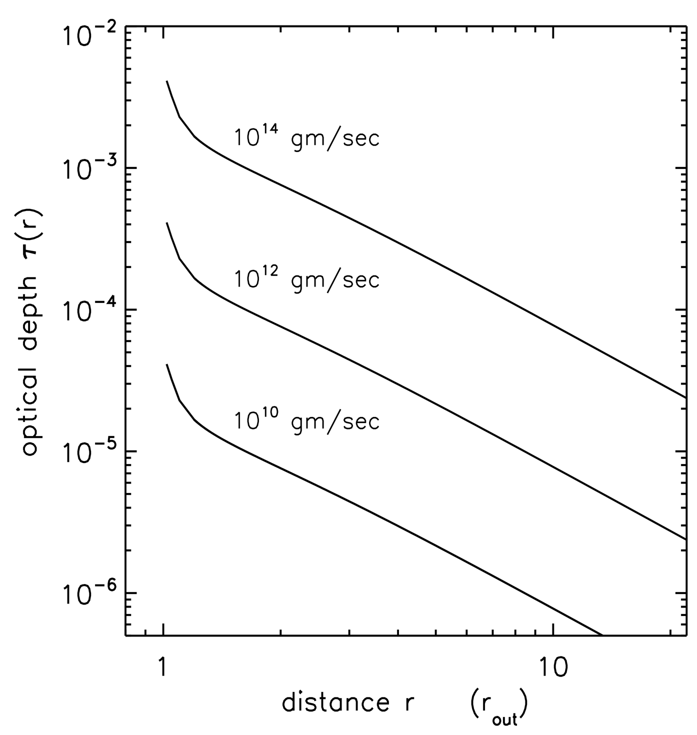

FIGURE 3: Optical depth $\tau(r)$ versus distance r for dust spawned in a narrow planetesimal ring of radius r_{out}, assuming three different dust production rates $\dot{M}_d$. All figures assume the central star is sunlike, and that the planetesimal disk's outer radius is $r_{out} = 50$ AU.

Collisional equilibrium

The system achieves collisional equilibrium when the rates of dust production and collisional destruction in Eqn. (2) balance, which takes about $t \sim 10^6$ yrs. (Fig. 2). Figure 3 shows that, once the system has settled into equilibrium, the debris disk's optical depth varies as $\tau \propto \dot{M}_d^{1/2}$ where $\dot{M}_d$ is the rate at which the planetesimal disk produces dust. The collisional lifetime of these grains is $T_c(R_i) = N_i/p_i$, which varies as expected as $T_c \propto \dot{M}_d^{-1/2}$ (see Fig. 4). Also note that $\tau(r) \propto r^{-3/2}$ outside the planetesimal disk's outer edge at $r \gg r_{out}$, consistent with Strubbe & Chiang (2006).

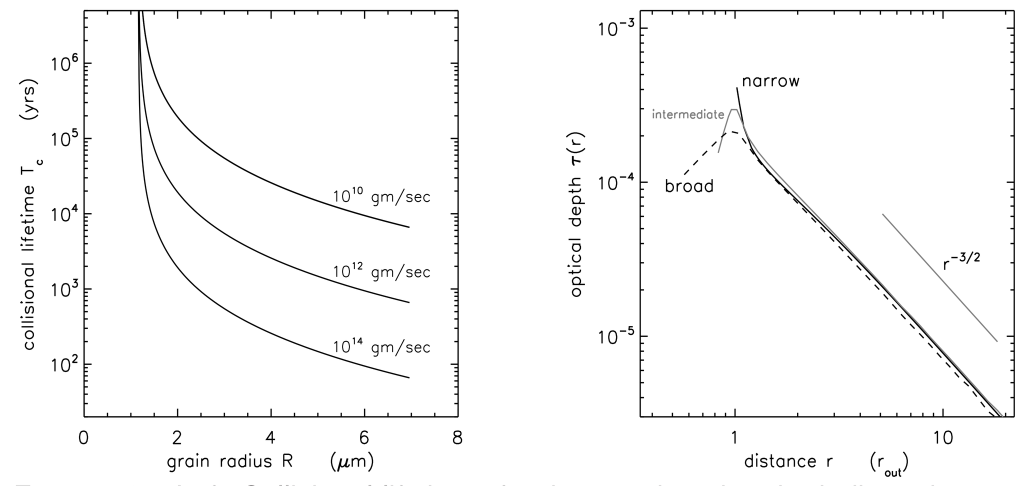

FIGURE 4: *Left:* Collisional lifetimes for dust produced at the indicated rates $\dot{M}_d$. *Right:* Optical depths $\tau(r)$ for dust produced by a narrow planetesimal ring of radius r_{out}, and a broad disk whose inner radius $r_{in} = 0.5r_{out}$, as well as an intermediate case with $r_{in} = 0.8r_{out}$.

Surface brightness

Many debris disks are observed edge-on (AU Mic, β Pic), and it is straightforward to calculate the surface brightness $B(x)$ versus projected distance x for a simulated debris disk that is viewed edge-on. Left Figure 5 demonstrates that $B(x) \propto \sqrt{\dot{M}_d}$ as expected, and that the outer disk's $B(x) \propto x^{-7/2}$ where $x \gg r_{out}$, consistent with Strubbe & Chiang (2006). Right Figure 5 also shows how the inner disk's surface brightness varies depends on the dust grains' scattering asymmetry factor g. Note that when the scattering of starlight is isotropic ($g = 0$) or moderately asymmetric ($g = 0.35$), the surface brightness $B(x)$ interior to the planetesimal disk's outer radius r_{out} is flat at $x < r_{out}$. However, the surface brightness of many edge-on debris disks instead show a knee-bend at $x \sim r_{out}$ (see Figs. 1 and 8), which suggests that light scattering by circumstellar dust is very asymmetric, with $g \sim 0.7$.

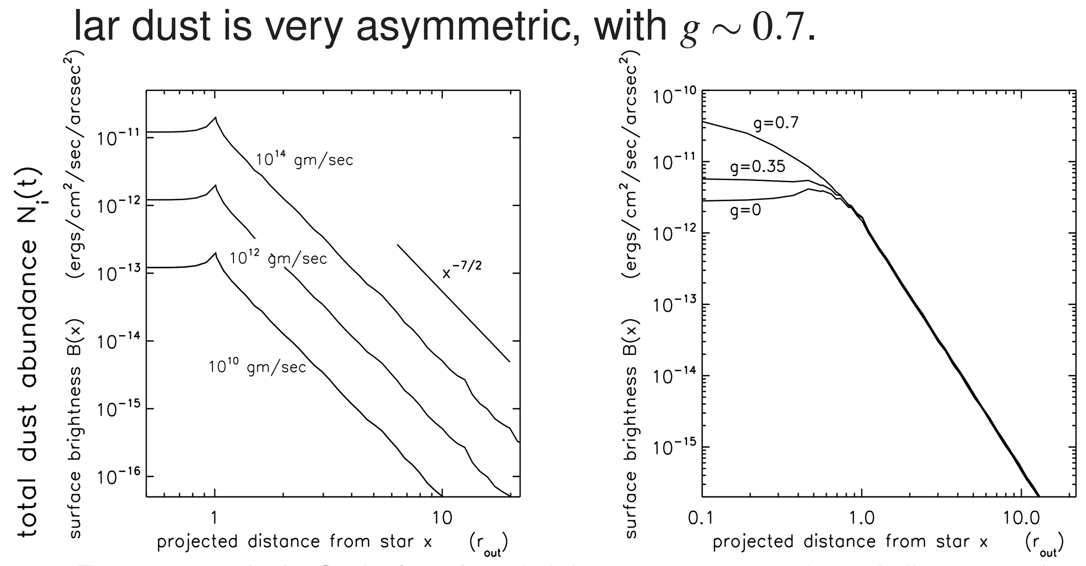

FIGURE 5: *Left:* Optical surface brightness versus projected distance x for dust produced by a planetesimal ring of radius r_{out} at the indicated dust production rates $\dot{M}_d$. *Right:* Surface brightness $B(x)$ of dust produced by a broad planetesimal disk whose inner radius $r_{in} = 0.5r_{out}$, with the dust grains' scattering asymmetry parameter g also indicated.

Evolution over time

Keep in mind that the preceding results are for a disk that is in collisional equilibrium, which probably occurs after a time $t \sim 10^{6\text{ to }7}$ yrs (see Fig. 2). However, there is evidence that some observed debris disks are not yet in equilibrium. The consequences are illustrated in Figs. 6-7, which shows the disk's optical depth $\tau(r)$ and surface brightness $B(x)$ at various times in a disk's history. As these Figures show, it is the outer disk that gets populated at later times, by the smaller dust having $\beta \sim 0.5$ that also have the larger apoapse distances $Q(\beta) \sim r_{out}/(1-2\beta)$. Note Fig. 2, which shows that it is the smaller dust in these wide orbits that do not get fully populated until later times. Figures 6-7 show that a younger disk will have an optical depth that is steeper than the canonical $\tau(r) \propto r^{-3/2}$ curve, and a surface brightness profile also steeper than $B(x) \propto x^{-7/2}$. Note that the surface brightness of the outer parts of the AU Mic and β Pic disks do tend to fall off faster than $B(x) \propto x^{-7/2}$ (Krist et al 2005, Golimowski et al 2006), which suggests that dust production in these disks may have only started or increased recently, relative to the $t \sim 10^7$ yr ages of these systems.

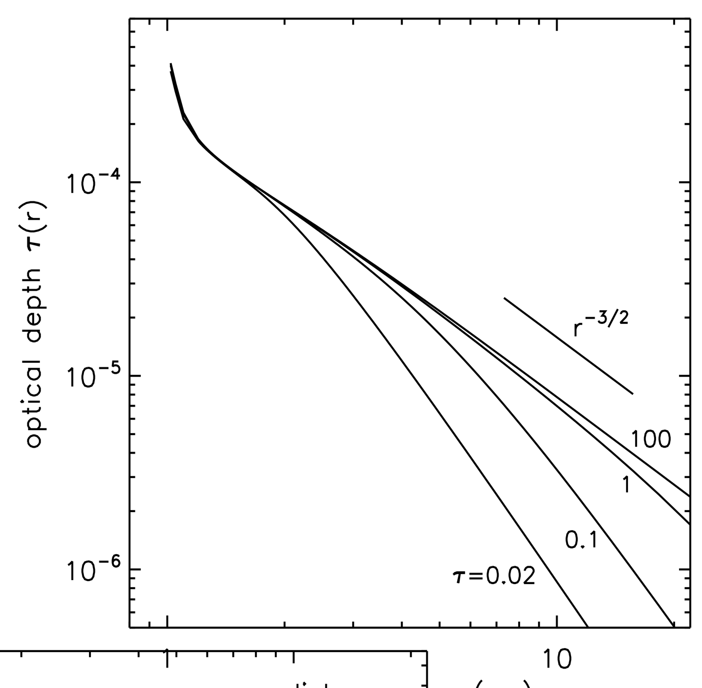

FIGURE 6: Optical depth $\tau(r)$ for a debris disk produced by a narrow planetesimal ring; shown are $\tau(r)$ at selected times t in units of Myrs.

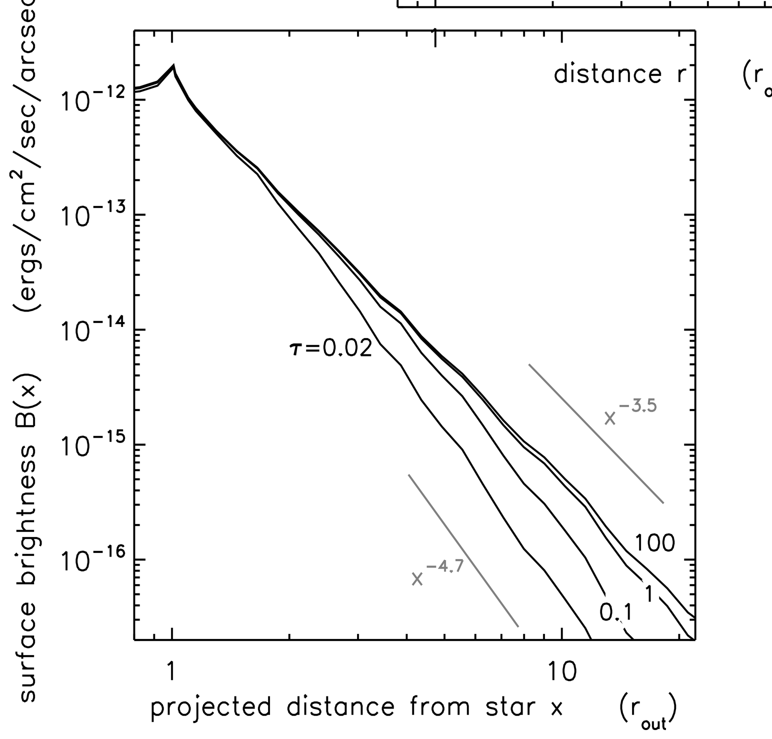

FIGURE 7: Surface brightness $B(x)$ for the same debris disk at the indicated times t in Myrs, with the disk viewed edge-on.

β Pictoris

The debris disk model developed here is applied to Hubble Space Telescope observations of this edge-on disk that were acquired by Golimowski et al (2006) at optical wavelengths. Figure 8 compares models to observations, which shows that the β Pic dust must be very asymmetric light scatters, with $g \simeq 0.67$. The model also indicates that the planetesimal disk that is the source of this dust is rather broad, with the disk's inner edge $r_{in} \lesssim 0.5r_{out}$, since models of narrower disks having $r_{in} \geq 0.8r_{out}$ are ruled out. The knee in the disk's surface brightness also indicates that the planetesimal disk's outer radius is $r_{out} \simeq 130$ AU.

The model's other key parameters are the dust grains albedo, and the planetesimal disk's dust production rate $\dot{M}_d$. Unfortunately, recent testing suggests that the simulated disk's surface brightness $B(x)$ are calculated incorrectly...that all such curves shown here may be off by the same constant factor $f \sim$a few. Consequently, a firm value for β Pic's dust production rate $\dot{M}_d$ cannot be quoted just yet, but it is probably considerable... Lastly, note that the simulated disk's surface brightness $B(x)$ gives very good agreement with the observed profile when the system is young, with $t \sim 0.2$ Myrs. (see Fig. 8). This suggests that the β Pic system may have experienced a recent increase in dust production. One then wonders whether a recent increase in dust production may be due to a recent alteration in the orbits of any unseen planets that might also be perturbing the planetesimal disk...

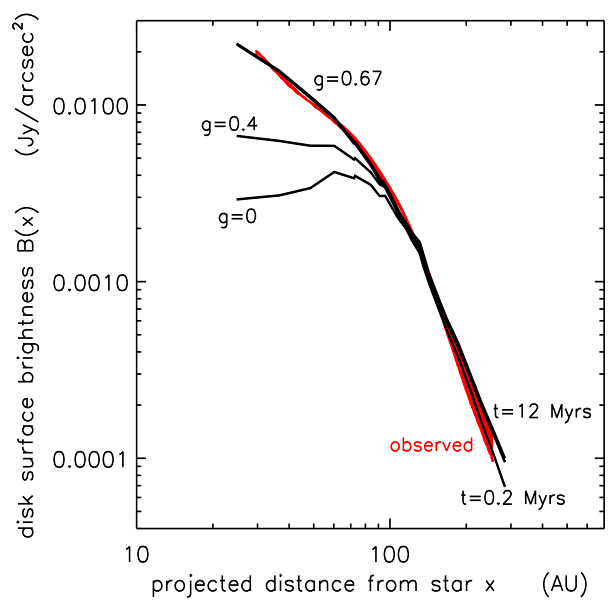

FIGURE 8: Optical surface brightness of edge-on disk orbiting β Pictoris (red curve, from Golimowski et al 2006) Black curves are models having light scattering parameters $g = 0$, 0.4, and 0.67. Dust inclinations are $I = 0.07$ rad $= 4.0°$, and a dust grain's density is $\rho = 1$ gm/cm³. The upper curve indicates the disk's surface brightness assuming steady dust production over the age of the β Pic system ($t = 12$ Myrs). Lower curve is if dust production only recently turned on at time $t = 0.2$ Myrs ago.

Findings thus far

- A rate equation for the abundance of dust of various sizes and orbits a debris disk is derived (Eqns. 2) and solved numerically (Fig. 2). Disk optical depths $\tau(r)$ and surface bright profiles $B(x)$ for a variety of disk conditions are then calculated (Figs. 2-7).
- Comparison of models to observations of β Pic shows that the unseen planetesimal disk that is the source of the observed dust is broad, with $r_{in} \lesssim 0.5r_{out}$ where $r_{out} \simeq 130$ AU. The dust grains are asymmetric light scatterers, with $g \simeq 0.67$. An improved model will also yield this system's dust production rate $\dot{M}_d$. The observed disk's surface brightness $B(x)$ are steeper than the canonical $x^{-7/2}$ curve, suggesting that the β Pictoris disk might be young, $t \sim 0.2$ Myrs, or that dust production there increased in recent times (Fig. 8).

Acknowledgments:
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References

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Preprint and software requests

A paper on the results is in preparation, and the disk-model software will be made public, once it is validated and documented. If interested, please leave an email address here.