Secular Planetary Perturbations: Explaining Structure Observed in Circumstellar Debris Disks

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Introduction

Circumstellar debris disks provide insight into the formation and evolution of planet-forming systems. These dust-disks are likely the by-product of collisions among unseen planetesimals. Since planetesimals are also the seeds of planets, it is not surprising that some debris disks, like those orbiting β Pictoris, Fomalhaut, and others, appear to be perturbed by unseen planets orbiting within. Signatures of planetary perturbations include: a central gap, warps and radial offsets, and other asymmetries in the disk.

The following describes a model of a circumstellar debris disk that is perturbed by an unseen planetary system. By modeling the disturbances observed in a circumstellar disk, one can then measure or else constrain the masses and orbits of the unseen planets that may be lurking within.

Secular perturbations

Note that nearly all of the circumstellar dust disks observed to date that hint at planetary perturbations (e.g., β Pic, Fomalhaut, and HR 4796) appear to be disturbed by the planets' *secular* perturbations. Secular perturbations are the slowly varying gravitational perturbations that can excite orbital eccentricities and inclinations in a dust disk, and they also drive a slow orbital precession. Secular planetary perturbations also have a broad reach across a disk, and tend to dominate a disk's large-scale structure. For instance, secular perturbations from an eccentric planet can give a disk a lopsided appearance (Wyatt *et. al.* 1999), while an inclined planet can warp the disk (Mouillet *et. al.* 1997).

A planet's mean-motion resonances can also account for the fine–structure also seen in some disks. A famous example is the dust ring at ε Eridani, whose clumpy appearance may be due to dust trapped at a planet's mean-motion resonances (Quillen & Thorndike 2002). However, mean-motion resonances are quite narrow, and their influence is limited to the innermost regions of a disk, so their effects are not modeled here.

Secular perturbation theory

To calculate the orbit of a dust grain that is perturbed by a planetary system's secular perturbations, express the grain's familiar orbit elements $(e, I, \Omega, \tilde{\omega})$ in terms of the more convenient set (h, k, p, q), where

$$h = e\sin\tilde{\omega} \qquad k = e\cos\tilde{\omega} \qquad p = I\sin\Omega \qquad q = I\cos\Omega. \tag{1}$$

Murray & Dermott (1999) show that the grain's motion is then the sum of two parts, a *free* part that is due to the grain's random motions, and a *forced* part that is due to planetary perturbations:

$$\begin{aligned} h(t) &= e_{\text{free}}\sin(At+\beta) + h_{\text{forced}}(t) \quad & k(t) &= e_{\text{free}}\cos(At+\beta) + k_{\text{forced}}(t) \\ p(t) &= I_{\text{free}}\sin(Bt+\gamma) + p_{\text{forced}}(t) \quad & q(t) &= I_{\text{free}}\cos(Bt+\gamma) + q_{\text{forced}}(t), \end{aligned} \tag{2}$$

where the forced motion is the sum of contributions from the N planets:

$$\begin{aligned} h_{\text{forced}} &= \sum_{i=1}^{N}\frac{\nu_i}{g_i - A}\sin(g_i t + \beta_i) \quad & k_{\text{forced}} &= \sum_{i=1}^{N}\frac{\nu_i}{g_i - A}\cos(g_i t + \beta_i) \\ p_{\text{forced}} &= \sum_{i=1}^{N}\frac{\eta_i}{f_i - B}\sin(f_i t + \gamma_i) \quad & q_{\text{forced}} &= \sum_{i=1}^{N}\frac{\eta_i}{f_i - B}\cos(f_i t + \gamma_i). \end{aligned} \tag{3}$$

Here, the ν_i and η_i are the amplitudes of the grain's horizontal and vertical motions that are excited by the i^{th} planet; they scale with the planet's mass, its eccentricity or inclination, and proximity to the planet. The g_i and the f_i eigenfrequencies describe the rates at which the i^{th} planet's orbit precesses due to perturbations from the other planets, and the A and B are the grain's free precession rates. The remaining angles β_i and γ_i are phases that describe the orientation of the planets' eccentric and inclined orbits. See Murray & Dermott (1999) for details.

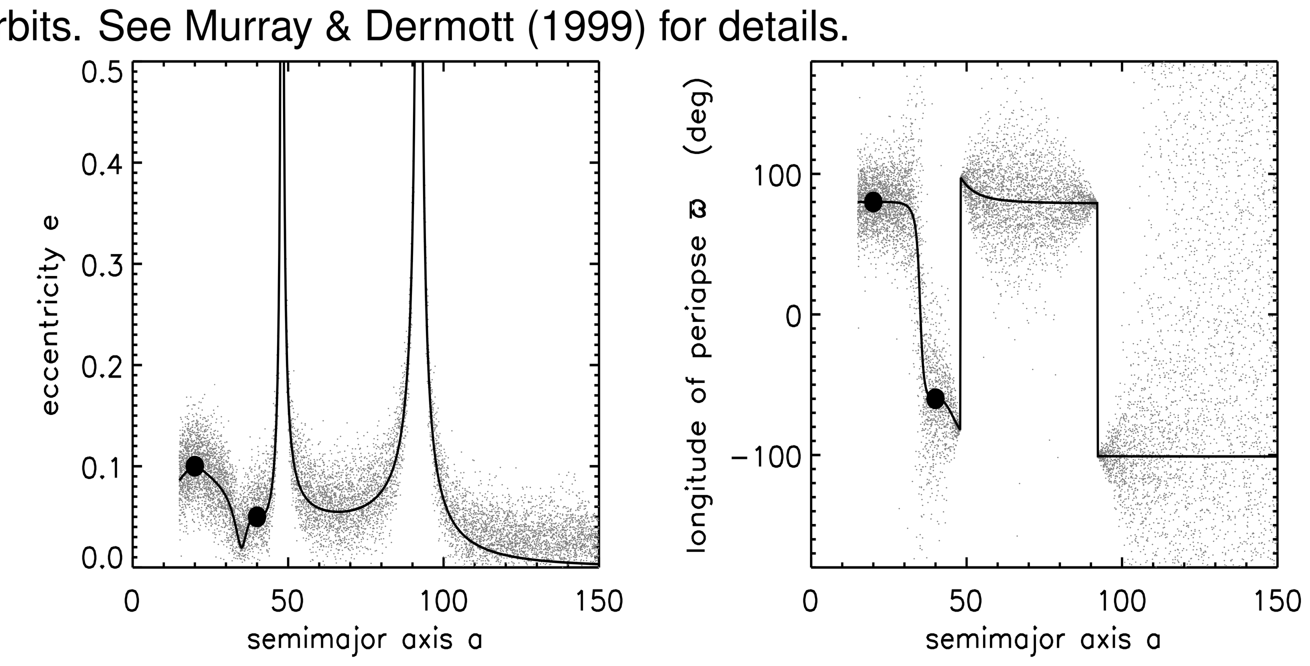

FIGURE 1: Solid curves are the dust grains' forced orbit elements (Eqns. 3) that are excited by a Jupiter and Saturn that are in rather wide orbits, as indicated by the black dots. Grey dots are the grains' total orbit elements e and $\tilde{\omega}$ when they also have some free eccentricity $e_{\text{free}} \sim 0.03$.

Secular resonances in a debris disk

Figure 1 shows the dust grains' forced orbit elements $e_{\text{forced}} = \sqrt{h_{\text{forced}}^2 + k_{\text{forced}}^2}$ and $\tilde{\omega}_{\text{forced}} = \tan^{-1}(h_{\text{forced}}/k_{\text{forced}})$ for circumstellar dust that is disturbed by a hypothetical planetary system. Note the two peaks in eccentricity at $a \simeq 50$ and 90 AU. These are *secular resonances*, which are sites where the grains' free precession rates A or B matches any of the planets' eigenfrequencies g_i or f_i, which results in large e's or I's (see Eqns. 3).

The disk seen in scattered starlight

A synthetic optical image of the model disk is generating by summing the starlight scattered by each grain towards an observer (Lester *et. al.* 1979):

$$F = \sum_{\text{all dust}} \frac{A_g \sigma_c \Phi F_\star}{\pi\Delta^2}, \tag{4}$$

where A_g is a grain's albedo, σ_c is its cross-sectional area, $F_\star$ is the incident flux of starlight, Δ is the distance to the observer, and Φ is its scattering phase function, assumed to be a Henyey-Greenstein phase function. Fig. 2 shows an optical image of the dusty planetary system whose orbits are given by Fig. 1. Dark arcs at $a \sim 50$ and 90 AU are sites that are partially evacuated by the planets' secular resonances in the disk.

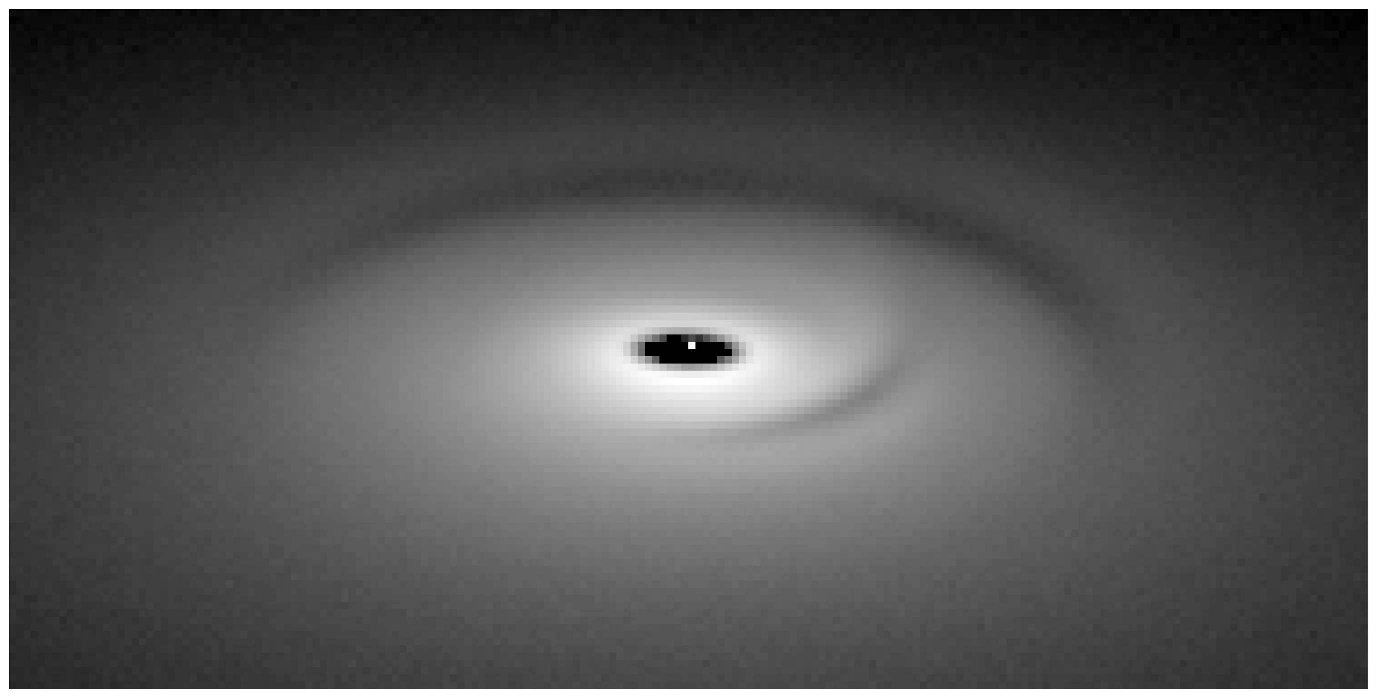

FIGURE 2: Logarithmic surface brightness of the dust from Fig. 1 seen via scattered starlight. The disk is viewed 20° from edge-on, and the grains have a scattering parameter $g = 0.4$. Dark arcs at $a \sim 50$ and 90 AU are partially evacuated by the planets' secular resonances; see Fig. 1. Note also that the star is offset from the disk's center due to the planets' secular perturbations.

β Pictoris

A first application of the model is the edge-on dust-disk orbiting β Pictoris. This disk is warped, with the inner $r \lesssim 100$ AU tipped $\sim 5^\circ$ from the outer disk (Heap *et. al.* 2000; see also Fig. 3). Mouillet *et. al.* (1997) show that this warp could be an outward-propagating kinematic wave launched by a single inclined planet. However, if the warp is instead in steady state (which we suspect is likely), then at least *two* inclined planets are required in order to warp the system's *Laplace surface*, which is the disk's warped midplane defined by Eqns. (2–3) with $I_{\text{free}} = 0$ (Novotny 2004).

Fig. 3 shows our best fitting two-planet model of a warped circumstellar dust-disk, which compares well with the HST image of β Pic. This result was arrived at via an automated search of a rather large parameter space: 10 parameters for the disk, plus $6N = 12$ parameters for $N = 2$ planets.

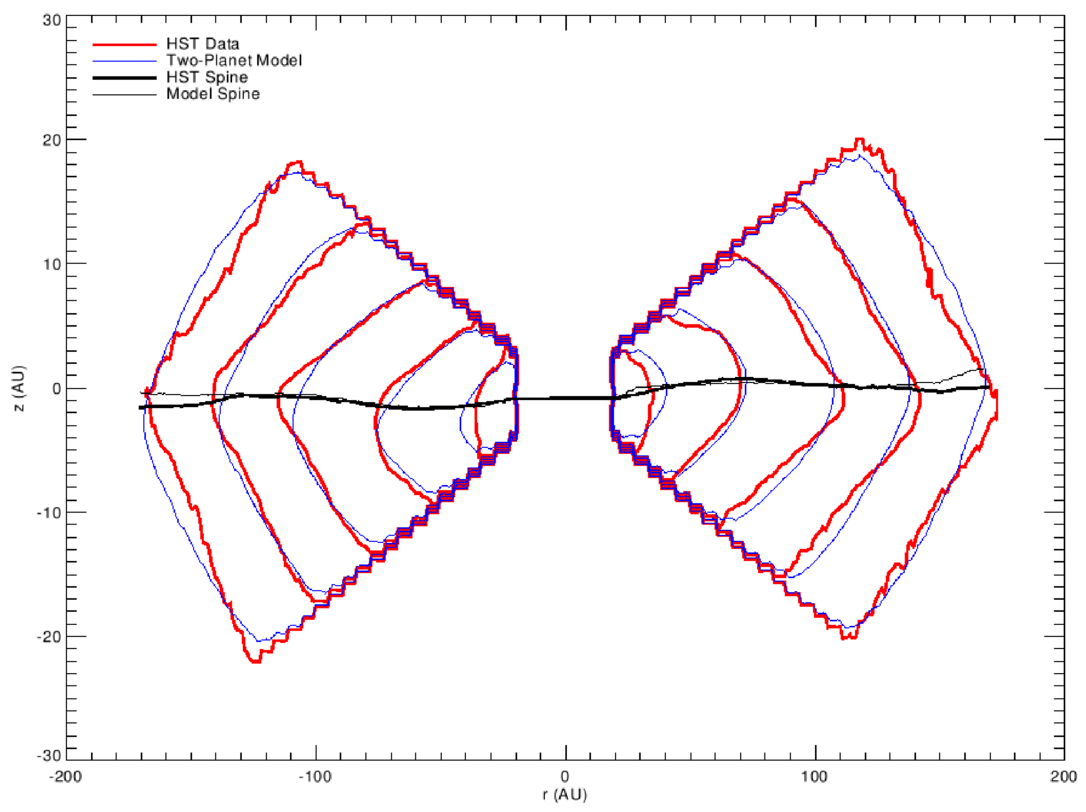

FIGURE 3: Red isophotes are the HST image of β Pic (from Heap *et. al.* 2000), masked at latitudes beyond $\pm 10^\circ$ and with the vertical axis is expanded by a factor of 5. Blue curves are the best-fitting dust-disk model, which is warped by two planets of mass 7 and 3 Jupiter masses, semimajor axes $a = 35$ and 130 AU, and a mutual inclination of 5° (Capobianco 2006). The warp is indicated by the horizontal curves that trace the disk's *spine*, which shows the height of the disk's peak brightness.

Future activities

- *examine the 'birth ring' scenario*

The surface brightness SB in the inner portions of several edge-on debris disks, such as β Pic and AU Mic, varies at $SB \propto r^{-1.5}$ out to $r_0 \sim 50$–100 AU, with the disks' outer portions diminishing much faster as $SB \propto r^{-4.5}$ or so. Strubbe & Chiang (2006) and Augereau & Beust (2006) show this may be due to a dust-producing ring of planetesimals orbiting at $r \sim r_0$, which launches smaller dust grains into the outer disk on very wide orbits having eccentricities

$$e = \beta/(1-\beta) \tag{5}$$

due to radiation pressure and/or solar wind, where β is the usual radiation–pressure/stellar gravity ratio that varies as grain size^{-1}.

We will examine this birth ring scenario using our dust-disk model composed of grains having a distribution of sizes and thus a range of eccentricities given by Eqn. (5). Inserting that free eccentricity into Eqns. (2–3) will yield a more realistic model of the birth ring scenario that is also perturbed by unseen planets. Fitting this model to the observations of β Pic will then determine whether the birth ring scenario alters the preliminary β Pic findings reported in Fig. 3.

- *modeling the disk's thermal emission*

We are also upgrading the dust-disk model so that it can generate an image of a model disk's thermal emission, which will then allow us to model multi-wavelength observations, such as the dust ring at Fomalhaut (see Fig. 4), and the β Pic disk (Figs. 3 and 5). The eccentric Fomalhaut ring ($e \sim 0.1$) is particularly interesting, since it may be forced by an unseen planet (Stapelfeldt *et. al.* 2004).

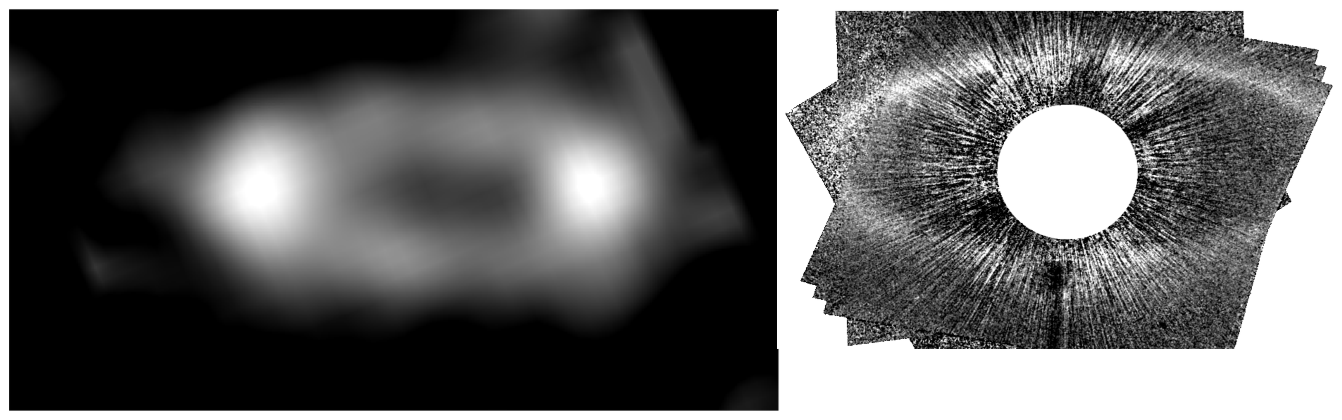

FIGURE 4: The $\lambda = 350\ \mu$m thermal emission from the $r \sim 150$ AU dust ring orbiting Fomalhaut (left, from Marsh *et. al.* 2005), and an optical HST image (right, from Kalas *et. al.* 2005).

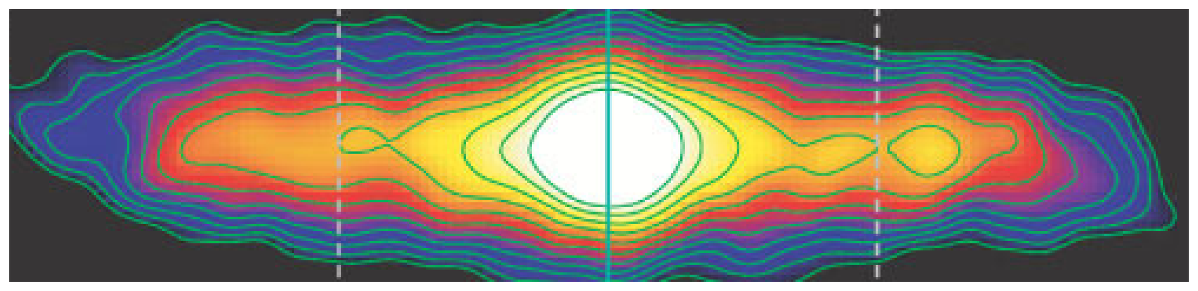

FIGURE 5: β Pic at $\lambda = 25\ \mu$m, from Telesco *et. al. 2005.*

- *structure due to secular resonances*

We will also determine whether secular resonances might account for other structures seen in circumstellar disks. Figure 6 shows the HST image of HD 141569A acquired by Clampin *et. al.* (2003), revealing rings and arcs that are both under- as well as over-dense. These structures have also been interpreted as spirals, and simulations by Quillen *et. al.* (2005) show that HD 141569A's stellar companions can in fact excite spirals in such disks. However, the structure seen in Fig. 6 could also be described as *arcs*, rather than spirals, which suggests to us that secular resonances might instead be operative here, as in Fig. 2. We shall use the dust-disk model to explore this scenario in greater detail.

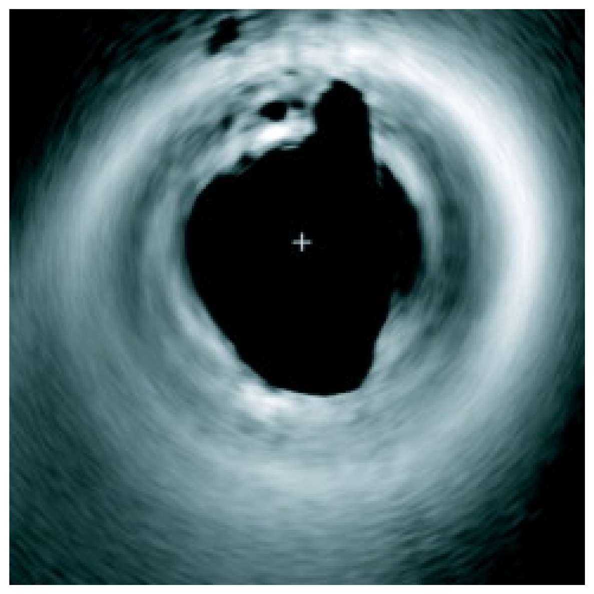

FIGURE 6: Opical HST image of the disk orbiting HD 141569A, from Clampin *et. al.* (2003).

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