THE SECULAR EVOLUTION OF THE PRIMORDIAL KUIPER BELT. J. M. Hahn, *Lunar and Planetary Institute, Houston TX 77058, USA, (hahn@lpi.usra.edu)*.

The Kuiper Belt is a vast swarm of comets orbiting at the Solar System's outer edge. This Belt is comprised of debris that was left over from the epoch of planet formation, and this swarm's distribution of orbit elements preserves a record of events that had occurred when the Solar System was still quite young. Although the common goal of most dynamical studies of the Kuiper Belt is to decipher this record, its interpretation is not entirely clear.

The ~ 400 dots in Fig. 1 represent the Kuiper Belt Objects (KBOs) eccentricities e and inclinations i plotted versus their semimajor axes a. This Figure reveals the KBOs' three major dynamical classes: the Plutinos which inhabit Neptune's 3:2 resonance at $a = 39.5$ AU, the Main Belt KBOs which are the nonresonant bodies orbiting between $40 \lesssim a \lesssim 48$ AU, and the more distant Scattered KBOs that live in eccentric, nearly Neptune crossing orbits. The Figure also shows that the Plutinos and the Scattered KBOs have inclinations that span $0 \lesssim i \lesssim 30^\circ$, while the Main Belt KBOs appear to have a bimodal distribution of inclinations centered on $i \simeq 2^\circ$ and $i \simeq 17^\circ$ [1]. Note that accretion models show that these large $\sim 100+$ km KBOs must have formed from much smaller planetesimal seeds that were initially in nearly circular and coplanar orbits having e and $\sin i \lesssim 0.001$ [2]. However the swarm's gravitational self stirring can not account for the Kuiper Belt's current excited state, so one or more mechanisms must also have stirred up the Kuiper Belt since the time of formation.

The various dynamical classes in the Kuiper Belt provide distinct insights into the history of the outer Solar System. For instance, the Scattered objects are KBOs that wandered close enough to Neptune to have had one or more close encounters with that planet which lofted these bodies into eccentric, inclined orbits [3]. However the Plutinos at Neptune's 3:2 resonance have had a different history; their concentration at this resonance suggests that that planet's orbit had expanded outwards some ~ 7 AU soon after its formation as Neptune began to vigorously scatter the local planetesimal debris [4]. Since this scattering process can result in an exchange of angular momentum between the planets and the planetesimal disk, this episode of disk clearing can result in a substantial rearrangement of all of the giant planets' orbits. This outward migration would also cause Neptune's mean motion resonances to sweep out across the primordial Kuiper Belt, and these migrating resonances are quite effective at accumulating KBOs and pumping up their eccentricities [4,5]. However if migration did indeed occur, then the Main Belt KBOs at $40 \lesssim a \lesssim 48$ evidently managed to slip through the 2:1 resonance as it advanced outwards to 48 AU. One of the outstanding outstanding mysteries then is to account for the Main Belt's high $i \sim 17^\circ$ component since models show that planet migration is quite ineffective at pumping inclinations up to this level [4,5].

It has been suggested that sweeping secular resonances, which are driven across the Solar System due to the removal of the primordial solar nebula gas, could account for the KBO's high inclinations. Indeed, a recent study shows that this mechanism can in fact pump the inclinations of massless KBOs up to the requisite amount [6]. However, when the Kuiper Belt has even just a modest amount of mass, a more recent model of a gravitating Kuiper Belt reveals a very different endstate due to the propagation of one–armed spiral density and spiral bending waves in the Kuiper Belt [7]. These waves are also known as apsidal density waves and nodal bending waves since their spiral patterns slowly rotate on precessional timescales. The properties of these waves are detailed in [8] and [9].

However the preliminary results reported in [7] had a deficiency in that it represented a continuous particle disk as a nested set of infinitesimally thin rings whose mutual gravitational perturbations cause the rings to elongate and tilt. Since thin rings have the same time-averaged gravitational potential as that due to an orbiting point mass [10], the classical Laplace–Lagrange solution for the secular evolution of the planets could then be applied to quickly obtain the secular evolution of this system of thin rings. The problem is that a real disk has a finite half–thickness h owing to the particles' dispersion velocities, so the model disk should instead be treated as a set of rings that also have a thickness h. As Tremaine points out, the effect of a ring's finite thickness is to soften its gravitational potential, and one consequence of this softening is a reversal of a massless body's periapse precession from prograde to retrograde when perturbed an axisymmetric disk [11, 12]. Since the strength of the ring–ring interactions are represented by Laplace coefficients, the rings' nonzero thickness thus requires the use of a softened Laplace coefficient

$$\tilde{b}_s^{(m)}(\alpha) = \frac{2}{\pi}\int_0^\pi \frac{\cos(m\phi)d\phi}{[(1+\alpha^2)(1+H^2) - 2\alpha\cos\phi]^s} \quad (1)$$

where $\alpha = a'/a$ is the ratio of the perturbing ring's semimajor axis a' to the perturbed ring's semimajor axis a, and the softening parameter $H^2 \equiv [(h/a)^2 + (h'/a')^2]/2$ depends upon the rings' thicknesses h and h'. The secular evolution of thick rings is thus readily obtained by replacing the unsoftened Laplace coefficients appearing in the classical Laplace–Lagrange solution with its softened counterpart, Eq. 1. However a more rigorous derivation of these assertions will also be given in [13].

A model of the secular evolution of the primordial Kuiper Belt has been developed, and results are summarized in Fig. 1. The model treats the Kuiper Belt as $N \sim 300$ gravitating rings typically spanning the interval $36 \le a \le 70$ AU, each having a thickness $h = \Delta a$ such that adjacent rings separated a distance Δa represent particles in crossing orbits. (Note that when $h \ge \Delta a$, a massless ring's periapse regresses at the expected rate given in [14]). The effects of the solar nebula gas are ignored in this treatment. The model also includes the four giant planets which are represented as rings having a zero thickness. The planets' initial orbits are their current orbits, and the Kuiper Belt rings are initially circular and coplanar with the system's invariable plane. When the system is evolved forwards in time, the giant planets launch spiral density and spiral bending waves at the Kuiper Belt's inner edge. These waves propagate outwards were they reflect at outer disk edge and propagate inwards to reflect again at the disk's inner edge. Eventually, a standing wave pattern is established in the Belt, and Fig. 1 shows the maximum eccentricities $e_{\max}$ and inclinations $i_{\max}$ that result. The rings' instantaneous e's and i's range over $0 < e < e_{\max}$ and $0 < i < i_{\max}$.

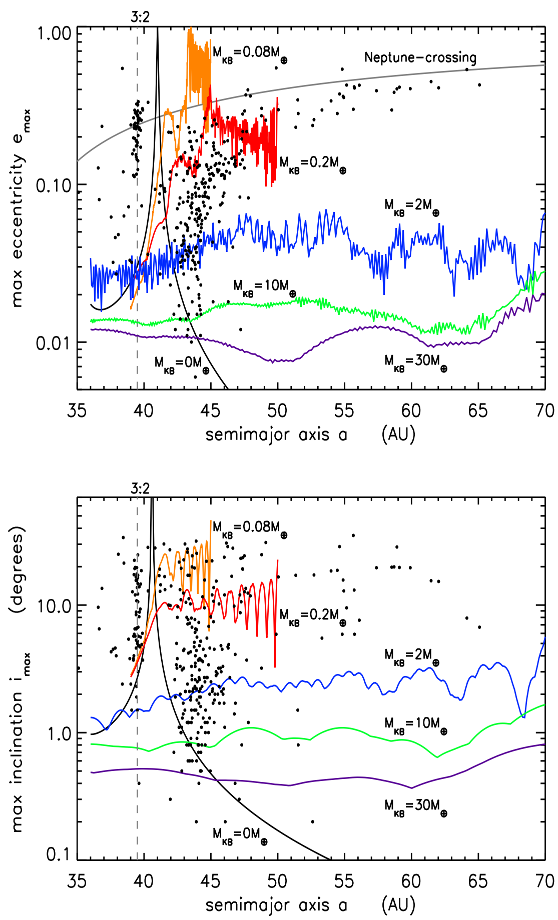

Figure 1: Dots indicate the eccentricities e and inclinations i plotted versus semimajor axes a of 373 multi-opposition KBOs reported by the Minor Planet Center. The dashed lines indicate Neptune's 3:2 resonances at 39.5 AU while the grey curve indicates the threshold for Neptune–crossing orbits. The colored curves are the maximum eccentricities $e_{\max}$ and maximum inclinations $i_{\max}$ achieved by various model Kuiper Belt's having a mass $M_{\rm KB}$ that would have been contained within the $30 < a < 50$ AU interval had the model Belt extended inwards that far. In these runs the disk rings have a thickness $h/a \sim 0.002$.

Each set of curves in Fig. 1 corresponds to simulations of a Kuiper Belt having a mass $M_{\rm KB}$. Although the present Kuiper Belt mass is $M_{\rm KB} \sim 0.2$ M$_\oplus$ [15], accretion models suggest the primordial mass was ~ 100 times larger [2]. Note that the excitation seen in the Belt is greater in a disk of lower mass (Fig. 1). This is due to the fact that in every simulation, the giant planets deposit roughly ~ 1% of their initial angular momentum deficit into the Kuiper Belt in the form of a spiral density wave, and ~ 10% of their initial in–plane angular momentum into the Belt in the form of a spiral bending wave. Since the angular momentum deposition is the roughly the same in all systems, the lower mass system reponds with higher amplitude waves. In fact, the model predicts that the rings in a low mass system having $M_{\rm KB} = 0.08$ M$_\oplus$ experience eccentricities of $e \sim 1$, which clearly violates the low eccentricity assumption that was built into the Laplace–Lagrange solution used here. Thus the orange curve in Fig. 1 should not be taken at face value. It should also be noted that lower mass disks admit shorter wavelength spiral waves.

Note, however, that wave–action ultimately must break down when one considers a system of sufficiently low mass. This breakdown likely occurs when the epicyclic motions h of the individual particles in each ring exceed the wavelength λ, in which case the system probably behaves as if it were massless, which is indicated by the $M_{\rm KB} = 0$ curve. In this instance, large e's and i's would occur only at the secular resonances near 41 AU, and the rest of the Belt would remain largely undisturbed.

Finally, it should be noted that if the Kuiper Belt was initially dynamically cold, then Fig. 1 suggests that, as the Belt's mass eroded down to its present mass of $M_{\rm KB} = 0.2$ M$_\oplus$, the Main Belt's inclinations would have been pumped up to $i \sim 10^\circ$ due to the propagation of spiral bending waves. This then suggests that the greater mystery is *not* the origin of the high i component in the Main belt (for recent work by Gomes may have the solution to that problem; see [16]), but understanding how the low i component manages to maintain its dynamically cold state. One might speculate that the cold component in the Main Belt achieves this by communicating its bending waves outwards into a more massive and as yet unseen Kuiper Belt orbiting beyond ~ 50 AU.

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