ORBITAL EVOLUTION OF PLANETS EMBEDDED IN A MASSIVE DEBRIS DISK. J. M. Hahn, *Lunar and Planetary Institute, Houston TX 77058-1113, USA, hahn@lpi.jsc.nasa.gov*, R. Malhotra, *renu@lpi.jsc.nasa.gov*.

The discovery of the Kuiper Belt has revealed that a sizable fraction of its members orbit at Neptune's mean motion resonances [1]. Such resonant configurations would have developed naturally had Neptune's orbit slowly expanded during an earlier time when there was still debris left over from the epoch of planet-formation [2]. Indeed, Monte Carlo models of planetary accretion have shown that the cores of Uranus and Neptune may have shifted over considerable radial distances due to an angular momentum exchange that develops when the giant planets toss planetesimals amongst themselves [3,4]. Here, the planet migration phenomenon is explored via direct integration of the giant planets' motions while embedded in a massive debris disk. Since the eccentricities and inclinations of resonantly trapped Kuiper Belt objects depend upon the rate at which Neptune's orbit may have expanded [2,6], which in turn depends upon the mass of the disk that drives this process, it is possible to infer the amount of planetesimal mass that was once present during the late stages of outer planet formation.

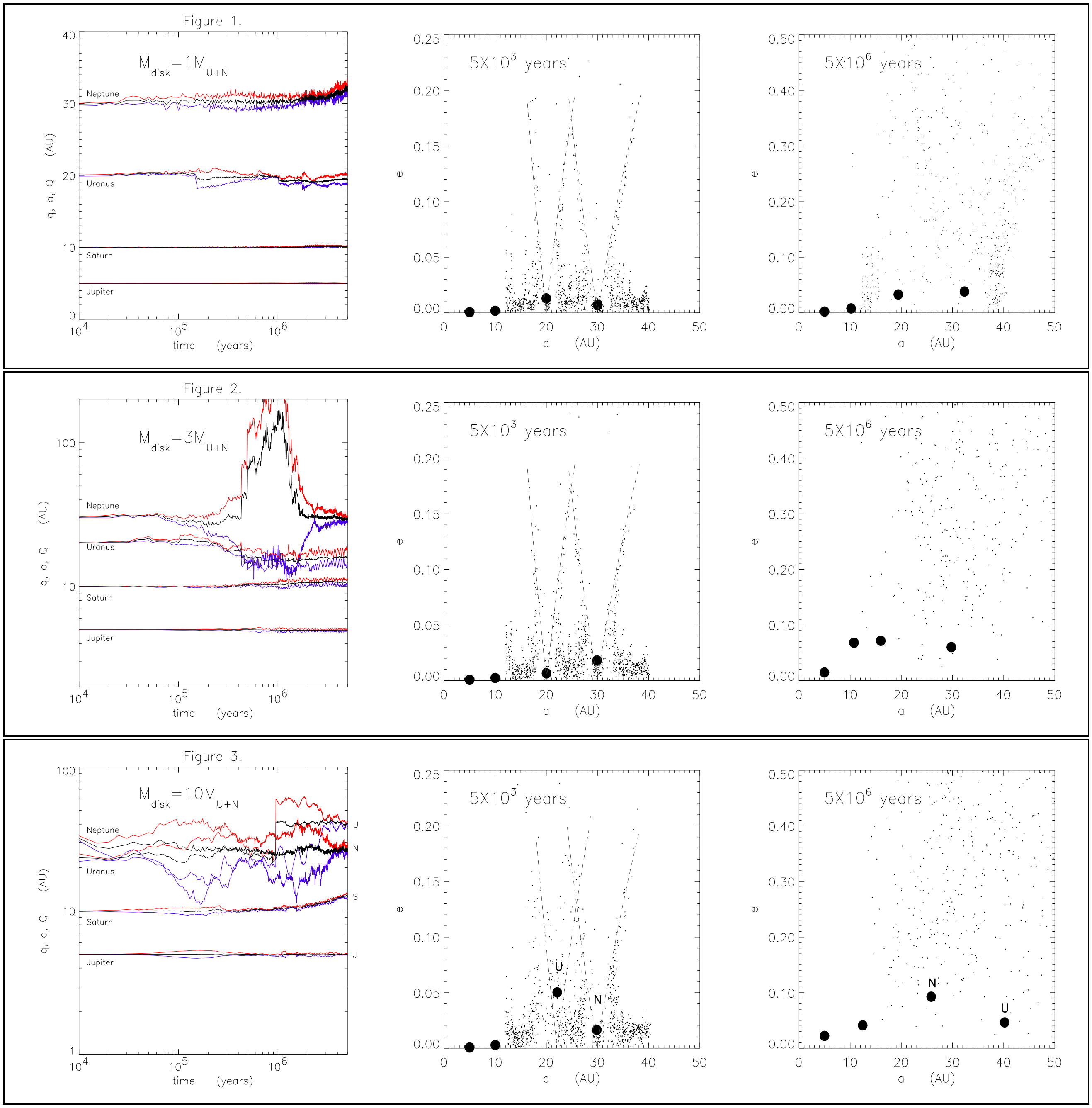
A second-order mixed variable symplectic (MVS) integrator [7,8] is employed to rapidly evolve the planets and disk particles trajectories numerically. Of particular interest here are the close planet–particle encounters that drive the planet migration process. As is well known, an MVS integrator is not ideally suited for particle systems that experience close encounters. To mitigate this problem, a particle's planet–centered trajectory is approximated as simple two–body motion when making a sufficiently close pass, with the planet reacting to the particle as required by momentum conservation. We find that a "sufficiently close" encounter is typically one that, during a single integrator timestep, changes the particle's fractional distance from the target planet by more than 50%, or varies its planet–centered anomaly by more than 90°. (These thresholds are optimized for resolving close encounters with a Jupiter at 5 AU and requires an integration timestep that samples Jupiter's motion at least 30 times per orbit.) Typically, this close encounter algorithm is triggered only when a particle gets nearer than about 20% of the planet's Hill sphere radius. For more distant encounters, particle trajectories are in fact evolved with greater accuracy with the MVS integrator rather than with a two–body approximation. The model computes all forces exerted between the four giant planets as well as the mutual forces exerted between the planets and the disk particles. But to keep the simulation runtime manageable it is necessary to neglect particle–particle interactions. Consequently, possible collective disk effects (e.g., spiral density waves [9]) are not treated. In this implementation all bodies are point masses, so collisional accretion is not (yet) treated.

This algorithm has the advantage over previous planet migration studies [3,4] that employ an Öpik formalism [5] since the MVS integrator accurately treats the long-range periodic and secular forcing exerted amongst the planets and the debris disk. These are additional paths by which the disk and planets can exchange angular momentum that is not accounted for in the Öpik calculations.

Results from an integration spanning 5×10^6 years are shown in Figs. 1-3. The four giant planets begin in initially circular coplanar orbits having semimajor axes $a_p = 5$, 10, 20, and 30 AU with their full masses. Although starting with fully formed planets effectively ignores the system's prior history of accretion/migration, this approximation focuses precious computer cycles on the latest stage of planet migration when particle scattering is most vigorous and the planets' orbital evolution is likely most rapid. The inner edge of the debris disk lies at 12 AU, near Uranus' 1:2 mean-motion resonance, and the disk extends out to 40 AU near Neptune's 3:2 resonance. The disk is composed of 1000 equal-mass particles distributed radially with a surface density that varies as distance^{-1}. For each simulation the particle masses are adjusted such that the total disk mass is 1, 3, and 10 times the combined mass of Uranus and Neptune ($\equiv M_{U+N} \simeq 32 M_\oplus$, where $M_\oplus$ is one Earth-mass), and the particles begin with small eccentricities/inclinations of 0.005.

From the figures it is evident that if the disk mass $\lesssim 1\ M_{U+N}$ there is only slight adjustment in the planets' orbits during the integration interval. But if the disk has mass $\gtrsim 3\ M_{U+N}$, then the planets are much more mobile and start to interact. As the planets excite their eccentricities, they quickly stir up and deplete the disk. It is evident that none of the simulations in Figs. 1-3 could be representative of the Solar System's early evolution since they are unlikely to produce the delicate resonant structure observed in the present Kuiper Belt. However if Neptune did indeed migrate a substantial radial distance over a 10^7 year timescale, then the required mass of the debris disk is likely at least $\sim 30 M_\oplus$ but probably not much more than $\sim 100 M_\oplus$. It should be noted that a $\sim 10^{6\text{ to }7}$ year migration timescale is favored by models that account for Pluto's unusually eccentric and inclined orbit as being due of its capture into the 3:2 resonance by an outward migrating Neptune [2,6,10]. Our findings should be regarded as tentative as the model planets' orbital histories are likely stochastic (*ie.*, different initial conditions may lead to different evolution) and may also depend upon the number of disk particles used as well as their size distribution, which is a subject of continued investigation. Simulations using greater numbers of particles will resolve the disk better, and final results shall be reported at conference time.

References: [1] Jewitt, D. *et al.* 1996, *AJ*, **112**, 1225. [2] Malhotra, R. 1995, *AJ*, **110**, 420. [3] Fernandez, J. A. and W.–H. Ip 1984, *Icarus*, **58**, 109. [4] Fernandez, J. A. and W.–H. Ip 1996, *Planet. Space Sci.*, **44**, 431. [5] Arnold, J. R. 1965, *ApJ*, **141**, 1536. [6] Gomes, R. S. 1997, *AJ*, **114**, 2166. [7] Wisdom, J. and M. Holman 1991, *AJ*, **102**, 1528. [8] Saha, P. and S. Tremaine 1992, *AJ*, **104**, 1633. [9] Ward, W. R. and J. M. Hahn 1998, submitted to *AJ*. [10] Malhotra, R. 1998, *Planet. Space Sci.*, in press. [11] Ida S. and J. Makino 1993, *Icarus*, **106**, 210.



Figures 1, 2, 3: On the left, the four giant planets' semimajor axis (black), periapse (blue), and apoapse (red) distances as a function of time for systems of various disk masses. To the right are snapshots of the disk particles' eccentricities versus semimajor axes, shown at two times, with the planets' orbits indicated with large dots. Initially, the planets clear gaps in the disk by scattering particles into orbits that lie along the dashed curves that approximately preserve their Jacobi integral [11], which for small inclinations is $J \simeq a_p/a + 2\sqrt{a(1-e^2)/a_p} \simeq 3$. Later, the disk becomes stirred/depleted once the planets achieve modest eccentricities. Note that in the $M_{disk} = 10M_{U+N}$ simulation, Uranus (U) and Neptune (N) exchange places.