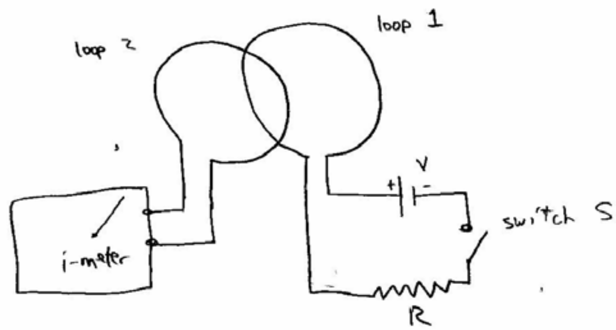
Chapter 31: Induction

See sections 1-11

induced current = a current that is produced by a $\vec{B}$ field that changes over time:

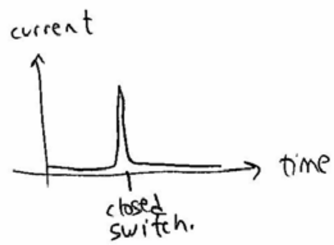
Consider this experiment:



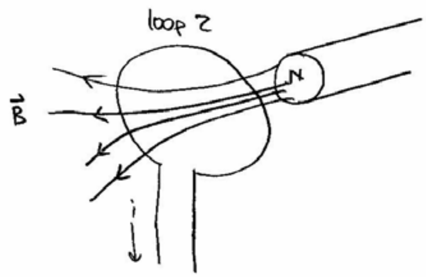
- closing the swith starts a current thru loop 1. This generates a $\vec{B}$ field.

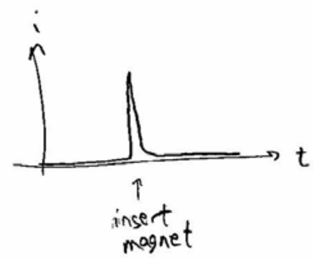
  which way does $\vec{B}$ point while inside loop 1?

- when S is closed, the ammeter reads a brief current pulse:



New experiment: remove loop 1,
and instead shove a magnet in loop 2:



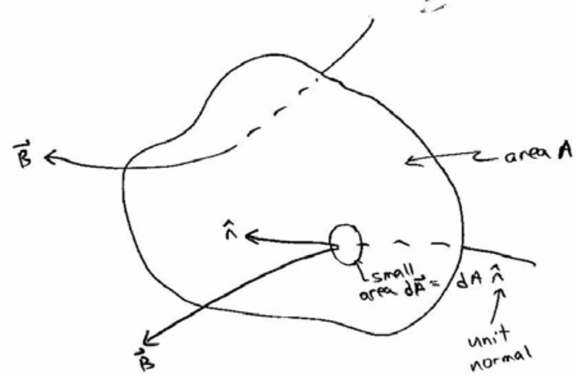


recall that emf $\mathcal{E}$ = potential difference
= iR ← wire's resistance

Note: wiggling the magnet causes $i(t)$ to wiggle.

Magnetic Flux:

B-flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$
↑ integrate over area A



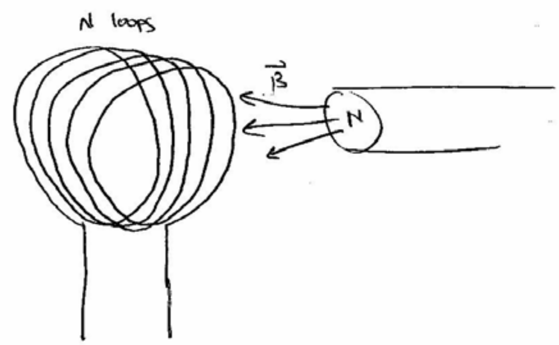
What is Φ_B if $\vec{B}$ is uniform and perpendicular to A?

Faraday's Law: emf $\mathcal{E}$ induced in a conducting loop obey's

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = iR$$

$\Rightarrow$ a time-changing $\vec{B}$-field induces an emf and a current flow

What is $\mathcal{E}$ if there are N coils of wire?



$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

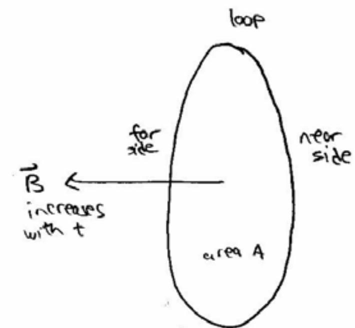
See "Induced Current" demo.

Lenz's Law - determines the direction of an induced current:

The induced current i generates an induced $\vec{B}_i$ that opposes changes in the magnetic flux Φ_B.

example:

suppose $\vec{B}$ points left, and increases over time



which way is the induced current flowing?
which way is the induced $\vec{B}_i$ field pointing?

since $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA$ increases with t,

$\vec{B}_i$ = induced B field must point right,
since its flux $\int \vec{B}_i \cdot dA = B_i A$ must be negative, so $B_i < 0$

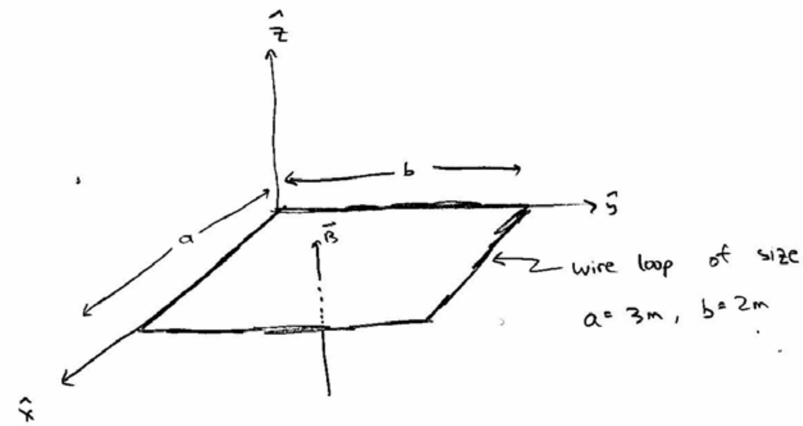
what if $\vec{B}$ decreased over time? $\vec{B}_i$ points to left.

Sample Problem 31-3.

A rectangular loop is embedded in an external B field that varies as

$$\vec{B} = B_0 \left(\frac{t}{t_0}\right)^2 \left(\frac{x}{x_0}\right)^2 \hat{z}$$

$B_0 = 4$ tesla $\qquad t_0 = 1$ sec $\qquad x_0 = 1$ m



what is the emf $\mathcal{E}$ that is induced in the loop?
which way is the induced current flowing?

Use Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$ = induced emf where $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA$ for $d\vec{A} = dA\,\hat{z} = dx\,dy\,\hat{z}$

$$= \underbrace{\int \int}_{\text{what are my integration limits ?}} B_0 \left(\frac{t}{t_0}\right)^2 \left(\frac{x}{x_0}\right)^2 dx\,dy$$

$$= B_0 \left(\frac{t}{t_0}\right)^2 \underbrace{\int_0^b dy}_{b} \ \underbrace{\int_0^a \left(\frac{x}{x_0}\right)^2 dx}_{\frac{1}{3}\frac{a^3}{x_0^2} \,=\, \frac{1}{3} x_0 \left(\frac{a}{x_0}\right)^3}$$

so $\Phi_B = \frac{1}{3} B_0\, b\, x_0 \left(\frac{t}{t_0}\right)^2 \left(\frac{a}{x_0}\right)^3$ = B-flux thru loop

so $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{2}{3}\dfrac{B_0 b x_0 t}{t_0^2}\left(\dfrac{a}{x_0}\right)^3$

$$= -144 \left(\frac{t}{t_0}\right) \text{ volts}$$

Which way is the induced field $\vec{B}_i$ pointed?

Since Φ_B increases for all time t,

the induced flux $\int \vec{B}_i \cdot d\vec{A}$ must <u>decrease</u> with time

$\Rightarrow \vec{B}_i$ points along $-\hat{z}$ direction.

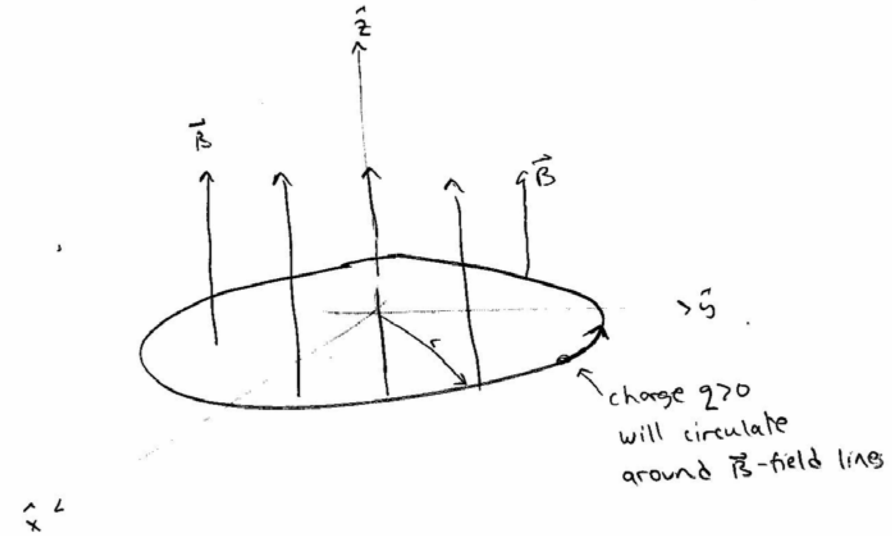
Which way does the induced current flow?

RHR $\Rightarrow$ i flows clockwise when viewed from $+\hat{z}$ zone

Induced $\vec{E}$-fields

Consider charge q as it travels thru $\vec{B}$ field that is time-varying but spatially uniform.

$$\vec{B} = B(t)\,\hat{z}$$



Lets consider the possibility that there is also an $\vec{E}$-field present... perhaps induced by the time-varying B field.

Then $W = \int \vec{F}\cdot d\vec{s} =$ work done by $\vec{E}$ on q during 1 orbit

$$= q\oint \vec{E}\cdot d\vec{s}$$

Recall that the emf $\mathcal{E} = \frac{dW}{dq} = \oint \vec{E}\cdot d\vec{s}$ (Eqn 28-1)

$$= -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)}$$

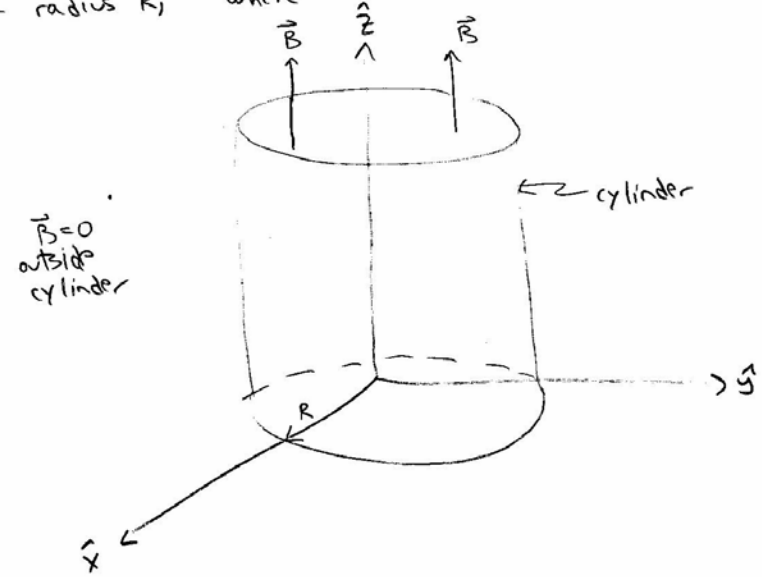
$\Rightarrow \quad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ = Faraday's Law, restated.

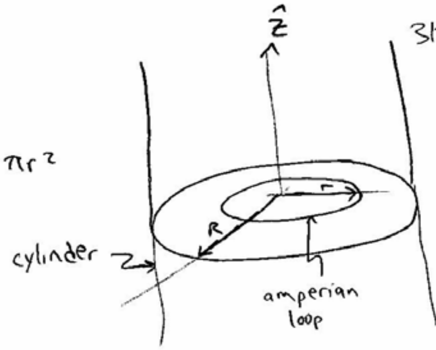
a time-changing $\vec{B}$ field will generate an $\vec{E}$-field

This eqn' holds even in the limit that $q \to 0$, ie, when there are no charges present.

Sample Problem 31-4

Suppose $\vec{B} = \frac{B_0 t}{\tau}\hat{z}$ inside a cylindrical region of radius R, where B_0, τ = constants





what is $\vec{E}$ inside cylinder?

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = B(t)\,A = B \cdot \pi r^2$$

↑ integrate area w/in amperian loop

so $$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt} = \frac{\pi r^2 B_0}{\tau}$$

while: $$\oint \vec{E} \cdot d\vec{s} = \oint E\, ds$$ due to symmetry

↑ integrate along loop

$$= E \cdot 2\pi r = -\frac{d\Phi_B}{dt} = -\frac{\pi r^2 B_0}{\tau}$$

$$\Rightarrow E = -\frac{r B_0}{2\tau}$$ at $r < R$, <u>inside cylinder</u>

what is E outside cylinder?

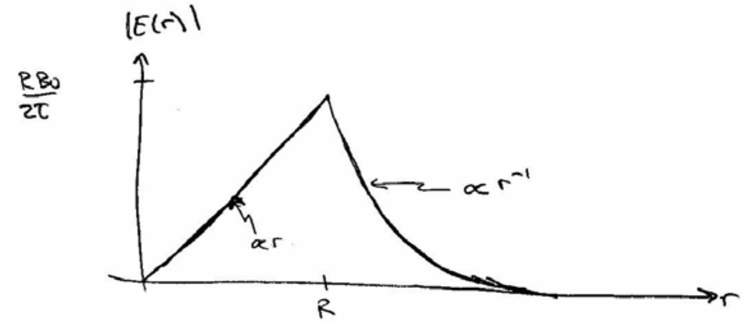
what is the flux Φ_B thru loop? $\Phi_B = B \cdot \pi R^2$

But $\oint \vec{E} \cdot d\vec{s} = E \cdot 2\pi r$ is unchanged

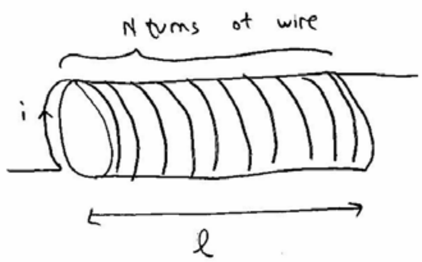
$$= -\frac{d\Phi_B}{dt} = \frac{-\pi R^2 B_0}{\tau}$$

$$\Rightarrow E = -\frac{R^2 B_0}{2 r \tau}$$ outside cylinder $r \geq R$

Plot $|E(r)|$:



<u>Inductor</u> = tightly-wound coil of wire that can store <u>magnetic</u> energy



← also known as a <u>solenoid</u>

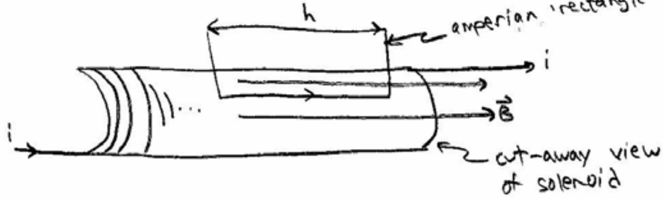
$$L = \frac{N\Phi_B}{i} = \text{inductance of a solenoid.}$$

L has units of $B \times \text{area}/\text{current}$, or $T \cdot m^2/\text{amp} \equiv 1$ Henry or 1 H.

we also call $N\Phi_B \equiv$ <u>magnetic flux linkage</u> = flux shared by N coils

Calculate Φ_B for a solenoid (from Section 30-4)

Use Ampere's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$



\# of enclosed coils

$$\oint \vec{B}\cdot d\vec{s} = B\cdot h = \mu_0 i_{enc} \quad \text{where} \quad i_{enc} = N' i$$

$$\text{so } B = \mu_0 \underbrace{\left(\frac{N'}{h}\right)}_{n = N'/h} i$$

$n = N'/h =$ coil density $= \dfrac{N}{\ell} = \dfrac{\text{total \# of coils}}{\text{total length}}$

$$\Rightarrow B = \mu_0 n i = \text{B-field in solenoid}$$

and $\Phi_B = \int \vec{B}\cdot d\vec{A} = BA =$ B-flux in solenoid

[cross-sectional area of circular amperian loop

Recall $L = \dfrac{N\Phi_B}{i} = \dfrac{NBA}{i} = N\mu_0 n A$

or $\dfrac{L}{\ell} = \mu_0 n^2 A =$ inductance per solenoid length.

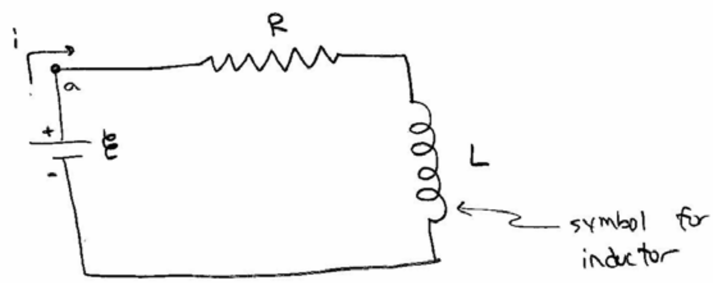
The emf that is induced in the inductor is

$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt} \quad \text{where } N\Phi_B = \text{flux shared by } N \text{ coils}$$

$$= -\frac{d(Li)}{dt} \quad \text{since} \quad L = \frac{N\Phi_B}{i} = \text{inductance}$$

or $\mathcal{E}_L = -L\frac{di}{dt}$ = emf induced across inductor

RL Circuits



Use Kirchoff's voltage law to analyze this circuit:

$$-iR - L\frac{di}{dt} + \mathcal{E} = 0 \quad \text{starting at a, going clockwise}$$

so $L\frac{di}{dt} + iR = \mathcal{E}$

which has solution $i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$

$$= \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

where $\tau = \frac{L}{R}$ = time constant for this RL circuit

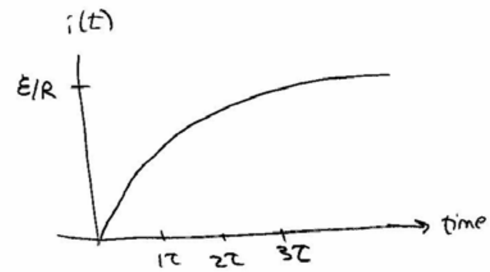
(you can confirm by calculating di/dt in Kirchoff's Law)

The voltage across the inductor is its emf, ie $V_L = |\mathcal{E}_L|$:

$$V_L = +L\frac{di}{dt} = \frac{L\mathcal{E}}{R}\left(\frac{1}{\tau}\right)e^{-t/\tau} = \mathcal{E}e^{-t/\tau}$$

= voltage across L

plot $i(t)$ and $V_L(t)$:



initially, when $(t \ll \tau)$, The conductor resists any current flow.

But at later times $(t \gg \tau)$, it acts like a conducting wire.