Chapter 28: Circuits

see sections 1-8

An EMF device = a charge pump - a battery, solar cell, fuel cell, etc, that pushes charge carriers thru a circuit

$$\mathcal{E} = \frac{dW}{dq} = \text{the emf of a device}$$

= work dW done by the emf device (often a battery) at it pushes charge dq thru circuit

$\mathcal{E}$ has units of energy/charge, or $J/c = 1$ volt in SI units

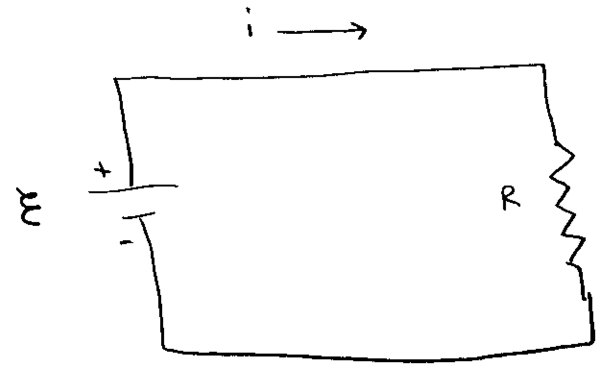
Example: a battery with a potential difference of 9 volts across its terminals has an emf of $\mathcal{E} = 9$ v.

The power of an emf device is

$$\mathcal{E} = \frac{dW}{dq} = \frac{dW/dt}{dq/dt} = \frac{P}{i}$$

so $P = i\mathcal{E}$ = rate at which the emf device does work on the charge carriers.

consider this simple circuit: an emf device (ie, a battery) hooked up to a resistor R:



since $dW = \mathcal{E}\, dq$ = small work done by battery on small charge dq as it is pushed thru the circuit

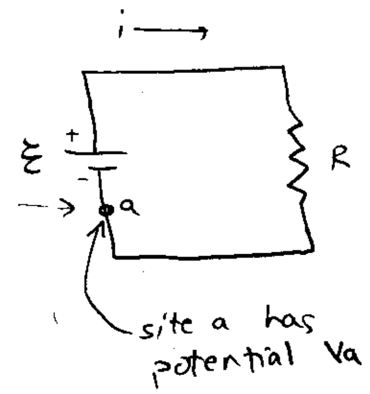
but that energy deposited at dq gets dissipated in the resistor R at the rate $P = i^2 R$ (27-22)

so $dW = \mathcal{E}\, dq = P\, dt = i^2 R\, dt$

so $\mathcal{E} \underbrace{\frac{dq}{dt}}_{i = \text{current}} = i^2 R$

$\Rightarrow i = \frac{\mathcal{E}}{R}$ = current thru this circuit

Kirchoff's voltage law.



mentally walk thru the circuit in the clockwise direction, starting at a →

Keep track of all changes in the circuit's potential, V.

When you again return to site a, the potential should be unchanged.

⇒ This is <u>Kirchoff's voltage law</u>: The sum of all changes in potential V about <u>any</u> loop in a circuit is zero.

Apply Kirchoff's Law to this circuit, starting at a, and travelling in the direction of the current flow

$$V_a + \mathcal{E} - iR = V_a$$

← back to where we began

initial potentia

potential <u>increase</u> as you cross the battery towards the + terminal

potential <u>drop</u> as you cross R in the direction of the current flow

$$\Rightarrow \quad i = \frac{\mathcal{E}}{R}$$ = current thru circuit, as expected.

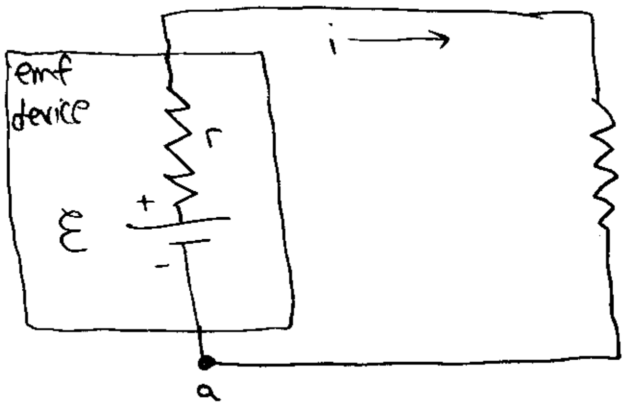
Note - you can travel in the direction opposite to the current.

In this case, $\mathcal{E} \to -\mathcal{E}$ and $-iR \to +iR$ } because the potential differences reverse as you travel in the opposite direction

the result is the same: $i = \frac{\mathcal{E}}{R}$

Internal Resistance:

Most emf devices also have some internal resistance r:



Start at a and use the Kirchoff voltage law, clockwise:

$$V_a + \mathcal{E} - ir - iR = V_a$$

$$\Rightarrow \mathcal{E} = i(r+R)$$

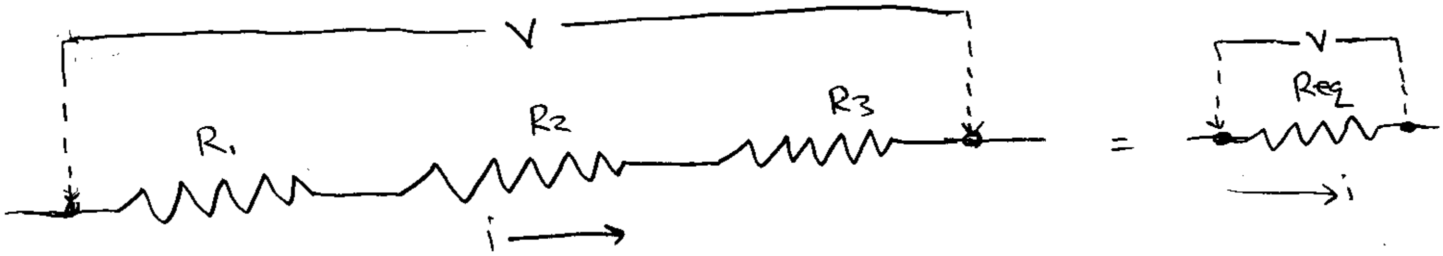
$$= i R_{eq} \quad \text{where } R_{eq} = r+R$$

= equivalent resistance

so $i = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{r+R}$ = current thru circuit.

Note that we can replace the two resistors r and R with an equivalent resistor $R_{eq} = r + R$

Resistors in series (ie, one after the other):

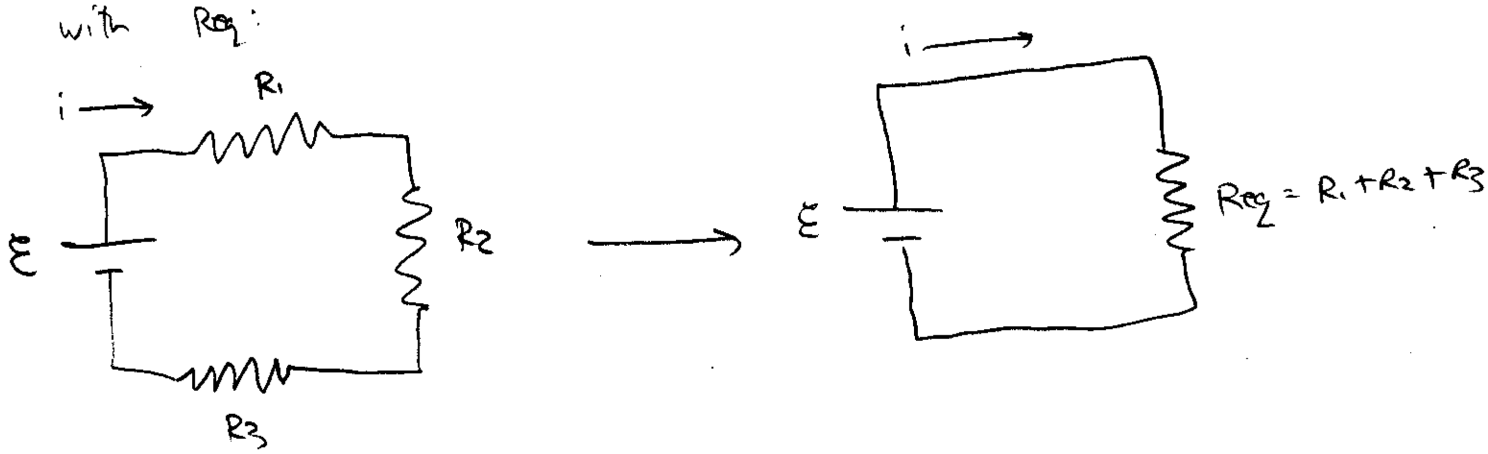


These resistors have an equivalent resistance

$$R_{eq} = \sum_{j=1}^{n} R_j = R_1 + R_2 + R_3 + \cdots$$

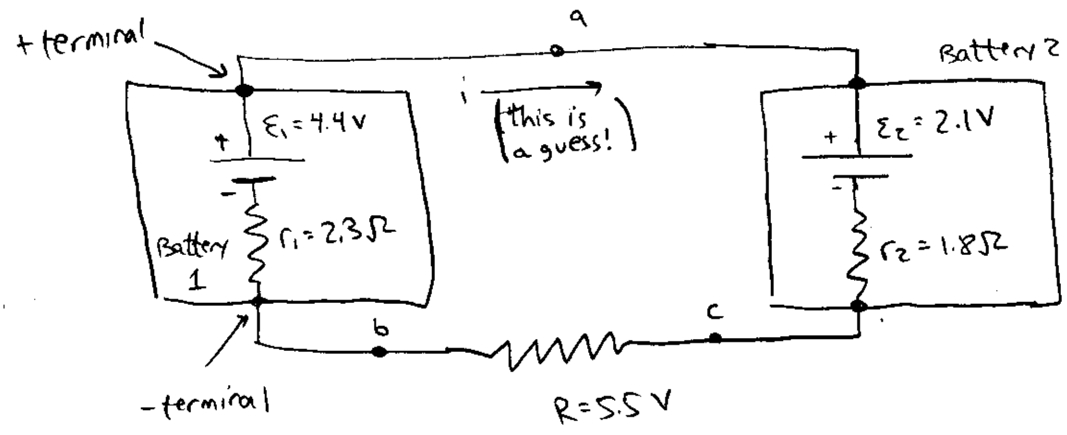
Both sets of resistors will have the same voltage V across them, and the same current i running thru them.

When analyzing circuits, simplify by replacing R_1, R_2, etc with R_{eq}:



Sample Problem 28-1

a.) what is The current thru This circuit?



Note That the batteries are connected + to + !
⇒ the emf of one battery is going to oppose the other!
(actually, one battery will tend to charge up the other)

use Kirchoff's voltage law. start at a, and travel counterclockwise, against The assumed current:

$$V_a - \mathcal{E}_1 + ir_1 + iR + ir_2 + \mathcal{E}_2 = V_a$$

so $\quad i(r_1 + R + r_2) = \mathcal{E}_1 - \mathcal{E}_2$

or $\quad i = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{R_{eq}}$ where $R_{eq} = r_1 + R + r_2 = 9.6\Omega$

$$= \frac{2.3\,V}{9.6\,\Omega}$$

so $i = 0.24$ amps = current thru circuit.

which way is this current flowing? Clockwise or Counter-clockwise?

b.) what is the potential difference across the terminals of battery 1?

That difference is $V_a - V_b$.

(Note that resistor r_1 is internal to the battery's terminals)

Again use K's voltage law.

This time, start at b, and go clockwise:

$V_b - ir_1 + \mathcal{E}_1 = V_a$

so $V_a - V_b = -ir_1 + \mathcal{E}_1 = 3.85$ volts

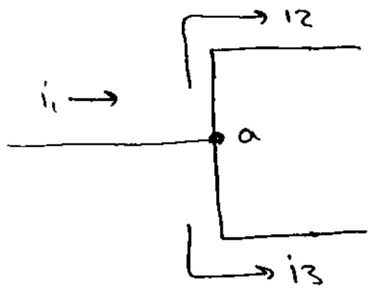
= voltage across terminals in battary 1

Note that $V_a - V_b$ is slightly less than $\mathcal{E}_1$, due to the battery's internal resistance r_1.

Multi loop Circuits

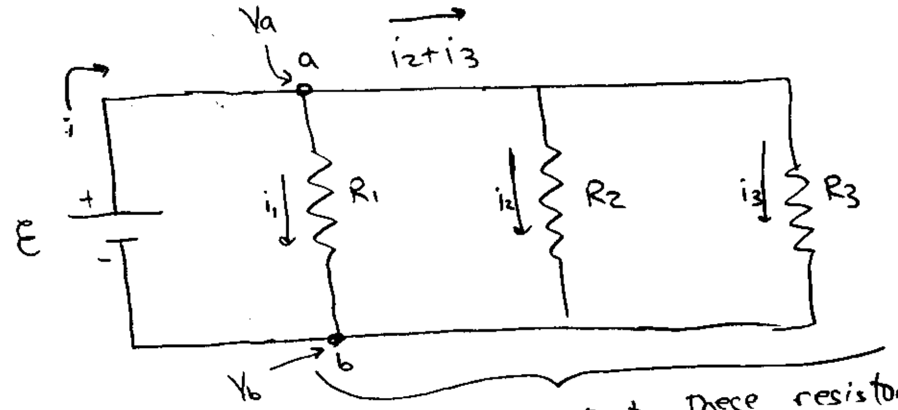
Kirchoff's current law: The sum of all currents entering a junction = sum of all exiting currents

example



$i_1 = i_2 + i_3$

Use this result to examine a multi loop circuit.



Note that these resistors are in "parallel"

$\Rightarrow$ all resistors in parallel have the <u>same</u> potential difference across them, $V_b - V_a \equiv V$

However the currents thru these resistors differ:

$i_1 = \frac{V}{R_1}$ $\qquad i_2 = \frac{V}{R_2}$ $\qquad i_3 = \frac{V}{R_3}$

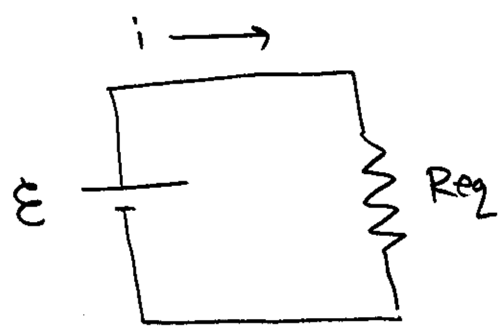
Kirchoff's current law at site a says:

$$i = i_1 + i_2 + i_3 = V \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}_{\text{replace with } \frac{1}{R_{eq}}}$$

$\Rightarrow$ the equivalent resistance for resistors in parallel is

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

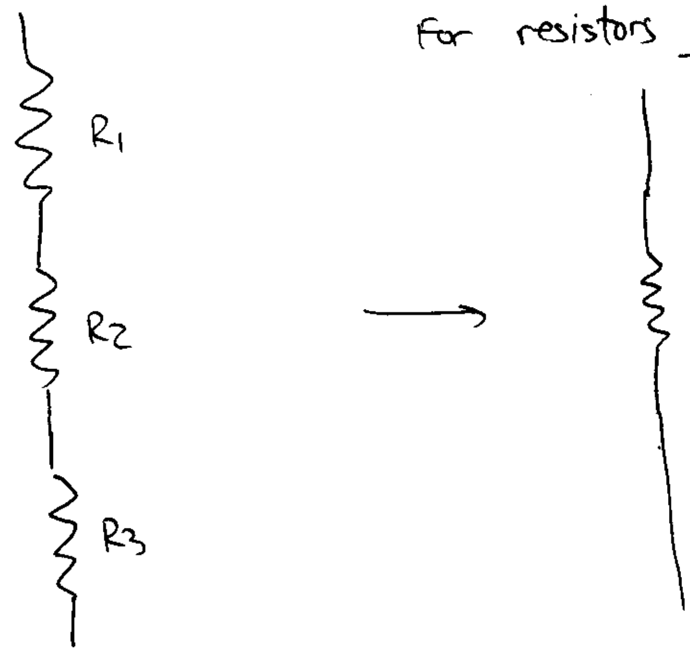
So we can replace our multi-loop circuit with the simpler circuit



Use this trick to simplify your analysis of circuits.

In Summary:

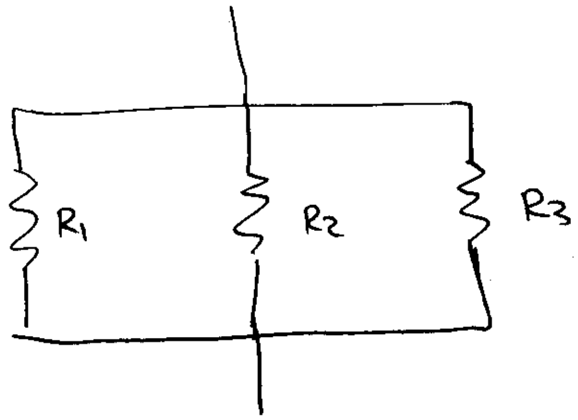
For resistors in series:

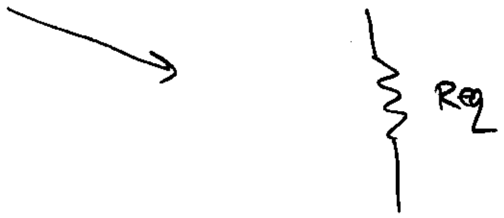


$$R_{eq} = R_1 + R_2 + R_3$$

For resistors in parallel

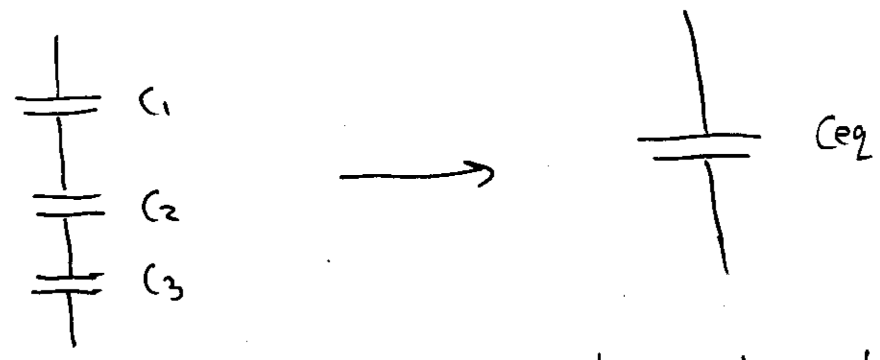
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$





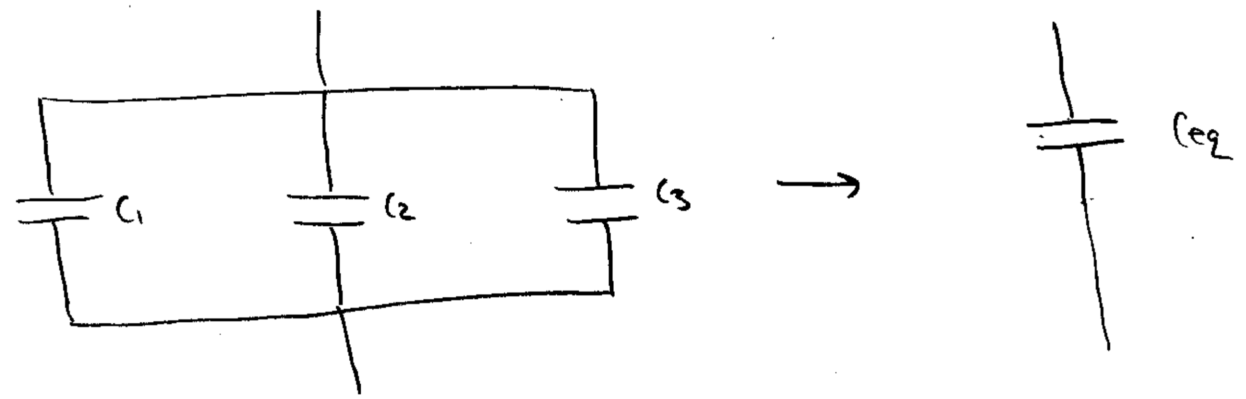
What about capacitors? See Section 26-4:

For C's in series:



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

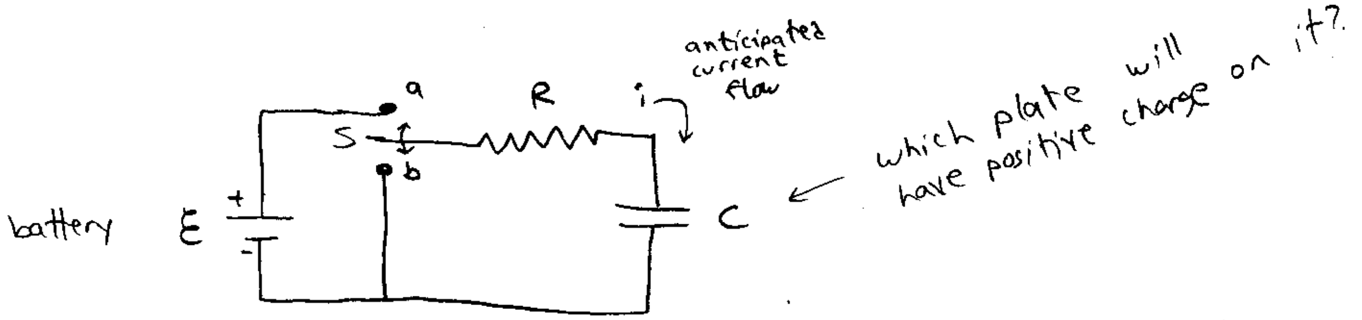
For C's in parallel:



where $C_{eq} = C_1 + C_2 + C_3$

RC Circuits

Lets examine this RC circuit:



Note: closing the switch S at point a completes the circuit. Current will flow, and charge up C.

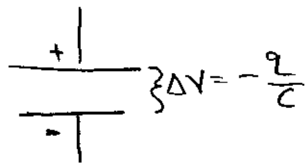
Then, closing the switch at point b completes another circuit. This discharges the capacitor.

1. charge the capacitor (ie, close switch on a).
Kirchoffs law (starting at - terminal, running clockwise):

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

recall that $V = \frac{q}{C}$ = potential across C

minus sign indicates that the potential decreases as you travel from + plate to - plate:



Since $i = \frac{dq}{dt}$,

$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$$ ← differential eqn' for charge $q(t)$ on capacitor

solution: $q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$

verify: $\frac{dq}{dt} = +\frac{C\mathcal{E}}{RC}e^{-t/RC}$

so $R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}e^{-t/RC} + \mathcal{E}\left(1 - e^{-t/RC}\right) = \mathcal{E}$ ✓

Also, the current that charges the capacitor is

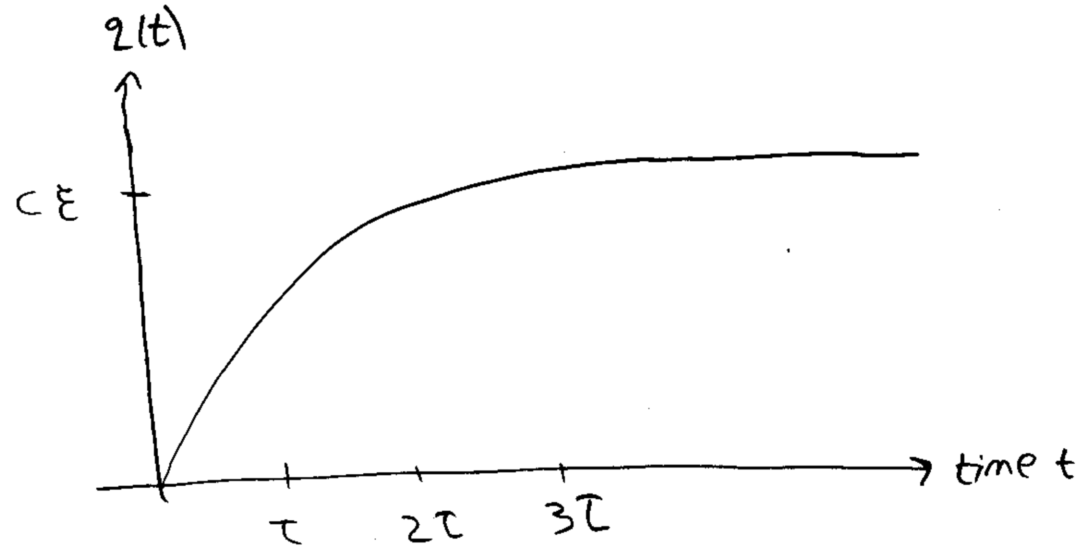
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

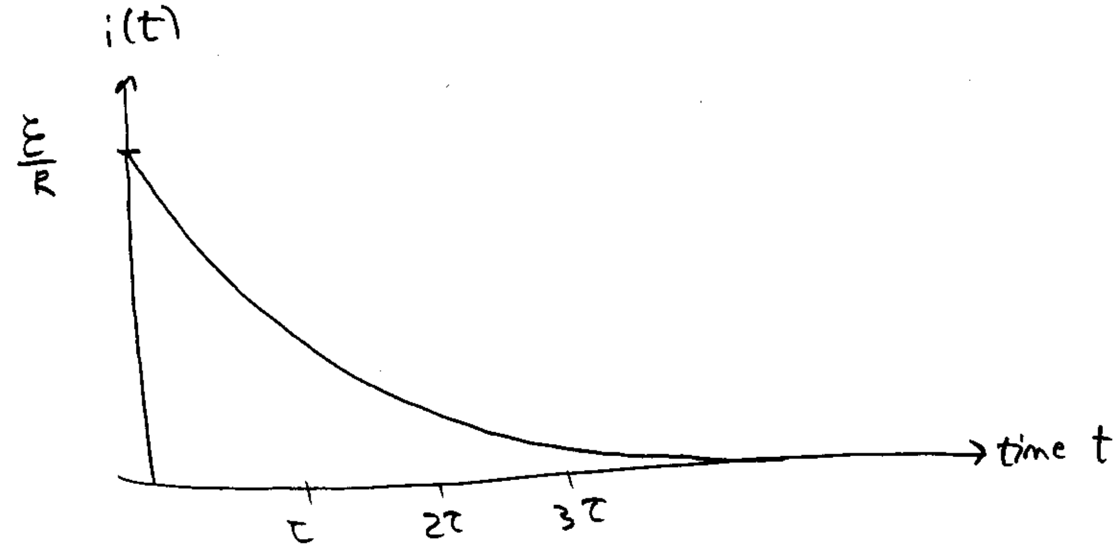
while the voltage V_C across the capacitor is

$$V_c = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

Note that the product $RC =$ time-constant τ

$\tau = RC =$ e-folding time $=$ time to increase $q(t)$ by a factor $e^1 = 2.72\ldots$





2. To discharge the capacitor:

Put switch in position 'b.'
This effictively removes the battery, $\mathcal{E}$

The equation for this circuit is obtained by setting $\mathcal{E}=0$:

$$R\frac{dq}{dt}+\frac{q}{C}=0$$

$$\text{or}\quad \frac{dq}{dt}=-\frac{q}{\tau}$$

this t = time since switched to 'b' position

which has solution $q(t)=q_0 e^{-t/\tau}$

What is q_0? q_0 = charge on fully charged C $= C\mathcal{E}$

The current is $i(t)=\frac{dq}{dt}=-\frac{q_0}{\tau}e^{-t/\tau}$

$$=-\frac{q_0}{RC}e^{-t/\tau}$$

$\Rightarrow$ discharging C also occurs over an exponential timescale $\tau = RC$