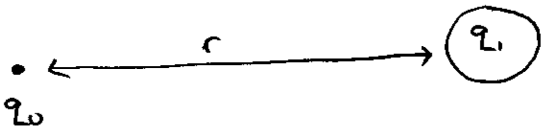
Ch 23: Electric Fields - see Sections 1-9

Recall that the electrostatic force law is

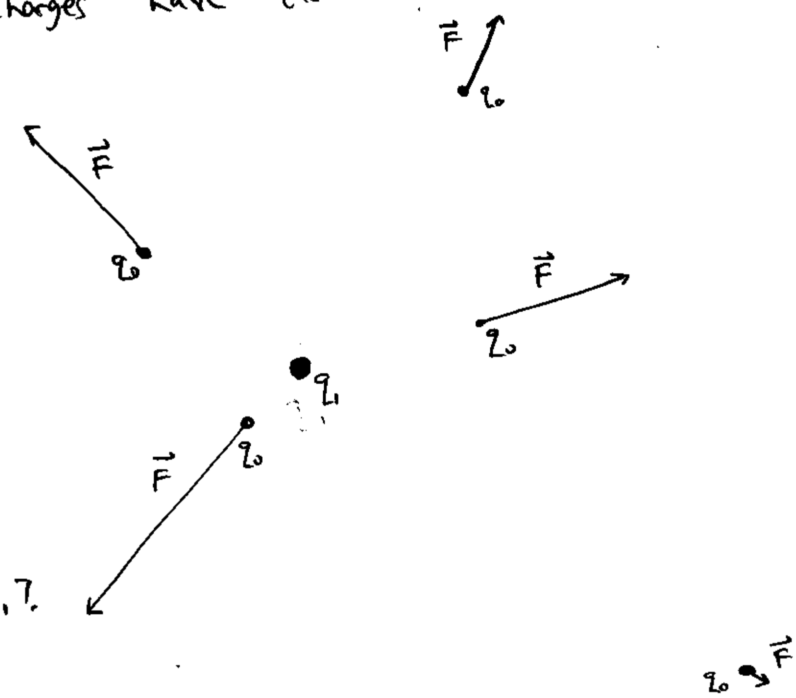
$$F = \frac{|q_0 q_1|}{4\pi\varepsilon_0 r^2}$$



= force on q_0 due to q_1

Suppose charge q_1 is nailed down so it cannot move (ie, it is <u>fixed</u> in place).

We can sample q_1's 'force-field' by moving q_0 around and measuring the force that q_1 exerts on it. If both charges have the same sign, then that force-field looks like:



Should the length of the $\vec{F}$ vectors get longer or shorter with distance from q_1?

Note that the magnitude of that electrostatic force-field is proportional to q_0, so we can write that field as

$$\vec{F} = q_0 \vec{E}$$

where $\vec{E}$ = electric field created by the fixed charge q_1:

$$E = \frac{F}{q_0} = \frac{|q_1|}{4\pi\varepsilon_0 r^2}$$ = magnitude of q_1's electric field.

[note that E has units of force/charge, usually N/C

q_1 is the "source" of the electric field that communicates/mediates any electrostatic forces felt by charge q_0

Electric Field Lines

You can sketch the electric field $\vec{E}$, which is a vector field, to indicate the direction & magnitude of the force that a test charge q_0 might feel due to a source charge q_1:

Convention for sketching field lines:

1. The $\vec{E}$-field lines originate at positive charges and terminate at negative charge.... or at infinity

2. the density of lines is greater where the $\vec{E}$-field is stronger:

example: the $\vec{E}$-field of a point source charge $q_1 > 0$:

$q_1 > 0$

How do I draw the electric field lines, using conventions 1&2?

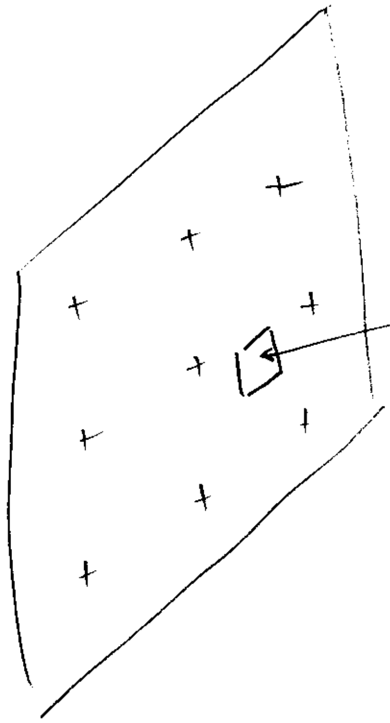
Recall that $\vec{E}$ is parallel (or antiparallel) to $\vec{F}$.

Hint: sketch $\vec{E}$ by mentally placing a positive test-charge $q_0 > 1$ all over, and use the fact that $\vec{E} = \vec{F}/q_0$

Is $|\vec{E}|$ proportional to the density of field lines?

How does this picture change if $q_1 < 0$?

Draw the electric field that is due to a very large sheet of charge having a uniform $\sigma =$ charge / area > 0



small box has area ΔA, so charge here is $= \sigma \cdot \Delta A$

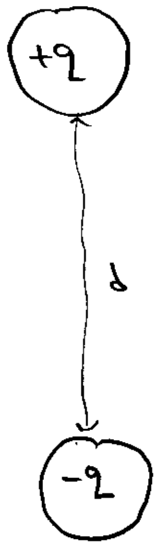
Again, place test charge $q_0 > 0$ everywhere, and note that $\vec{E} = \vec{F}/q_0$

Do you expect the field lines to have any components parallel to the sheet, or will they be perpendicular to the sheet?

What if $\sigma < 0$?

Draw $\vec{E}$ for a

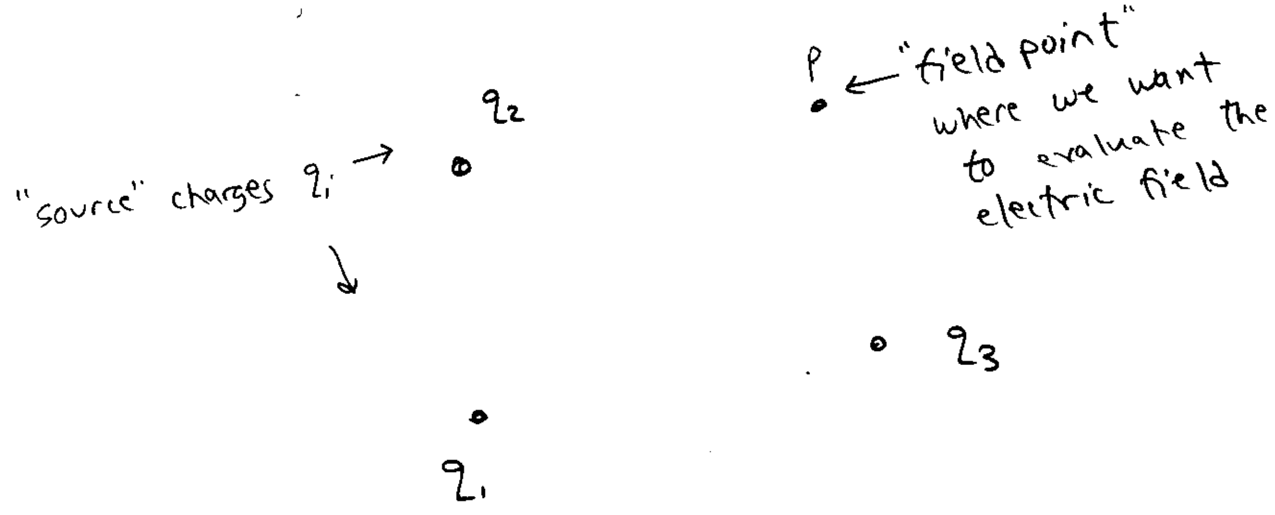
dipole = a pair of charges $\pm q$ separated by a distance d:



If you were to place a positive charge in this sketch, the field lines would indicate the direction of force on the charge.

More on dipoles later...

The electric fields due to several charges add vectorially:

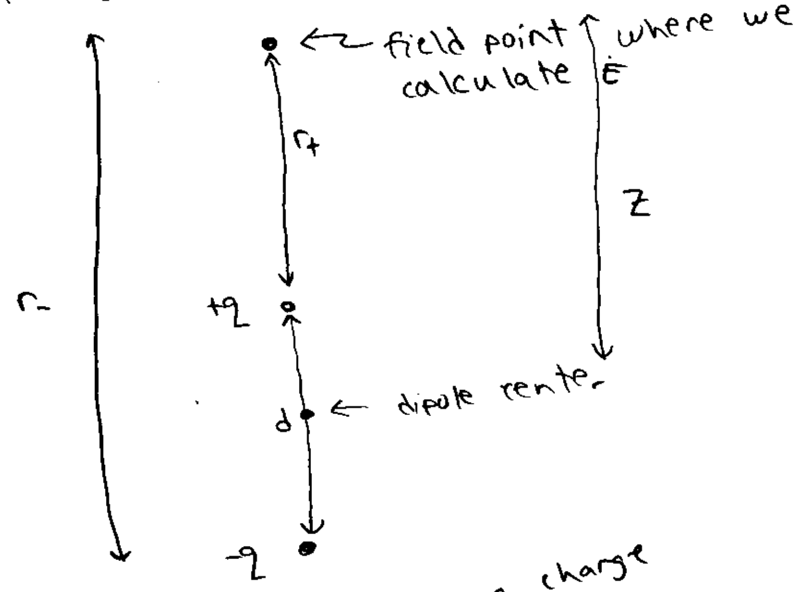


if $\vec{E}_i$ = electric field exerted by charge q_i,

then $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$ = total field exerted by all 3 charges.

Electric Field Due to a Dipole

Let's calculate the electric field along the dipole axis.



the total field is $E = E_+ + E_-$ (← due to $+q$ charge; ← due to $-q$)

It will be convenient to write $E = E(z)$

$$= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

Usually we are interested in the field at spots far from the dipole, ie, where $|z| \gg d$,

so $\left|\frac{d}{2z}\right| \ll 1$

We can obtain a simpler expression for E if we apply the binomial theorem of Appendix E:

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2 \cdot 1}x^2 + \cdots$$

for $n=-2$,

$$(1+x)^{-2} \simeq 1 - 2x + \underbrace{3x^2 + \cdots}_{\text{can ignore small terms when } |x| \ll 1}$$

So $\left(1+\frac{d}{2z}\right)^{-2} \simeq 1 - \frac{d}{z}$

$\left(1-\frac{d}{2z}\right) \simeq 1 + \frac{d}{z}$

and $E \simeq \frac{q}{4\pi\varepsilon_0 z^2}\left[1+\frac{d}{z} - \left(1-\frac{d}{z}\right)\right]$

or $E \simeq \frac{qd}{2\pi\varepsilon_0 z^3}$ = magnitude of the electric field along the dipole axis

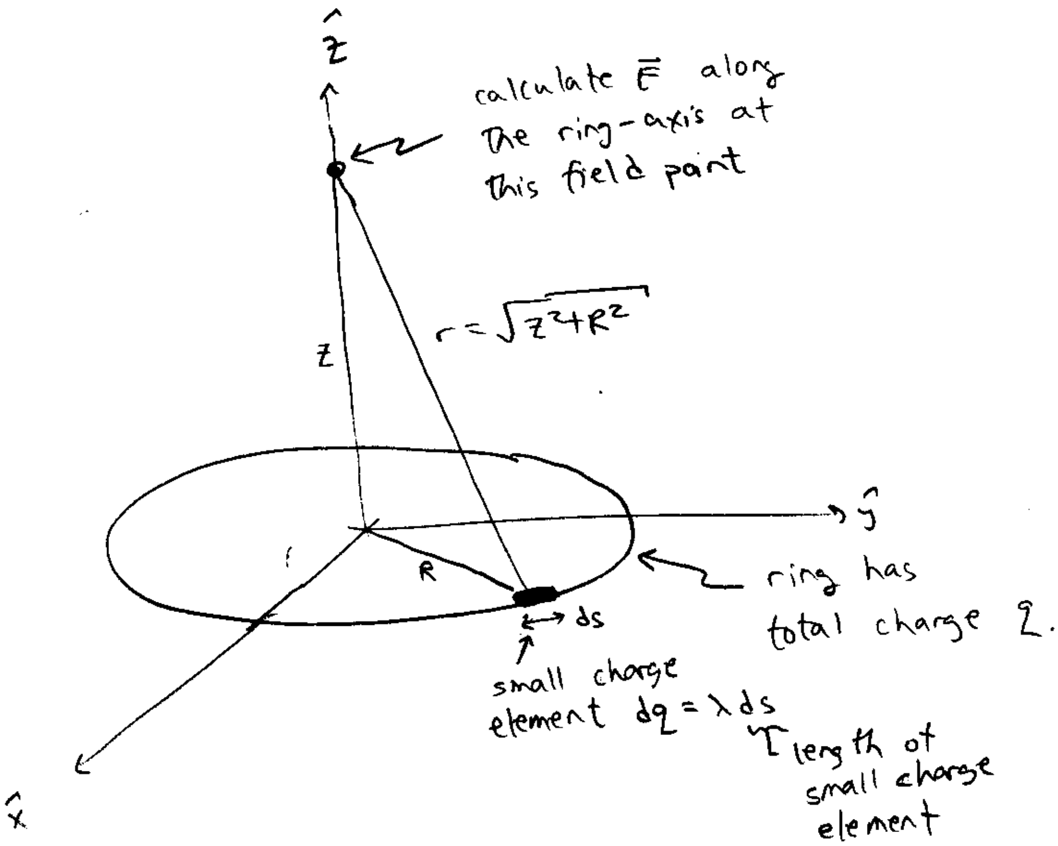
we often identify $p \equiv qd$ = electric dipole moment, which tells you the 'strength' of the dipole.

Note that the field strength of a dipole falls off as $\frac{1}{z^3}$, which is faster than the $1/r^2$ dependance of a 'bare charge'.

The $1/z^3$ dependance is due to the partial but incomple cancellation of the electric fields exerted by the $\pm$ charges in the dipole.

What happens if the dipole separation $d \to 0$?

Now lets calculate the $\vec{E}$-field due to a uniform ring of charge



How do you calculate E ?

Hint: Think about Assign #10 / PHY210 final. Problem 5 asks you to calculate the gravitational field of a ring of mass m...

Let λ = ring's linear charge density = ?

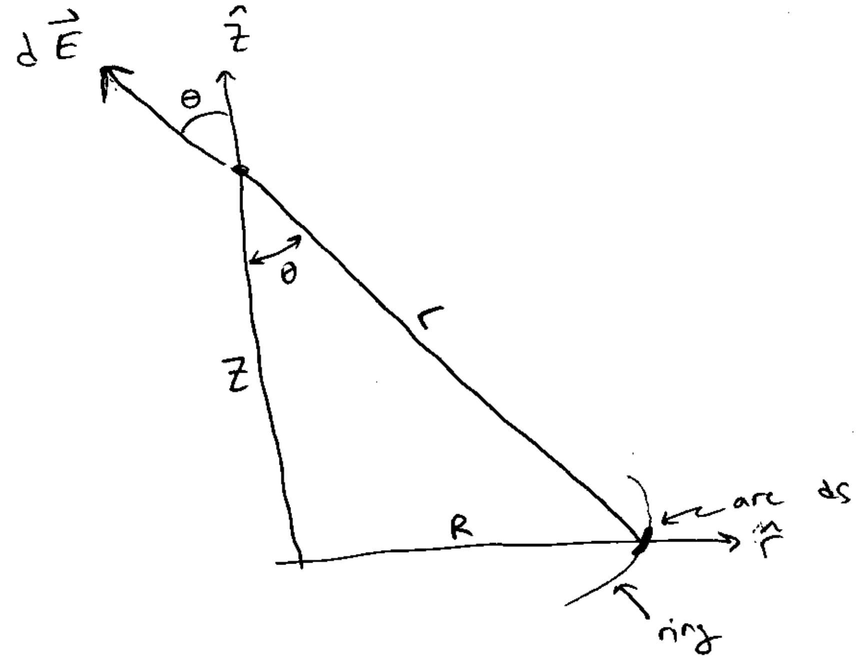
$= \frac{q}{2\pi R}$ = charge-per length

so $dq = \lambda\, ds$ = charge carried by small arc of length ds

so $dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda\, ds}{4\pi\varepsilon_0 (R^2+z^2)}$

= magnitude of the electric field contributed by small arc ds

Which way is the field $d\vec{E}$ pointing?



Break $d\vec{E}$ up into horizontal & vertical components.

$$d\vec{E} = dE_r \hat{r} + dE_z \hat{z}$$

$$= dE \sin\theta \, \hat{r} + dE \cos\theta \, \hat{z}$$

Note that the total $\vec{E}$-field is

$$\vec{E} = \underbrace{\int}_{\text{integrate about ring's circumference}} d\vec{E}$$ = sum of all the dE's contributed by all the ring arcs ds in the ring

What do you think the horizontal components of $\vec{E}$ sums to – will $\vec{E}$ have a component that points in the $\hat{x}$-$\hat{y}$ plane?

$\Rightarrow \quad d\vec{E} = dE \cos\theta \, \hat{z}$

so $$\vec{E} = \int d\vec{E} = \int_0^{2\pi R} \frac{\lambda \cos\theta \, ds}{4\pi\varepsilon_0 (R^2+z^2)}$$

$$= \frac{\lambda \cos\theta}{4\pi\varepsilon_0 (R^2+z^2)} \underbrace{\int_0^{2\pi R} ds}_{\text{ring's circumference } = 2\pi R}$$

Note that $q = \lambda \cdot 2\pi R$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

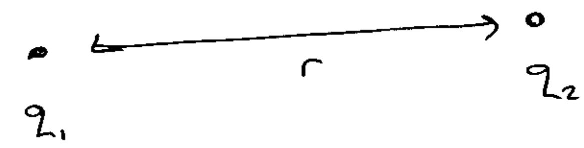
so $$\vec{E} = \frac{qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}\hat{z}$$ = E-field along the ring-axis

What if the field point is far from the ring, say, $|z| \gg R$:

$$\vec{E} \simeq \frac{q}{4\pi\varepsilon_0 z^2}\hat{z}$$

which is just the E-field due to a point-charge q.

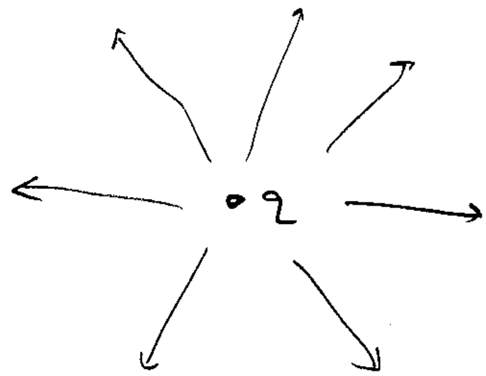
Summary of Electrostatic Force



$$F_{12} = \frac{|q_1 q_2|}{4\pi \varepsilon_0 r^2}$$ = magnitude of force on q_1 due to q_2

ε_0 = permittivity constant

Direction of $\vec{F}_{12}$ is determined by noting that 'opposites attract'.



Is $q > 0$?
or $q < 0$?

If $\vec{F}$ = electrostatic force
$= q\vec{E}$

where $\vec{E}$ = electric field

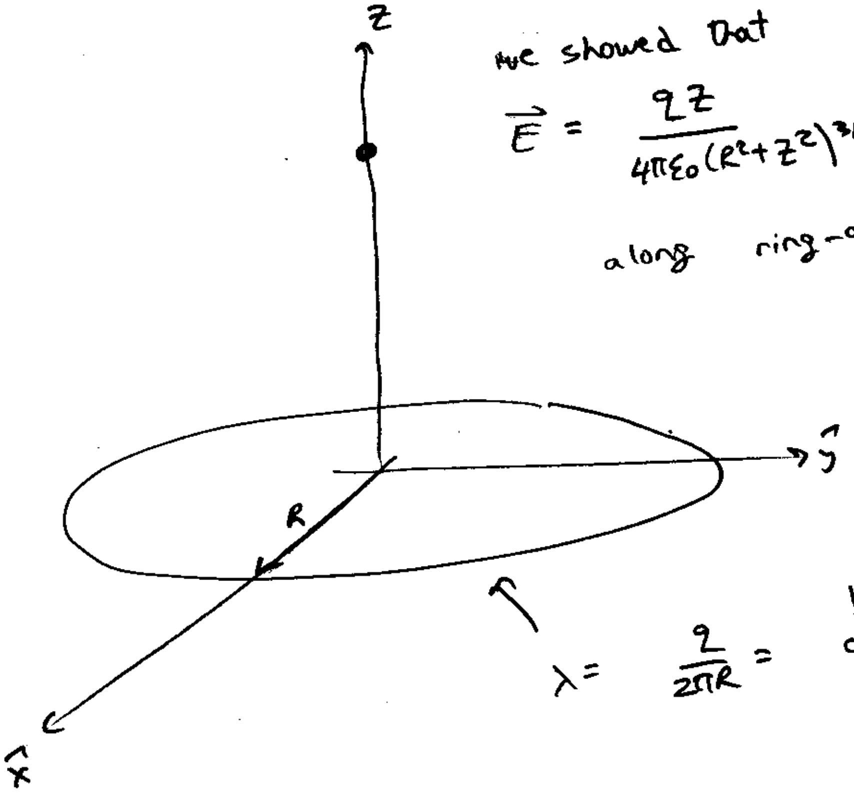
For a bare charge q, $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$\vec{E}$-field due to a ring of charge

we showed that

$$\vec{E} = \frac{qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} \hat{z}$$

along ring-axis

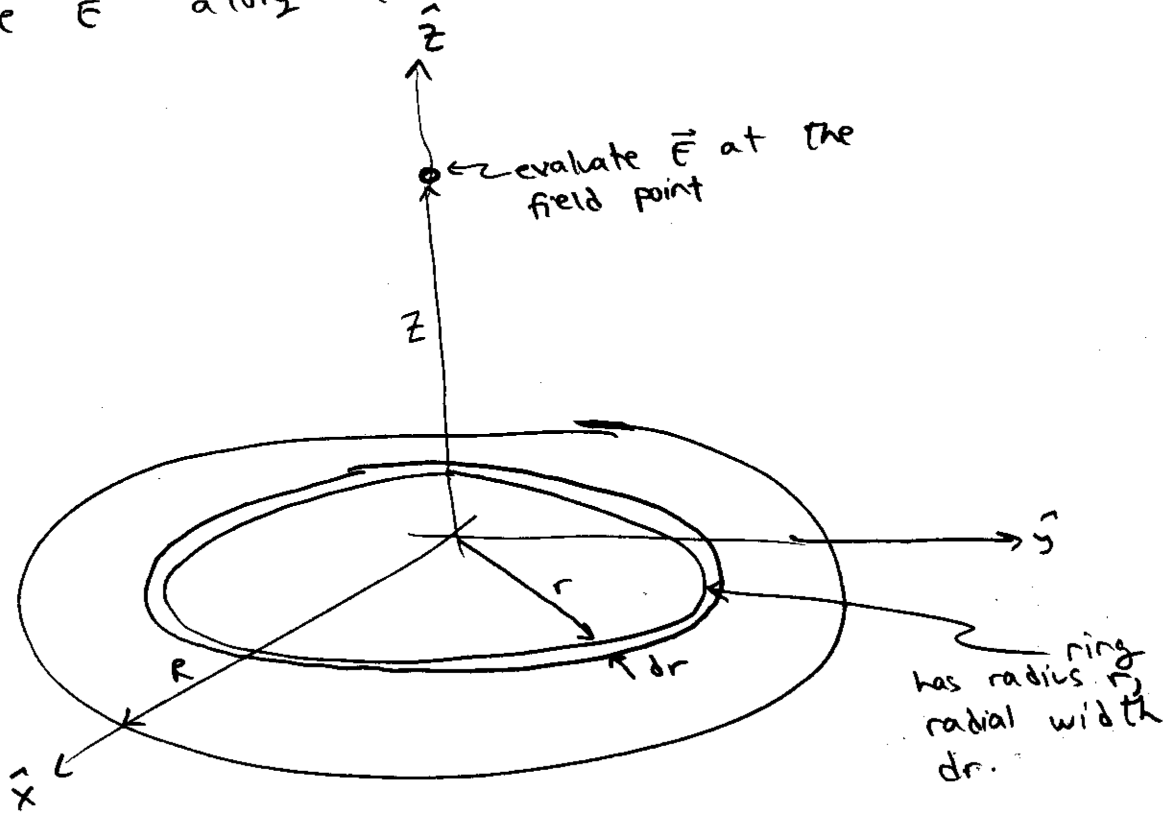


$\lambda = \frac{q}{2\pi R}$ = linear charge density on ring

E-field due to a charged Disk

Disk has radius R and a uniform surface charge density $\sigma = \frac{2q}{\pi R^2} = \frac{\text{charge}}{\text{area}}$ of disk.

Calculate $\vec{E}$ along the disk's axis



How do I use our previous results for a ring to calculate $\vec{E}$ due to a disk?

Treat the disk as a set of concentric rings

Let narrow ring have radius r & width dr.
What is the charge dq on this ring?

$dq = \sigma \cdot dA$ where $dA =$ area of ring

$= 2\pi\sigma r\,dr$ $= \underbrace{2\pi r}_{\text{circumf.}} \cdot dr$ ← width

What is the field $d\vec{E}$ due to ring of charge dq?

$$d\vec{E} = \frac{z\,dq}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}\,\hat{z}$$

$$= \frac{z \cdot 2\pi\sigma r\,dr}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}\,\hat{z}$$

The total field is

$$\vec{E} = \int d\vec{E} = \frac{2\pi\sigma z}{4\pi\varepsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}\,\hat{z}$$

↑ integrate over all rings having radii $0 \le r \le R$

Do u-substitution to solve the integral:

$u = r^2 + z^2$

$du = 2r\,dr$

$$\text{so} \quad \vec{E} = \frac{\sigma z}{2\varepsilon_0} \int_{\underbrace{z^2}_{u \text{ when } r=0}}^{\overbrace{R^2+z^2}^{u \text{ when } r=R}} \left(\frac{1}{2} u^{-3/2}\, du\right)\hat{z}$$

$$= \frac{\sigma z}{4\varepsilon_0} (-2)\, u^{-1/2} \Big|_{z^2}^{R^2+z^2} \hat{z}$$

$$= -\frac{\sigma z}{2\varepsilon_0} \left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{|z|} \right] \hat{z}$$

Note that $\frac{z}{|z|} = \pm 1$

$$= \frac{\sigma}{2\varepsilon_0} \left(\pm 1 - \frac{z}{\sqrt{R^2+z^2}} \right) \hat{z}$$

= electric field along axis of uniformly charged disk what if the disk has an infinite radius, ie, it is a very large sheet of charge?

in this case, $R \gg |z|$ so

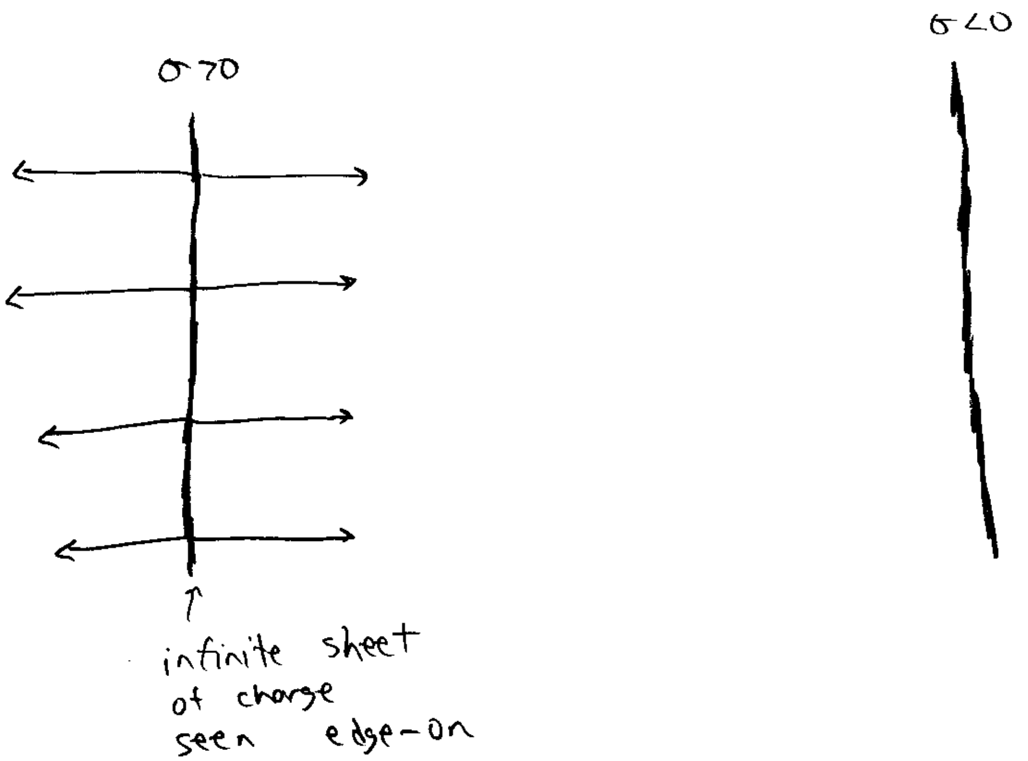
$$\vec{E} \simeq \pm \frac{\sigma}{2\varepsilon_0} \hat{z}$$

use − sign if $z<0$

see ch. 24 for a much easier solution to this problem...

Note that the E-field of an infinite sheet is constant everywhere, independant of z

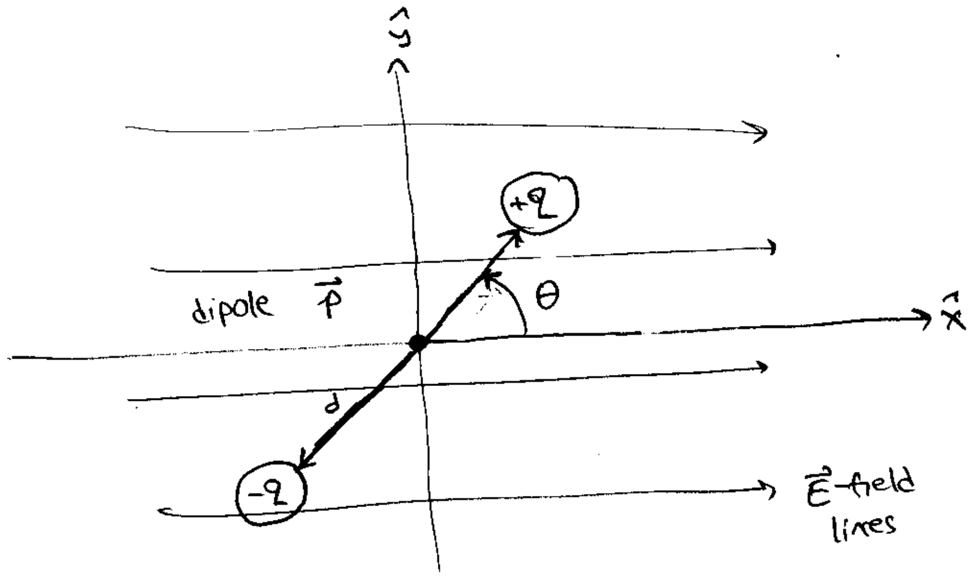
sketch the field lines for infinite sheet having $\sigma > 0$ and $\sigma < 0$:



This of course is the sketch we obtained earlier

Dipole in an Electric Field

Lets consider what happens when we place a dipole in a uniform $\vec{E}$-field



$\vec{E} = E\,\hat{x}$

$\hat{z}$ points up, out the page.

Recall that the dipole is composed of 2 charges $\pm q$, and has length d, magnitude $|\vec{p}| = qd$, and a direction $\vec{p}$.

↑ dipole moment

What is the net force that the $\vec{E}$-field exerts on the dipole?

what does that tell you about the dipole center?

Are there forces at the dipole ends?

what will those forces do to the dipole?

The $\vec{E}$-forces will twist, or torque the dipole:

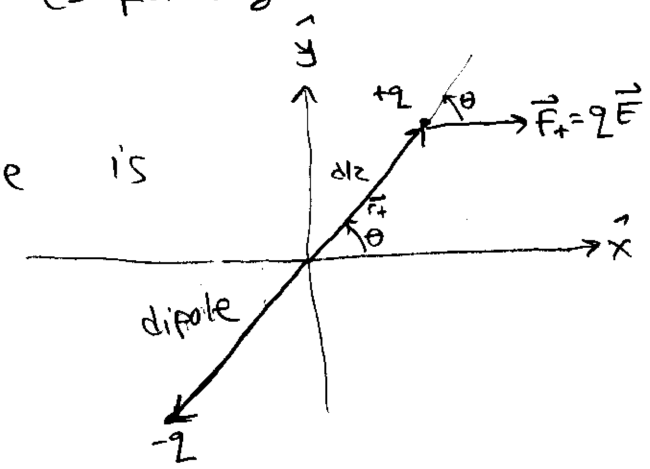
$\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_-$ = total torque on individual charges

$= \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$

position & force on + charge

torque on $-q$

Which direction is $\vec{\tau}_+$ and $\vec{\tau}_-$ pointing



The magnitude of the torque is

$$|\vec{\tau}_+| = \left(\frac{d}{2}\right) qE\sin\theta$$

so $\vec{\tau}_+ = -\frac{d}{2} qE\sin\theta\ \hat{z}$

likewise, $\vec{\tau}_- = -\frac{d}{2} qE\sin\theta\ \hat{z}$

so $\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_- = -qdE\sin\theta\ \hat{z}$

Suppose this dipole is initially stationary.

what happens when it is released?