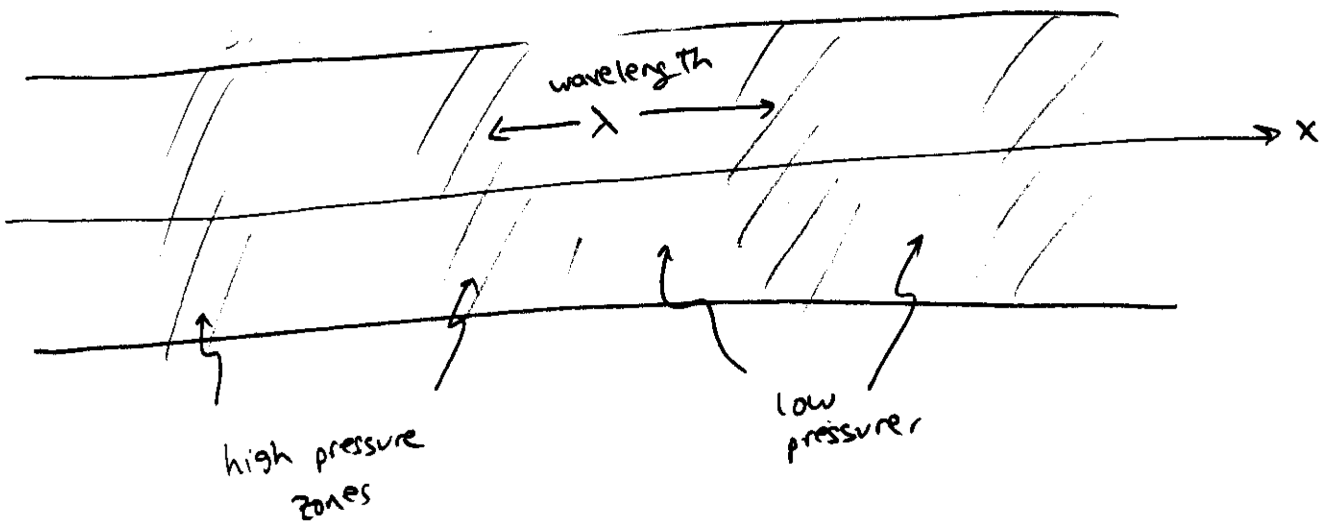
Chapter 18: Waves II

lecture covers sections 1-8

sound wave - longitudinal wave of alternating regions of high/low pressure and/or density

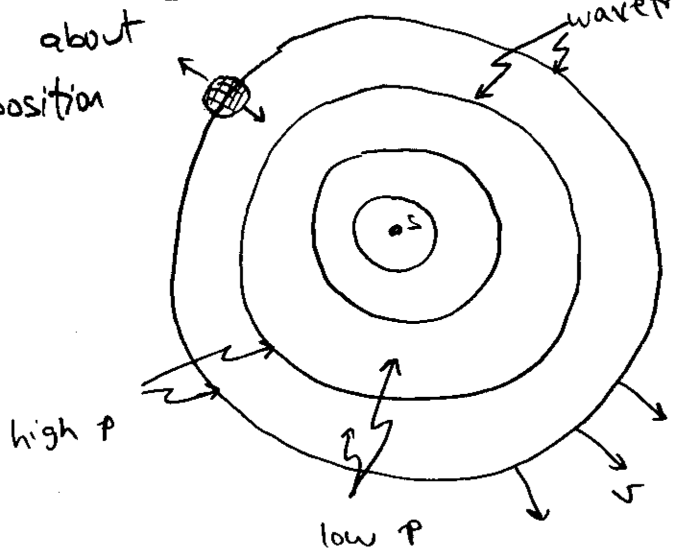
1D sound wave in an air-filled cylinder:



But in general, sound waves are 3D:

S = source of soundwave

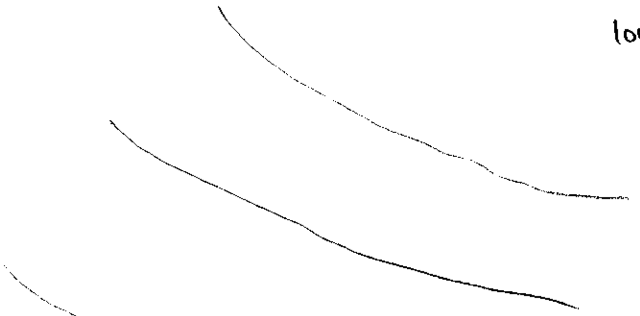
Note that an individual air-parcel merely oscillated about a fixed position



wavefronts = <u>spherical</u> surfaces where air-elements have the <u>same</u> <u>phase</u>, ie, are acting 'in-sync'.

If these wavefronts represent the high-pressure zones, then the regions between are the low-pressure zones

However the <u>wavefronts</u> <u>expand radially</u> at the wave speed v



Note that when <u>far from the source</u>, the wavefronts become parallel

Speed of sound:

Note that Sections 18-2,3 employ bulk modulus B, which you have not studied...

The following is an alternate, qualitative derivation of the sound speed. A rigorous derivation is given in PHY 417, Thermal Physics.

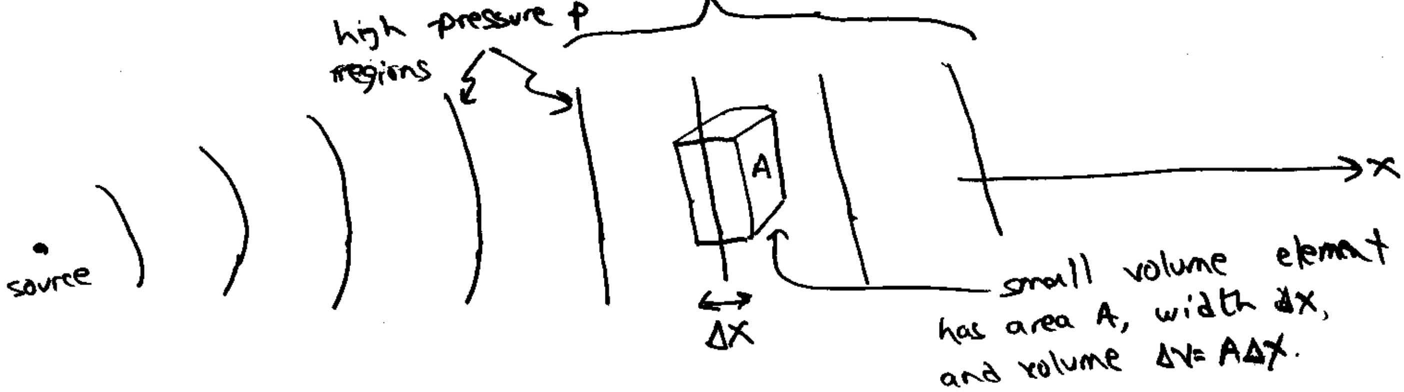
Recall that the speed of waves on a string is

$$v = \sqrt{\frac{T}{\mu}} = \text{speed of wave on string}$$

where T = tension (force) in string
μ = string's mass/length (string's inertia)

Consider sound wavefronts:

parallel wavefronts far from source



The force on the small volume element is

$$F = \underset{\text{area of volume}}{\overset{\text{pressure in volume}}{p A}} \sim T$$

while $\mu = \frac{\text{mass}}{\text{length}}$ of volume element

$$= \frac{\text{density} \times \text{volume}}{\text{length}} = \frac{\rho A \Delta x}{\Delta x}$$

$$= \rho A$$

$\Rightarrow$ we anticipate that sound wave will propagate at the speed

$$v = \sqrt{\frac{T}{\mu}} \sim \sqrt{\frac{p}{\rho}}$$ where p = air pressure

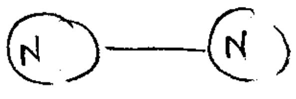
ρ = air density

(approximately)

However the <u>exact</u> calculation is not done until PHY 417, Thermal Physics:

$$v = \sqrt{\frac{\gamma p}{\rho}} \qquad \text{(exact)}$$

where $\gamma = \frac{7}{5}$ diatomic gases (N_2, O_2)



Average atmospheric pressure and density at sea level:

$p = 1.01 \times 10^5 \ N/m^2$ (1 atm = "1 atmosphere")

$\rho = 1.29 \ kg/m^3$ at STP = standard temp' and pressure

$$\Rightarrow \quad v = \sqrt{\frac{\left(\frac{7}{5}\right)\left(1.01 \times 10^5 \ N/m^2\right)}{1.29 \ kg/m^3}}$$

$1N = 1 kg \cdot m/sec^2$

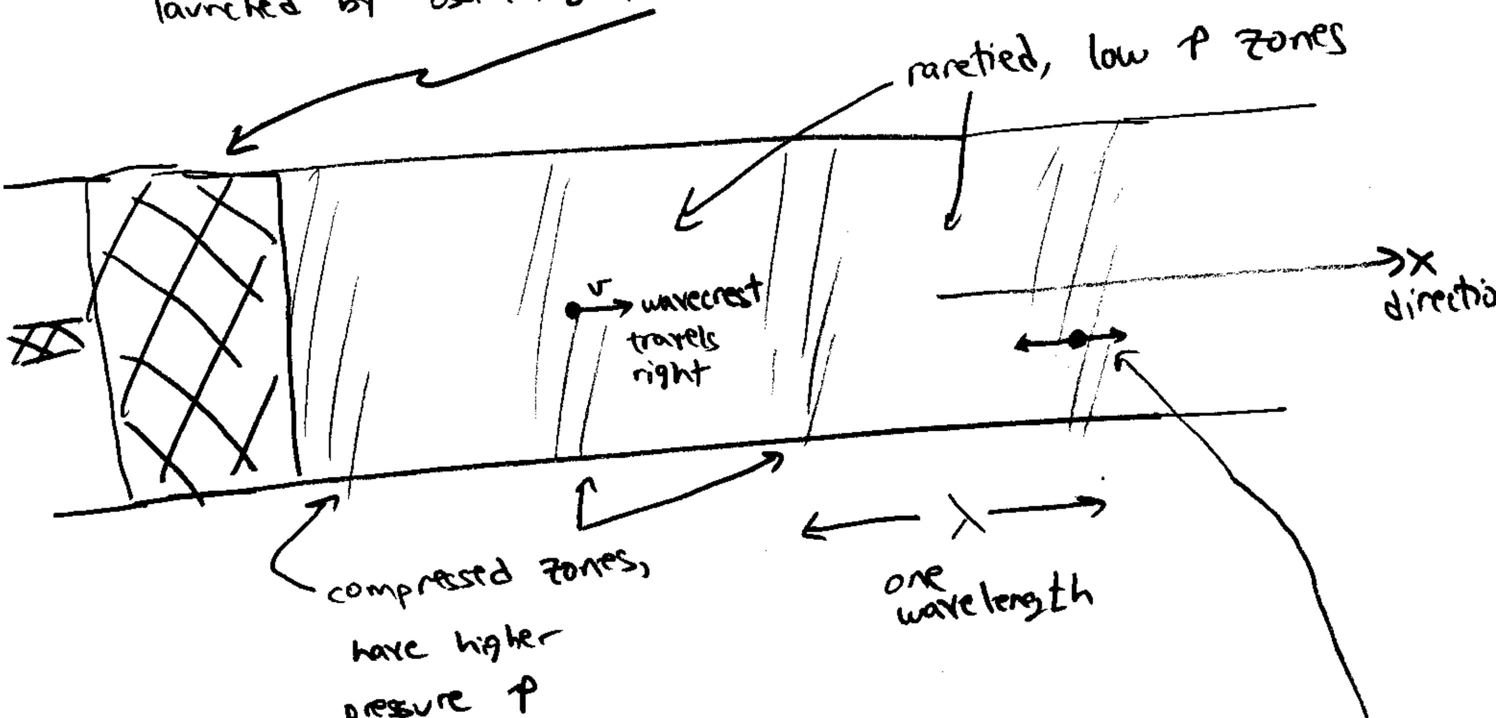
$$= \sqrt{1.10 \times 10^5 \ (m/sec)^2}$$

speed of sound $v = 331$ m/sec in air at Temp = $0°C$

(343 m/sec when $T = 20°C$)

Traveling Sound Waves

consider sound waves traveling down an air-filled pipe, launched by oscillating piston



The oscillating piston displaces the air, so that each air parcel moves the small distance

$$s(x,t) = s_m \cos(kx - \omega t)$$

The air parcel oscillates left-right as the waves travel to right

Note that we use $s \propto \cos(kx-\omega t)$ for convenience; I could also have written $s \propto \sin()$ instead, as in Chapter 17

<u>Intensity</u> ≐ power per area transmitted by a sound wave:

$$I = \frac{\bar{P}}{A}$$

$\bar{P}$ ← power (energy/time)

A ← area

~ a measure of how loud/quiet a sound is

Recall that waves on a string transmit energy at the rate

$$\bar{P} = \frac{1}{2}\mu v \omega^2 y_m^2 \qquad \text{(Eqn 17-32)}$$

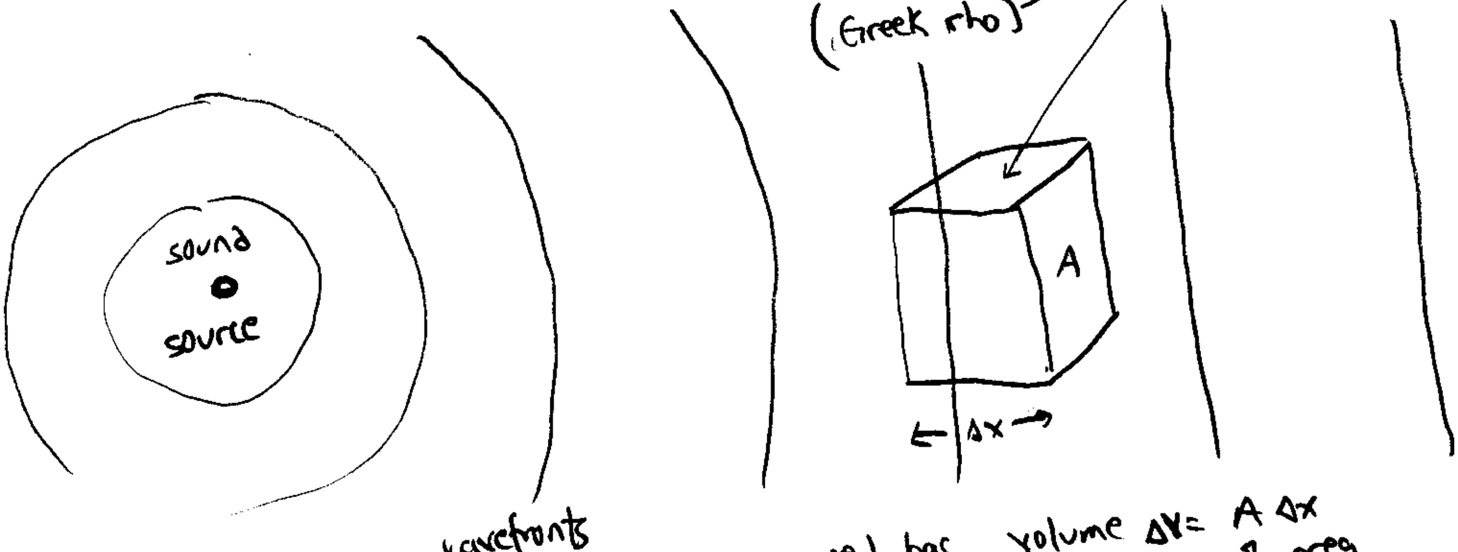
v ← wave speed

ω^2 ← angular frequency²

μ ← string's mass/length

y_m^2 ← string's vertical amplitude²

Apply this result to soundwaves. Consider this parcel of air having <u>density</u> ρ (Greek rho)



parcel has volume $\Delta V = A\,\Delta x$ ← area

and mass $\Delta m = \rho\,\Delta V = \rho A \Delta x$

So the parcel's equivalent

$$\mu = \frac{\text{mass}}{\text{length}} = \frac{\rho A \Delta x}{\Delta x} = \rho A$$

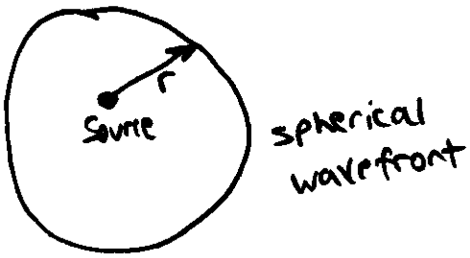
likewise, displacement $y_m \rightarrow s_m$ for sound

so $\bar{P} = \frac{1}{2}(\rho A) v \omega^2 s_m^2$

and the intensity of soundwaves is

$$I = \frac{\bar{P}}{A} = \frac{1}{2} \rho v \omega^2 s_m^2$$

Note also if $A =$ area of entire spherical wavefront:
$= 4\pi r^2$



Then $I = \dfrac{\bar{P}}{4\pi r^2} = \frac{1}{2} \rho v \omega^2 s_m^2$

$\Rightarrow s_m \propto \frac{1}{r}$ ← the amplitude of the air's displacement, s_m, falls off as $1/r$, while the sound intensity I falls off as $1/r^2$

Human ear can sense sounds having displacements of (see Sample Problem 18-2)

$$\overbrace{10^{-11}\,m \lesssim s_m \lesssim 10^{-5}\,m}^{\text{dynamic range of } 10^6}$$

displacement of faintest detectable sound, $\sim\frac{1}{10}$ size of atom

displacement of loudest tolerable sound is $\sim\frac{1}{10}$ thickness of paper

let I_0 = intensity of faintest sound detectable

$= \frac{1}{2}\rho v w^2 s_m^2$

$\rho = 1.21\ kg/m^2$ = density of air

$v = 343\ m/sec$ = sound speed

$f = 1100\ Hz$ = sound frequency (a relatively low tone)

$w = 2\pi f = 7000\ rad/sec$ = angular frequency

$s_m = 10^{-11}\,m$

$\Rightarrow I_0 = 10^{-12}$ watts/m^2

UNITS = Power (in watts, or J/sec) per area

The Decibel Scale

Since people hear sounds having displacements s_m that vary over 6 orders of magnitude (ie, by factors of 10^0 to 10^6), then $I \propto s_m^2$ varies over 12 orders of magnitude (ie, by factors of 10^0 to 10^{12})

It is convenient to use a logarithmic scale to handle the human ear's large dynamic range:

$$\beta = \underbrace{10 \log}_{\text{log base 10, NOT ln base e}} \left(\frac{I}{I_0} \right) \; dB$$

unit = decibels, or dB

deci = power of 10

bels = Alexander Graham Bell, for his studies of human hearing

For $I = I_0$ (faintest hearable sound)

$\log (I/I_0) = \log (1) = 0$

$\Rightarrow \beta = 0 \; dB$ = intensity of faintest audible sound, in decibels.

Note that $\beta = 0$ corresponds to sound having a nonzero intensity.

The loudest tolerable sound has an intensity 10^{12} times larger:

$$I = 10^{12} I_0$$

so

$$\beta = 10 \log (I/I_0) \text{ dB}$$
$$= 10 \log(10^{12}) \text{ dB}$$
$$= 10 \times 12 \text{ dB}$$
$$= 120 \text{ dB} \quad = \quad \text{pain threshold}$$

Table 18-2:

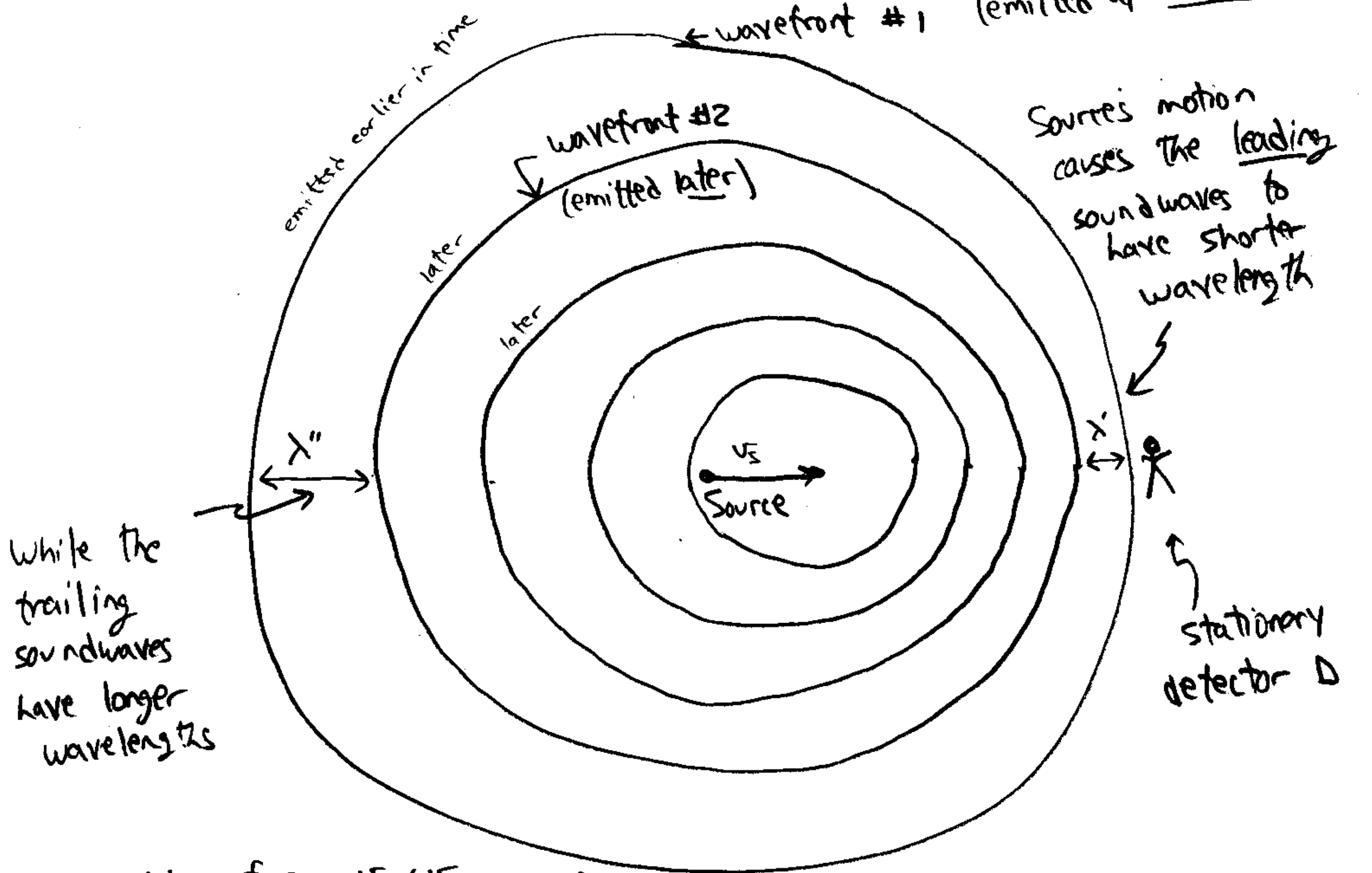
sound	intensity (dB)
hearing threshold	0
rustling leaves	10
conversation	60
rock concert	110
pain threshold	120
jet engine	130

Please read sections 18-6 (Musical Sounds) &
18-7 (Beats)

Dopple Effect = change in a sound's frequency (or pitch) due to relative motion

A source of sound waves, S, moves right with velocity v_s, while a stationary detector D listens.

The source's motion causes its spherical wavefronts to 'bunch-up'



picture for $v_s < v$

what happens if $v = v_s$?. OR $v > v_s$? (supersonic motion). See section 18-9

Recall soundspeed $v = \lambda' f'$ (Eqn 17-12)

wavelength at detector D

sound frequency & wavelength at detector D

Let T = time interval between emission of wavefronts #1 and #2 = sound wave period
$= 1/f$: where f = sound frequency

So $v_s T$ = distance traveled by source S in that time

source speed

While vT = distance traveled by wavefront #1 in that time

$\Rightarrow$ separation between wavefronts #1 & 2 near detector D is $= vT - v_s T$

$$= (v - v_s)T$$
$$= (v - v_s)/f$$
$$= \lambda'$$

so $f' = \dfrac{v}{\lambda'} = f\left(\dfrac{v}{v - v_s}\right)$

$\Rightarrow$ The detector hears a higher-frequency sound, ie, $f' > f$, due to the source's motion.

How would you revise this result if source S moves away from D?

$$v_s \to -v_s$$

$$f' \to f\left(\frac{v}{v+v_s}\right)$$

Does D hear a higher or lower frequency sound?

More generally,

if the detector is also moving with velocity v_D,

$$f' = f\left(\frac{v \pm v_D}{v \pm v_s}\right)$$

Choose signs so that $f' > f$ when S and D approach each other, and that $f' < f$ when S and D move away.

where v_s, v_D = source, detector's speed relative to the stationary air

v = sound speed

f = frequency emitted by source.

e.g. if S & D are approaching each other,

$$f' = f\left(\frac{v+v_D}{v-v_s}\right)$$