Chapter 17: Waves - I

This lecture will cover sections 1-9 & 11-12

Wave = a disturbance that transmits energy, and/or momentum, information, etc.

mechanical waves: sound waves, seismic waves, water waves } will be studied here

electromagnetic waves: light, TV & radio signals radar, ... } studied PHY 355, E&M

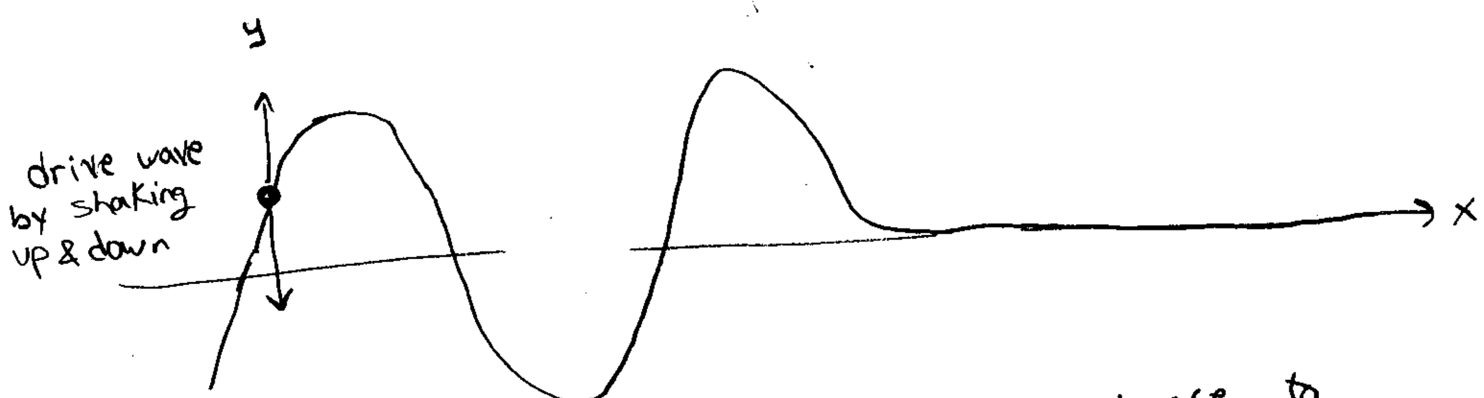
waves propogate as one particle pushes or pulls its neighbor, which in turn pushes/pulls its neighbor, etc.

All particles oscillate about some mean position, so there is no net displacement in the matter that is transmitting the wave.

Two <u>waveforms</u> (or shapes):

<u>transverse wave</u>: The matter transmitting the wave moves <u>perpendicular</u> to the direction of wave propagation.

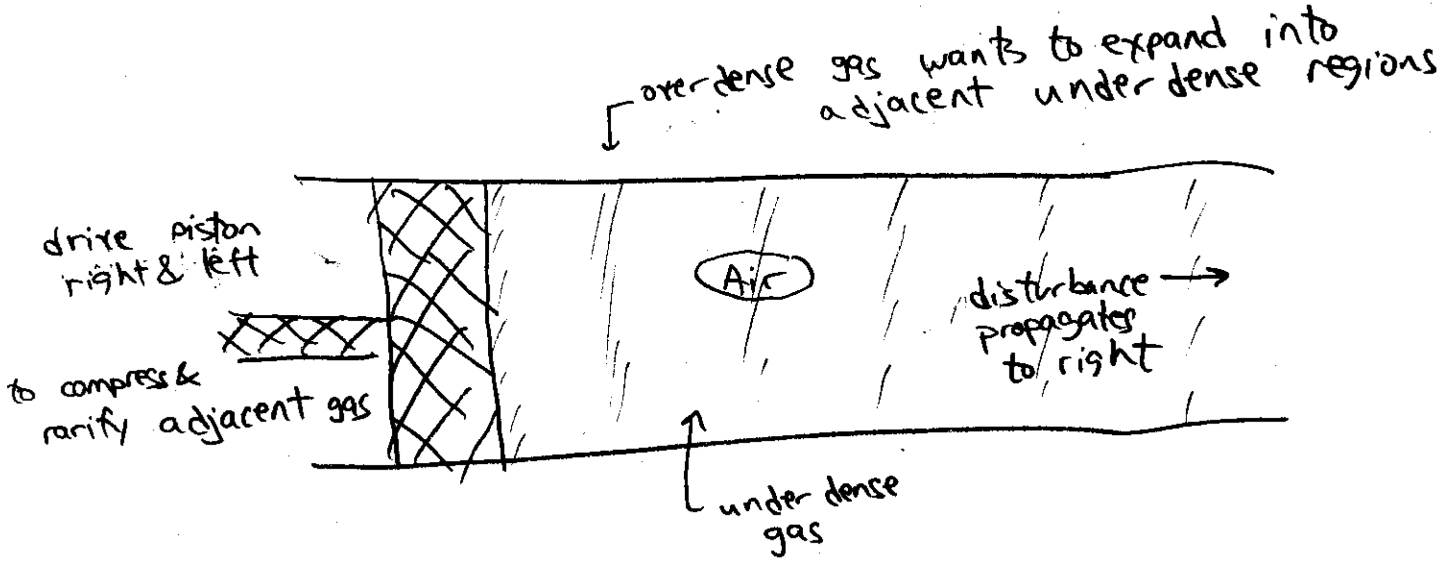
Example: waves on a string



This raises/lowers adjacent string elements, causing the disturbance to propagate to the right.

longitudinal wave: The displacement is parallel to the wave propagation.

Example: sound wave



Both waveforms are known as traveling waves, since their wavecrests move about.

See 'transverse & longitudinal waves' video

Wavelength & Wave Speed

A particle's displacement due to a wave can be written

wavenumber, angular frequency

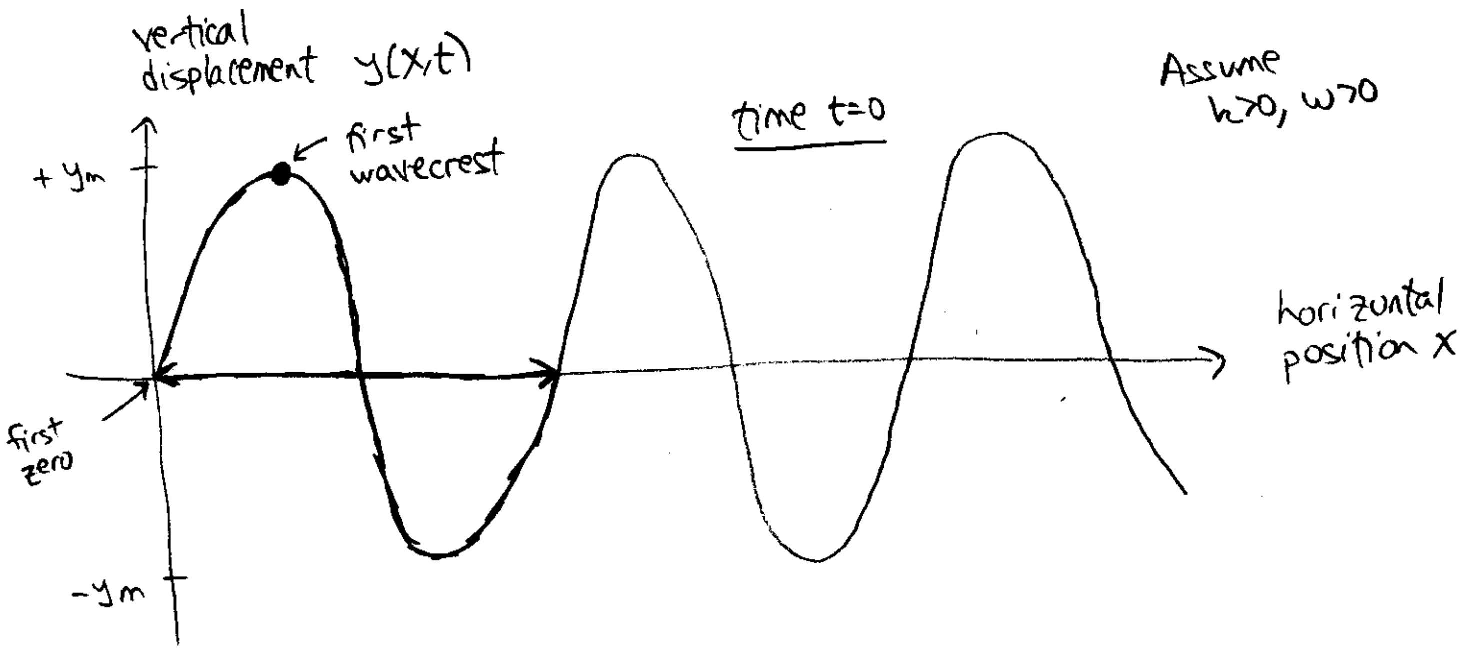
displacement $y(x,t) = y_m \sin(kx - \omega t)$

amplitude, position, time

also, The phase $= kx - \omega t$

k, ω are constants

Plot a wave on a string at time $t=0$:



wavelength λ = distance beyond which the waveform repeats

= distance over which phase increments by 2π

$$\Rightarrow \quad \text{phase} = k(x+\lambda) - \omega t = \underbrace{kx - \omega t}_{\text{starting phase}} + 2\pi$$

$$\Rightarrow \quad k\lambda = 2\pi$$

$$\text{so} \quad \lambda = \frac{2\pi}{k} = \text{wave length}$$

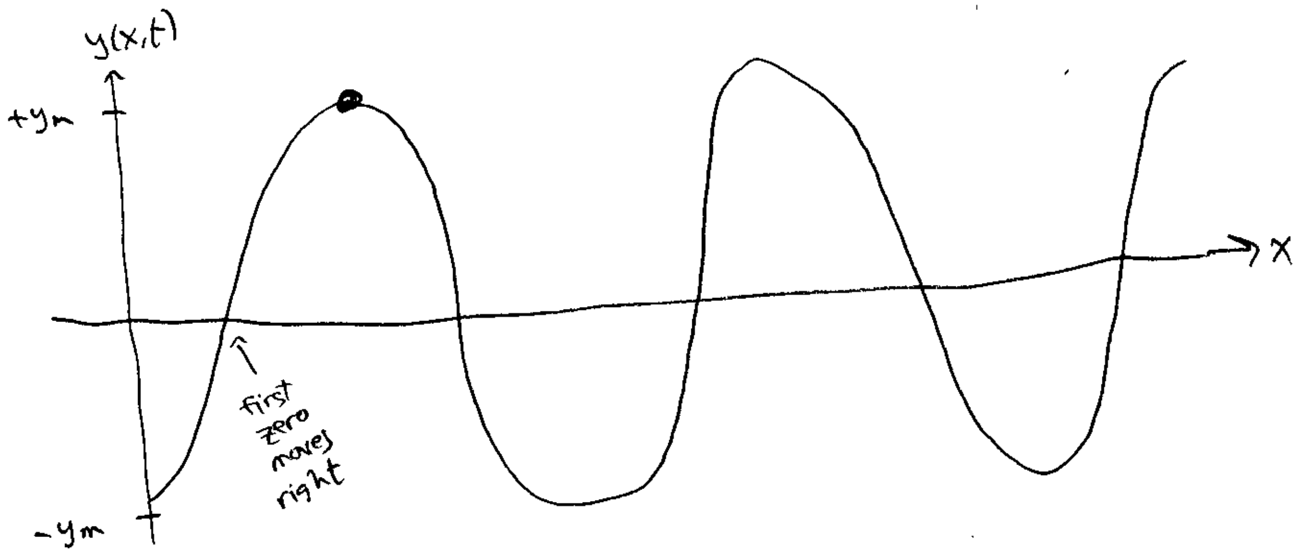
To calculate the wave speed, place a dot on The first wavecrest

What is The phase of The wave at This spot?

$$\text{phase:} \quad kx - \omega t = \frac{\pi}{2}$$

Note That The dot lies at $x = \frac{\pi}{2k}$ when $t=0$

Plot wave at later time $t>0$:



The dot's phase $= kx - \omega t = \frac{\pi}{2}$

$\Rightarrow$ The dot has advanced to $x = \frac{\pi}{2k} + \frac{\omega t}{k}$

In what direction is the wave propagating?

How do you calculate the wave's speed?

$$v = \frac{dx}{dt} = \frac{\omega}{k}$$

- wave propagates to right if ω & k have same sign,
- and left otherwise.

What kind of wave is this: transverse or longitudinal?

Period & Frequency

$$y = y_m \sin(kx - \omega t)$$

where $\omega =$ angular frequency (as for Chap 16)

$f = \frac{\omega}{2\pi}$ frequency

$T = \frac{2\pi}{\omega} = \frac{1}{f}$ = period

Note that wave speed $v = \frac{\omega}{k} = \frac{2\pi f}{k} = \lambda f$

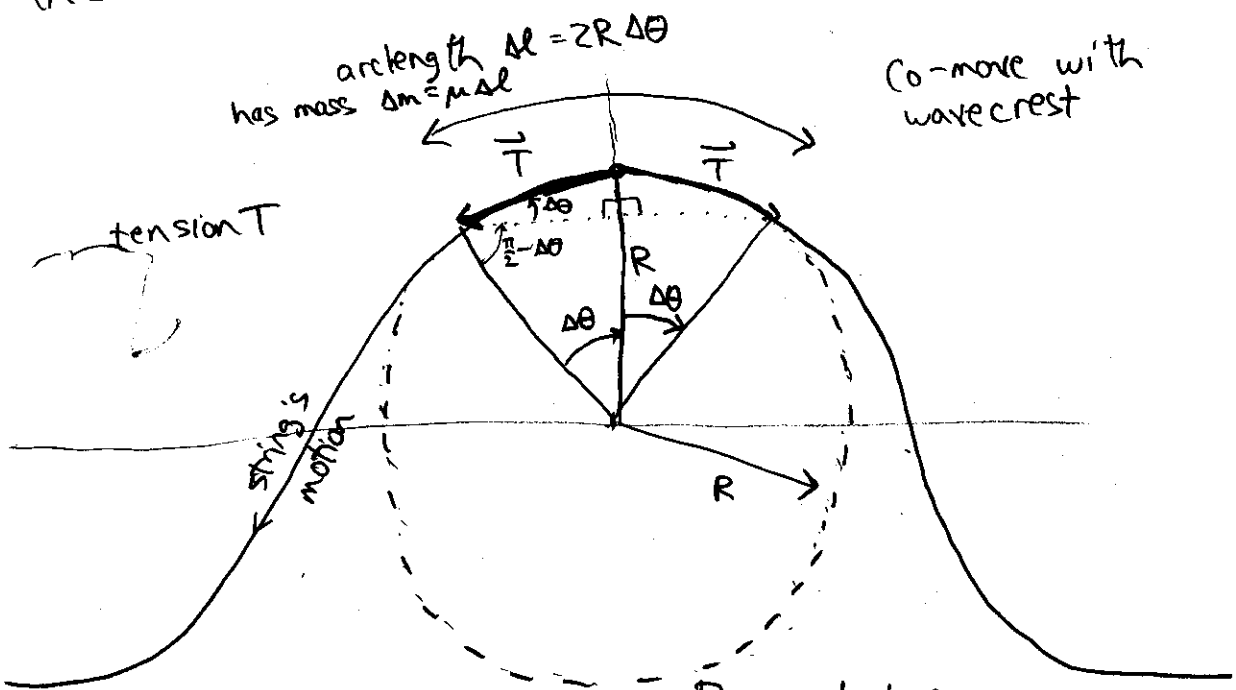
or $\lambda = \frac{v}{f} = vT$

Traveling Waves on a Stretched String

Derive the wave speed v:

Suppose a transverse wave propages to the right along a string that is pulled tight by tension T.
Let μ = string's linear density (its mass-per-length)

Keep your eye on a single wave crest. In the reference frame that co-moves with the wave, the string appears to move left:



Consider the tension in the string at the wavecrest

The horizontal components cancel.

What about the vertical components?

the vertical force at that spot is

$F = 2T \sin \Delta\theta \simeq 2T\Delta\theta$ since $\Delta\theta$ is small

$= \frac{\Delta l}{R} T$

$= \Delta m \, a$

where $a =$ centrifugal acceleration (in this reference frame, the string-element is rolling along a circle of radius R)

$= \frac{v^2}{R}$ (see Eqn 6-17)

so $F = \frac{\Delta l}{R} T = \Delta m \, a = (\mu \Delta l) \frac{v^2}{R}$

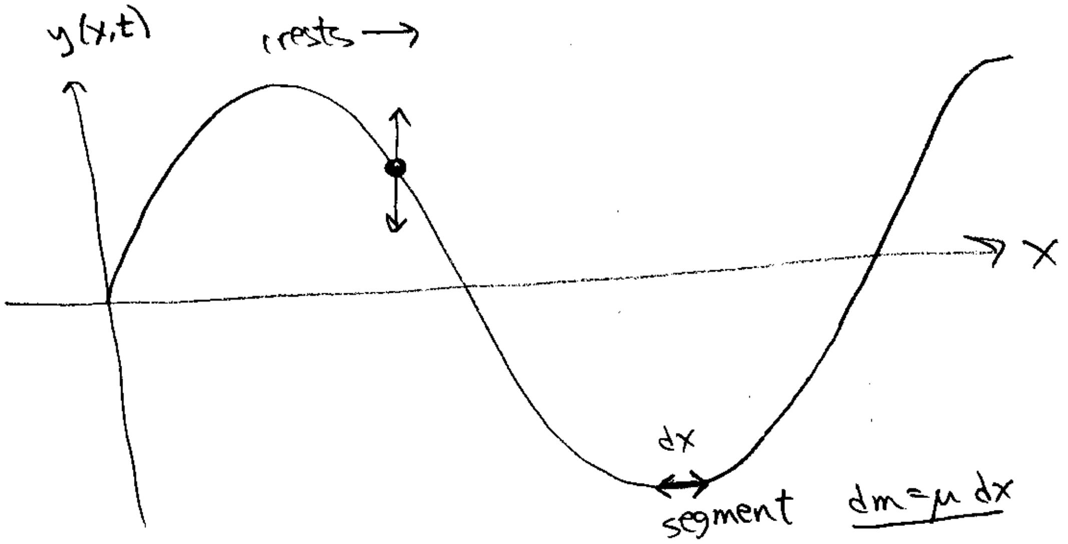
$\Rightarrow v^2 = \frac{T}{\mu}$

or $v = \sqrt{\frac{T}{\mu}}$ = speed of a wave on a string

See "wave speed" video

Energy & Power

When we launch a wave on a string, we are depositing energy within the string, which the string then transports away.



let $dK = \frac{1}{2} dm\, u^2$ = KE of small mass element dm
(Kinetic Energy)

↑ vertical velocity² of segment (not wave speed v)

recall $y(x,t) = y_m \sin(kx - \omega t)$ = vertical displacement

so $u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t)$

partial derivative, $\frac{dy(x,t)}{dt} = \left.\frac{dy}{dt}\right|_{x=\text{constant}}$

and $dK = \frac{1}{2}(\mu\, dx)\, \omega^2 y_m^2 \cos^2(kx - \omega t)$

so $\frac{dK}{dt} = \frac{1}{2}\mu \left(\frac{dx}{dt}\right) \omega^2 y_m^2 \cos^2(kx - \omega t)$

but $\dfrac{dx}{dt} = v =$ wave's horizontal speed,

so $$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \underbrace{\cos^2(kx-\omega t)}_{\text{varies between } 0-1}$$

Next - average this result over 1 wavelength λ: the average rate at which KE is transported is

$$\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 \underbrace{\frac{1}{\lambda}\int_0^\lambda \cos^2(kx-\omega t)\,dx}_{I = \frac{1}{2} \text{ when averaged over 1 wavelength..}}$$

$$= \frac{1}{4}\mu v \omega^2 y_m^2$$

= average rate at which KE is transported by the wave.

Likewise, $\left(\dfrac{dU}{dt}\right)_{avg} = \dfrac{1}{4}\mu v \omega^2 y_m =$ avg rate at which wave transports PE

(we obtained a similar result in our earlier discussion of oscillations)

$$\Rightarrow P = \left(\frac{dK}{dt}\right)_{avg} + \left(\frac{dU}{dt}\right)_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2$$

= power (energy per time) transmitted by wave

Superposition of waves (2 waves co-add)

Two distinct wavetrains travel along the same string. The string's net displacement is simply

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= \text{wave } 1 + \text{wave } 2$$

Wave Interference

Let two waves having same wavelength & amplitude travel along the same string in the same direction

wave 1 : $y_1(x,t) = y_m \sin(kx - \omega t)$

wave 2: $y_2(x,t) = y_m \sin(kx - \omega t + \phi)$

ϕ = phase constant allows for possibility of waves being out of phase

The string's net displacement = superposition of 2 waves:

$$y(x,t) = y_m[\sin\underbrace{(kx-\omega t)}_{a} + \sin\underbrace{(kx-\omega t+\phi)}_{b=a+\phi}]$$

Use trig identity in Appendix E:

$$\sin a + \sin b = 2\sin\left[\tfrac{1}{2}(a+b)\right]\cos\left[\tfrac{1}{2}(a-b)\right]$$

so $y(x,t) = y_m\, 2\sin\left\{\frac{1}{2}\left[2(kx-\omega t)+\phi\right\}\cos\left[\frac{1}{2}(-\phi)\right]\right]$

or $y(x,t) = \underbrace{2y_m \cos\left(\frac{1}{2}\phi\right)}_{\text{constant amplitude}} \sin\left(kx-\omega t+\frac{1}{2}\phi\right)$

$\Rightarrow$ two waves combine to form a new wave whose amplitude is controlled by phase constant ϕ

← when $\phi = 0$, $\cos\phi = +1$, the waves are exactly in phase, which results in fully constructive interference. The net wave amplitude is doubled.

← What happens when $\phi = \pi$? fully destructive inteference, the 2 waves cancel each other out, there is no net wave.

Intermediate values of ϕ result in intermediate interference.

Show "wave interference" clip

Standing Waves

What if two waves of identical λ (wavelength) and y_m (amplitude) travel in opposite directions?

You get standing waves:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= \underbrace{y_m \sin(\overbrace{kx - \omega t}^{a})}_{\text{propagates in } +x \text{ direction}} + \underbrace{y_m \sin(\overbrace{kx + \omega t}^{b})}_{-x \text{ direction}}$$

$$= y_m (\sin a + \sin b)$$

$$= 2y_m \sin\left[\tfrac{1}{2}(a+b)\right] \cos\left[\tfrac{1}{2}(a-b)\right]$$

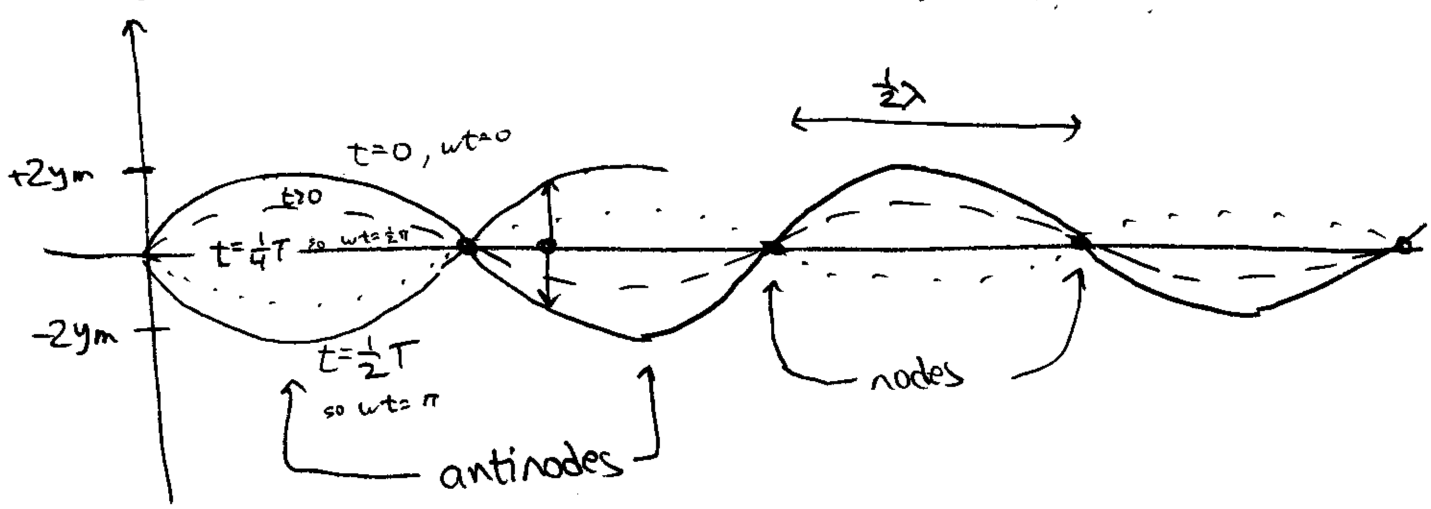
$$= 2y_m \sin(kx) \cos(\omega t)$$

this is an oscillator ($\cos \omega t$ term) whose amplitude $2y_m \sin(kx)$ depends on position x

This is a standing wave

because the disturbance doesn't travel anywhere.

Standing waves at times
$t = 0, \frac{1}{2}T$, etc ...



<u>Nodes</u> occur where $y(x,t) = 0$

so $\sin(kx) = 0$

Note: adjacent nodes are separated by ½ wavelength

$\Rightarrow kx = n\pi$ where $n = 0, 1, 2, \ldots$

so $x = \frac{n\pi}{k} = \frac{1}{2}n\lambda$

= sites where nodes occur

<u>Antinodes</u> are where $|y(x,t)|$ have maxima

so $\sin(kx) = \pm 1$

or $kx = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi \ldots$

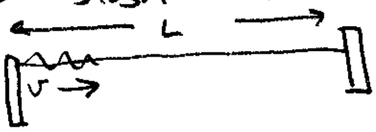
so $x = \left(n + \frac{1}{2}\right)\frac{\pi}{k} = \frac{1}{2}\left(n + \frac{1}{2}\right)\lambda$

for $n = 0, 1, 2 \ldots$

adjacent nodes have $\Delta n = 1$ so they are separated by a distance $\Delta x = \frac{1}{2}\lambda$

Standing Waves & Resonances

Suppose we fix a guitar string between 2 clamps, launch traveling waves from one end, which reflect & forever slosh back-and-forth (if frictionless...)



Soon we have many traveling waves which are interfering with each other.

However we can 'tune' the waves so that there are 2 or more equally spaced nodes (one at each end, and maybe more along string)
⇒ This results in a standing wave

WHY?

since node separation = $\frac{1}{2}\lambda$

string length ⇒ $L = \frac{n}{2}\lambda$ requirement for standing waves

Recall that wave speed $v = \lambda f$

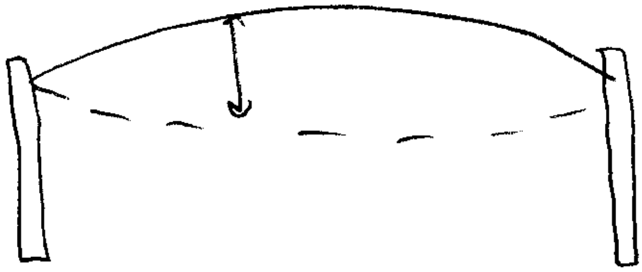
so $L = \frac{n}{2}\frac{v}{f}$

or $f = \frac{n}{2}\frac{v}{L}$ = frequency at which <u>you must drive the string</u> to establish the standing wave

also known as the string's <u>resonant frequency</u>.

$L = \frac{n}{2}\lambda$

$f = \frac{n}{2}\frac{v}{L}$



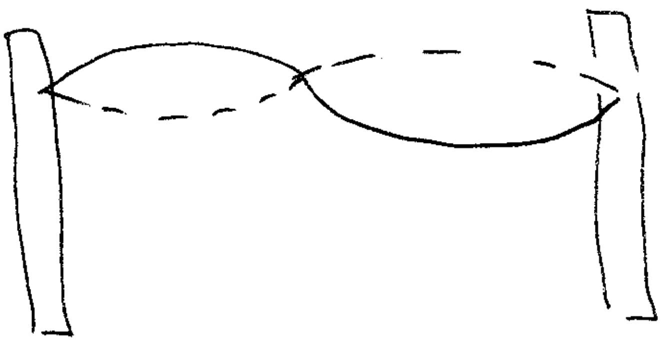
$n=1$,

fundamental mode,

or first harmonic

$L = \frac{1}{2}\lambda$

$f_1 = \frac{1}{2}\frac{v}{L}$ where $v = \sqrt{\frac{T}{\mu}}$



$n=2$

second

harmonic

$L = \lambda$

$f_2 = \frac{v}{L}$

where $f_1, f_2, f_3, \ldots$ = string's harmonic frequencies

$n = n^{th}$ harmonic

Play "standing waves & resonance / two-clamp" clip

Note that the standing wave is generated by
launching a traveling wave at left,
which reflects at right boundary,
so incoming + reflected waves establish the
standing wave.