Bill Ward @ 60:
Still Makin' Waves
After All These Years

Joseph M. Hahn
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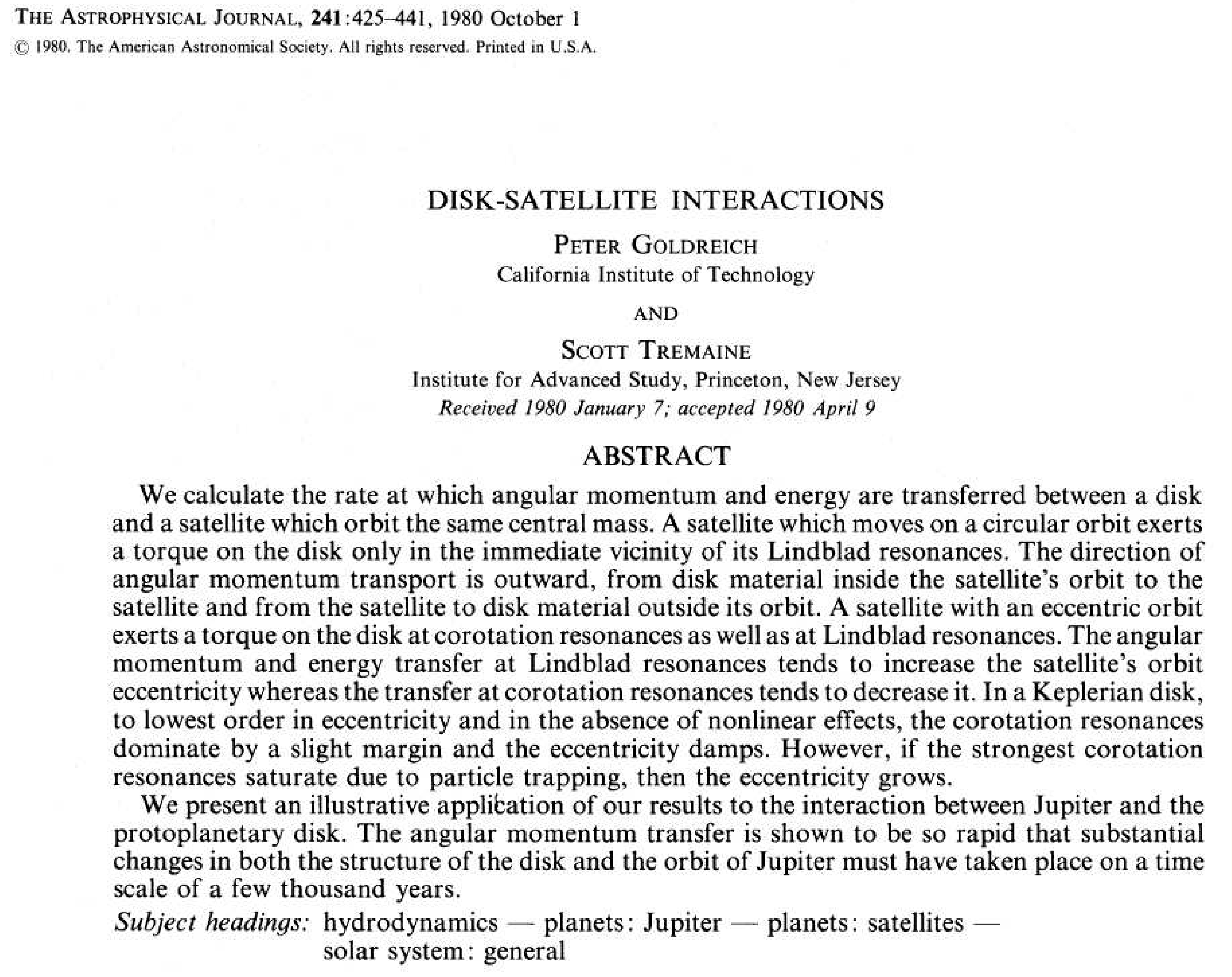
Bill Ward,

a.k.a.,

the Silver Surfer of Spiral Waves



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DISK-SATELLITE INTERACTIONS

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ABSTRACT

We calculate the rate at which angular momentum and energy are transferred between a disk and a satellite which orbit the same central mass. A satellite which moves on a circular orbit exerts a torque on the disk only in the immediate vicinity of its Lindblad resonances. The direction of angular momentum transport is outward, from disk material inside the satellite's orbit to the satellite and from the satellite to disk material outside its orbit. A satellite with an eccentric orbit exerts a torque on the disk at corotation resonances as well as at Lindblad resonances. The angular momentum and energy transfer at Lindblad resonances tends to increase the satellite's orbit eccentricity whereas the transfer at corotation resonances tends to decrease it. In a Keplerian disk, to lowest order in eccentricity and in the absence of nonlinear effects, the corotation resonances dominate by a slight margin and the eccentricity damps. However, if the strongest corotation resonances saturate due to particle trapping, then the eccentricity grows.

We present an illustrative application of our results to the interaction between Jupiter and the protoplanetary disk. The angular momentum transfer is shown to be so rapid that substantial changes in both the structure of the disk and the orbit of Jupiter must have taken place on a time scale of a few thousand years.

Subject headings: hydrodynamics — planets: Jupiter — planets: satellites —
solar system: general

BW's interest in spiral waves is probably due to last sentence in G&T (1980), which used spiral wave theory to examine the torque that a disk exerts on a planet.

G&T suggest that the torque from the solar nebula can drive rapid planet migration, but at this stage the direction of migration was still unknown.

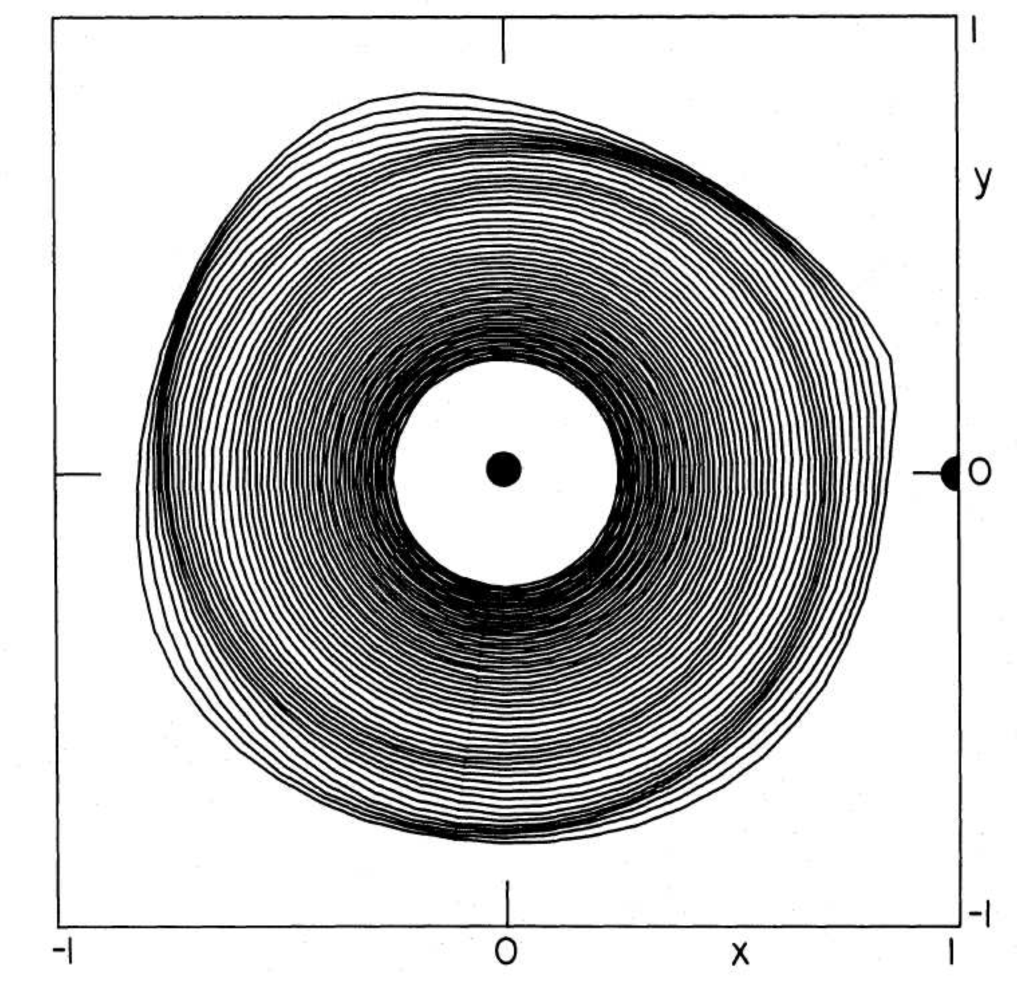
What is a spiral density wave? Why do waves drive migration?

- a disturbance excited by a planet, often launched at resonance in disk, makes disk's streamlines eccentric

- eccentric streamlines perturb neighbor steamlines via disk gravity or pressure

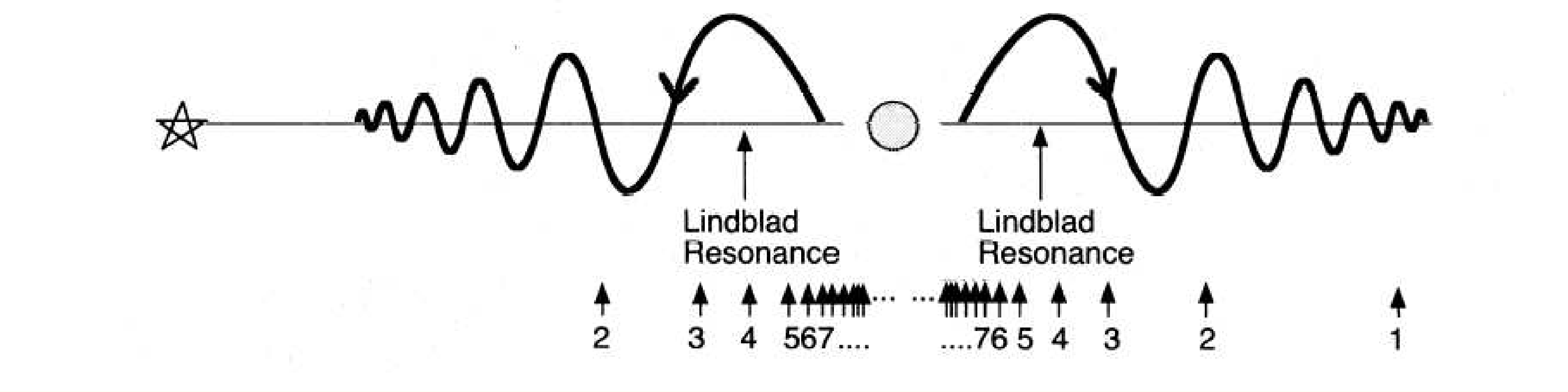
- note that ellipses' orientations $\tilde{\omega}(r)$ varies slowly with distance r, which causes the spiral pattern

- overdense where streamlines crowd, underdense elsewhere



from Lin & Papaloizou (1986)

Type I migration: follow the angular momentum



from Takeuchi *et al.* 1996.

- a planet in a gas disk launches spiral density waves at Lindblad resonances.
- interior waves transport negative $\mathbf{L}$ (*i.e.*,planet gains **L** from inner disk),
- exterior waves transport positive $\mathbf{L}$ (planet loses **L** to outer disk),
- no migration thus implies a very delicate torque balance—unlikely!
- $m = 1$ ILR is special—a secular resonance—later...

Torques due to wave–action are powerful!

torque that disk @ m^{th} resonance and satellite exert *on each other is* (G&T 1978)

$$|T_m| \sim m^2 \left(\frac{M_{disk}}{M_{primary}}\right)\left(\frac{M_{satellite}}{M_{primary}}\right)^2 r^2\Omega^2 M_{\text{primary}}$$

for Mimas, which launches waves in Saturn's rings,

$$T_{total} \sim \sum_m |T_m| \sim 10|T_1| \quad = \quad \text{total torque}$$

$$\text{migration rate} \quad |\dot{r}| \quad = \quad \frac{2T_{total}}{r\Omega M_{satellite}}$$

the migration timescale is

$$\tau_{migrate} \quad \sim \quad \left|\frac{r}{\dot{r}}\right| \sim 10^{-3}\left(\frac{M_{disk}}{M_{primary}}\right)^{-1}\left(\frac{M_{satellite}}{M_{primary}}\right)^{-1} \text{orbits}$$

At Saturn, $M_{disk} \sim M_{Mimas} \sim 10^{-7} M_{Saturn}$

$\Rightarrow \tau_{migrate} \sim 10^{11}$ orbits $\sim$ few hundred million years,
which is the time for Mimas to migrate outwards *and* collapse A→B

Since $\tau_{migrate} \ll$ age of SS, why aren't rings & satellites already decoupled?

Why do the wave & perturber torque each other?

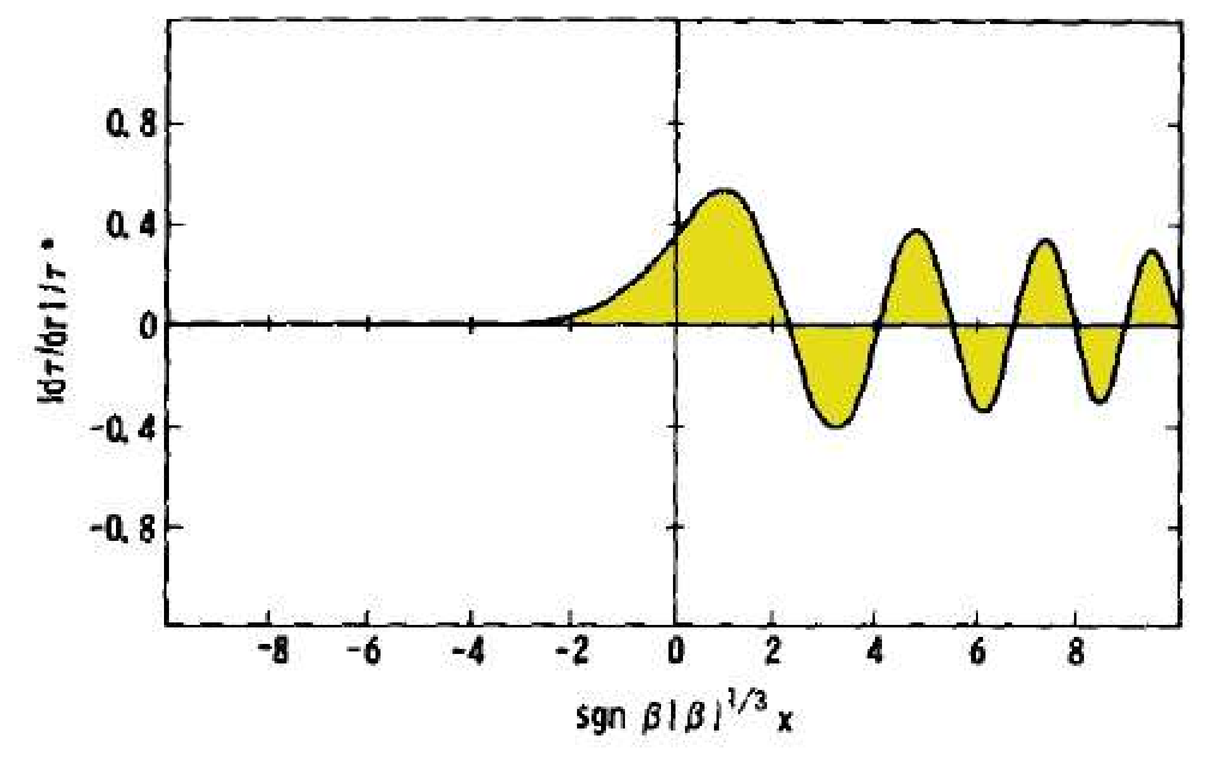
- the planet (or satellite) is attracted to wave's surface density variations $\sigma(r, \theta)$ which oscillates with r,

- as does the torque density $\frac{dT}{dr}$

- the total torque from $\mathbf{m}^{\mathbf{th}}$ resonance is

$$|T_m| = \int \frac{dT}{dr} dr \neq 0$$

Ward (1986)



Torque on protoplanet embedded in disk

- calculate the torque differential ΔT_m:

$$= T_m(inner) + T_m(outer) < 0$$

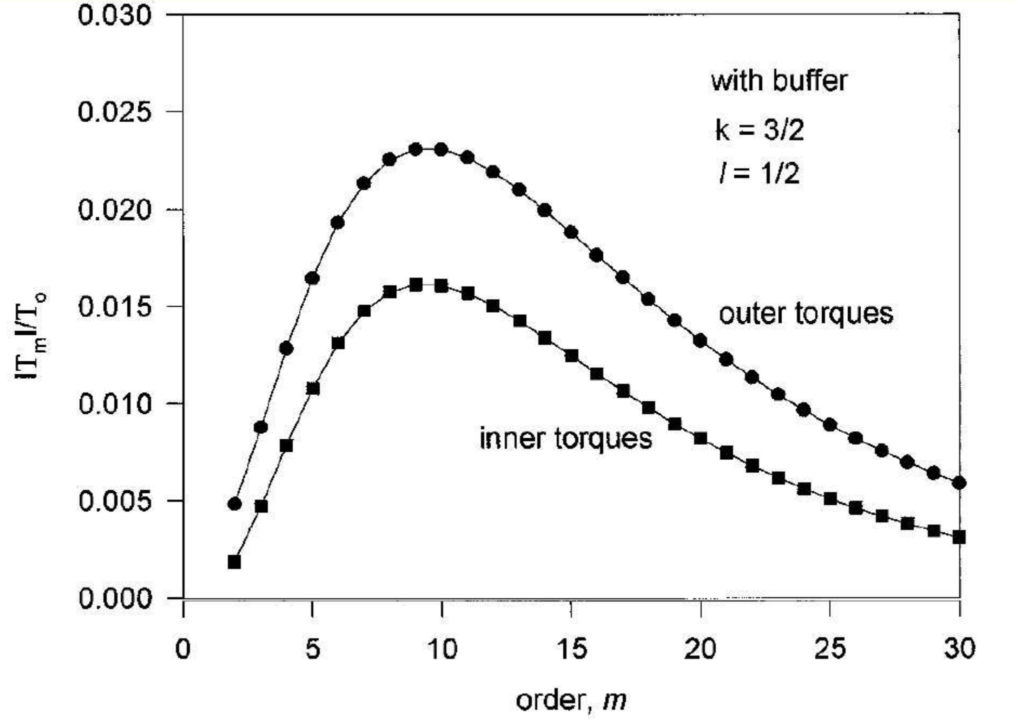
- the total torque on protoplanet is

$$T = \sum_m \Delta T_m$$

and is evaluated in Ward (1986, 1997)

- $T < 0$ so the protoplanet's orbit decays

- note that the important resonances are those with $m \sim (h/r)^{-1} \sim 10$ and lie a distance $\Delta r \sim h$ away.



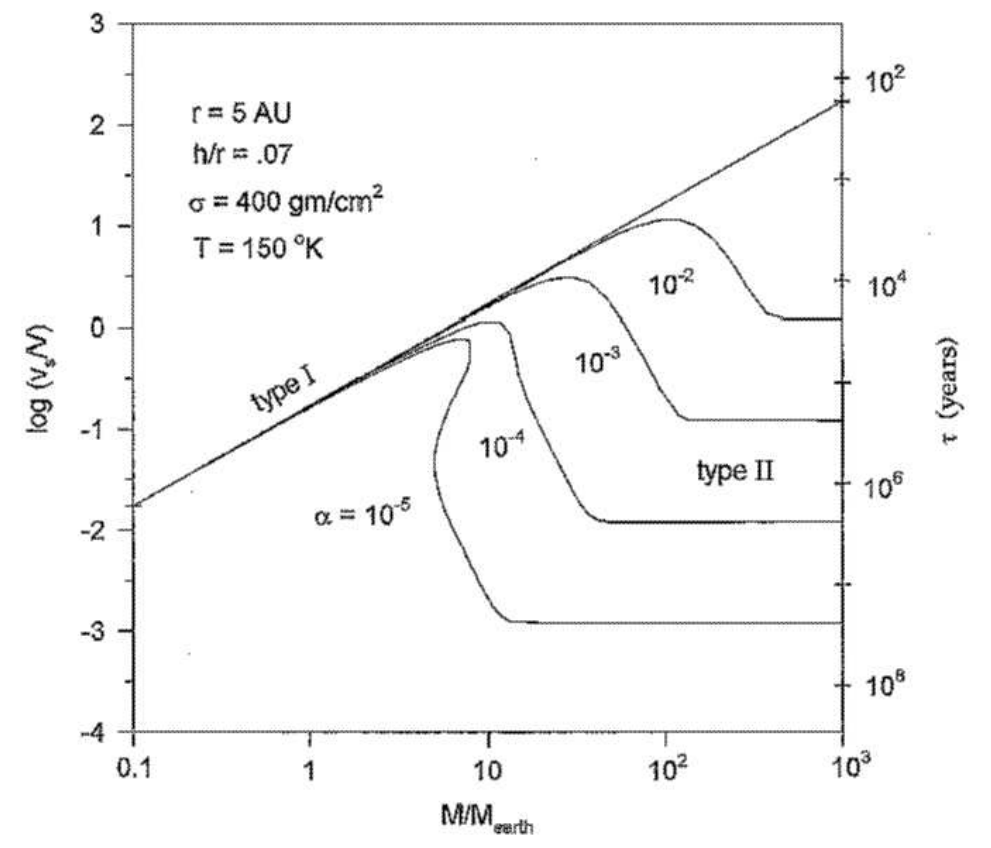
from Ward (1997)

Type I orbit decay timescales

- The timescale for orbit decay is

$$\tau_{typeI} \sim \left|\frac{r}{\dot{r}}\right| = \frac{r^2 \Omega M_{planet}}{2T}$$

- this motion is known as type I migration
- results were confirmed numerically in 2D hydro model by Korycansky & Pollack (1993),
  and 3D model by Tanaka, Takeuchi, & Ward (2002)



from Ward (1997)

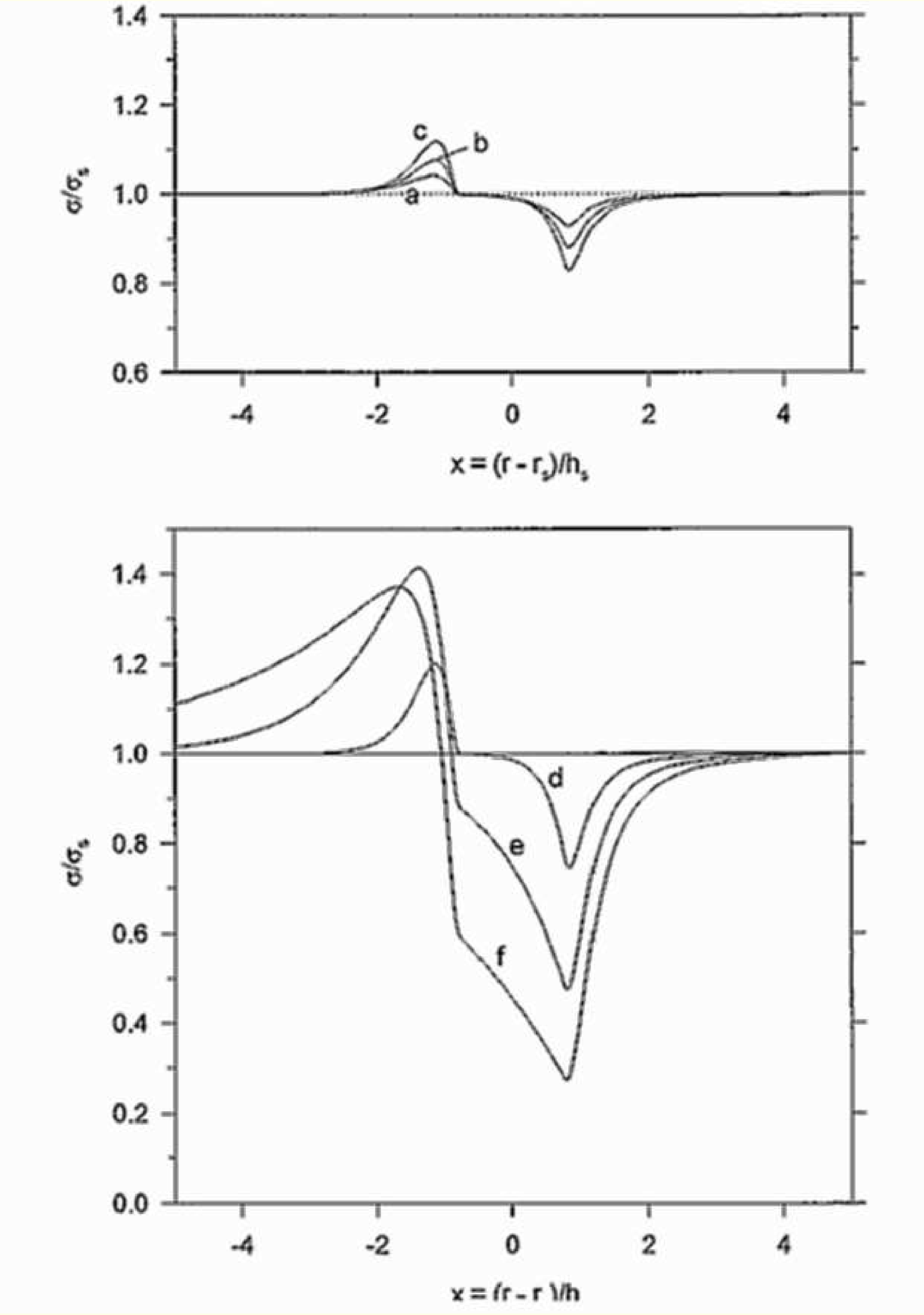
- result: massive planets decay faster
- figure might suggests that a $10M_\oplus$ protoplanet will survive for $\tau_{typeI} \sim 10^4$ years before spiraling into the Sun
- actually, the protoplanet can save itself by carving open a gap in the disk around its orbit, which stalls type I migration
- however the protoplanet is still susceptible to slower type II migration...

Stalling type I migration by gap formation

protoplanet opens a gap when
(Hourigan & Ward 1984)

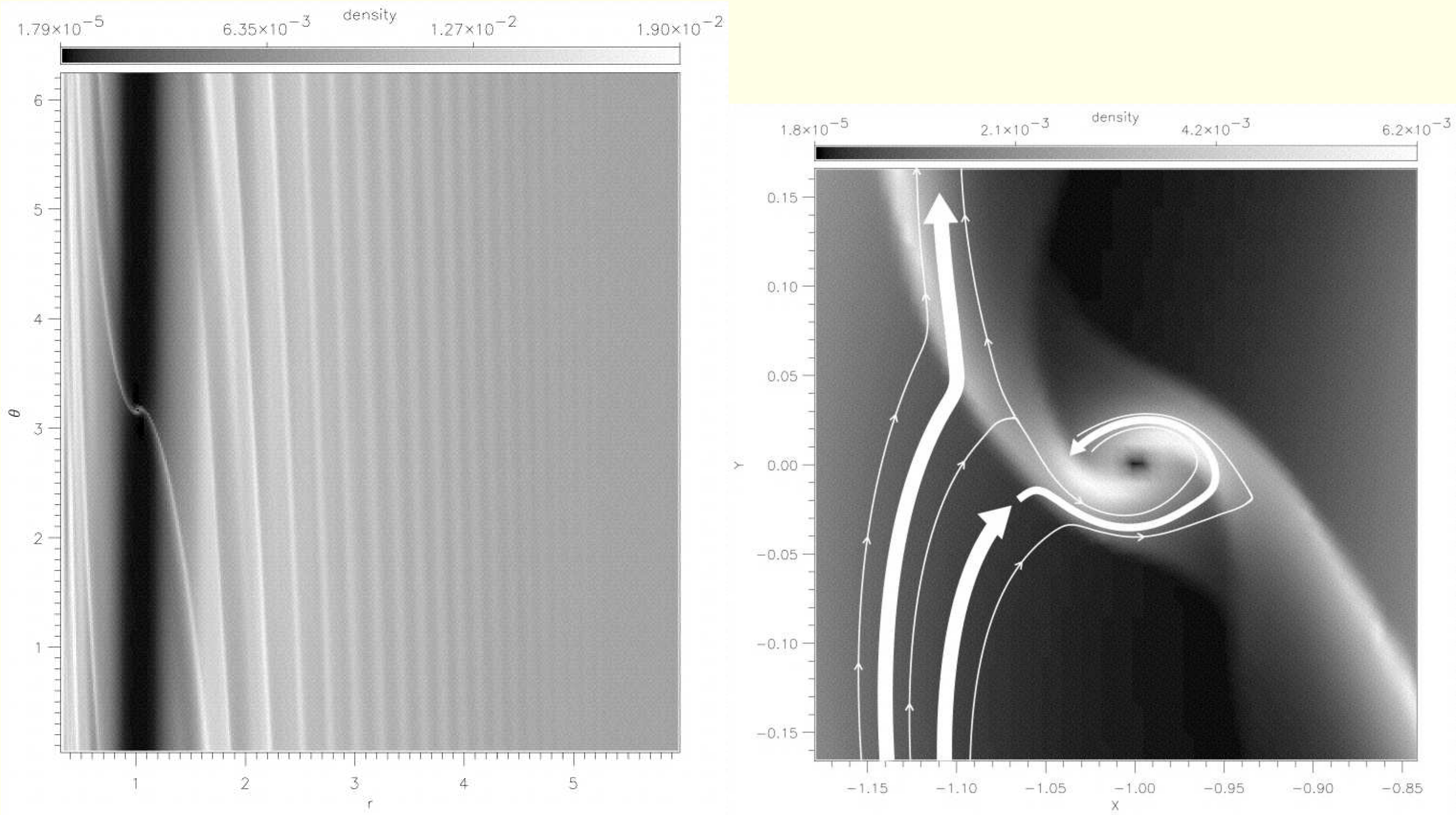
$$M_{planet} \gtrsim \sqrt{\frac{\nu}{r^2\Omega}}\left(\frac{h}{r}\right)^{3/2} \sim 10\sqrt{\frac{\alpha}{10^{-4}}}M_{\oplus}$$

- migrating protoplanet's *plows* the disk
- this strengthens the torques from ILRs, which slows the migration
- note that the ILRs draws $\mathbf{L}$ from inner disk and deposits $\mathbf{L}$ in outer disk
- gap opens by *pushing* the disk's gas away the protoplanet does not accrete the gas!
- gap width $\Delta r \sim \pm h$,
  zone where strongest LRs live



←planet migration from Ward (1997)

Simulations of gap formation



Lubow, Seibert, & Artymowicz (1999)

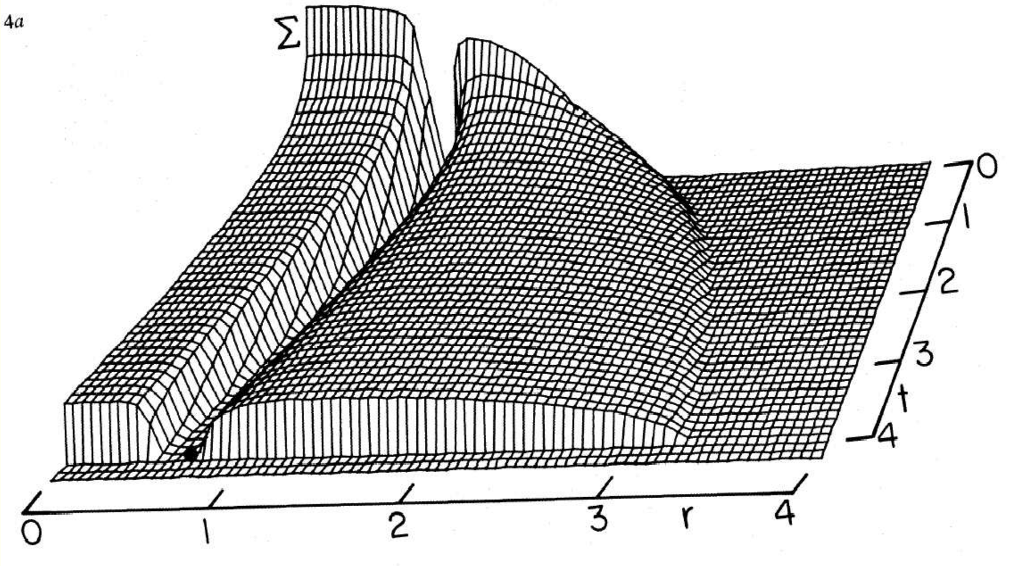
Thus even a $\sim \mathbf{10}M_{\oplus}$ protoplanet core can open a gap and still accrete nebula gas, become a full–sized gas giant planet

Type II migration

Protoplanet & its gap is locked into the disk which evolves on a viscous timescale

$$\tau_{mig} \sim \frac{r^2}{\nu} \sim \frac{1}{2\pi\alpha}\left(\frac{h}{r}\right)^{-2} P_{orb}$$

$$\sim 10^5 \left(\frac{\alpha}{10^{-4}}\right)^{-1} \text{orbits.}$$



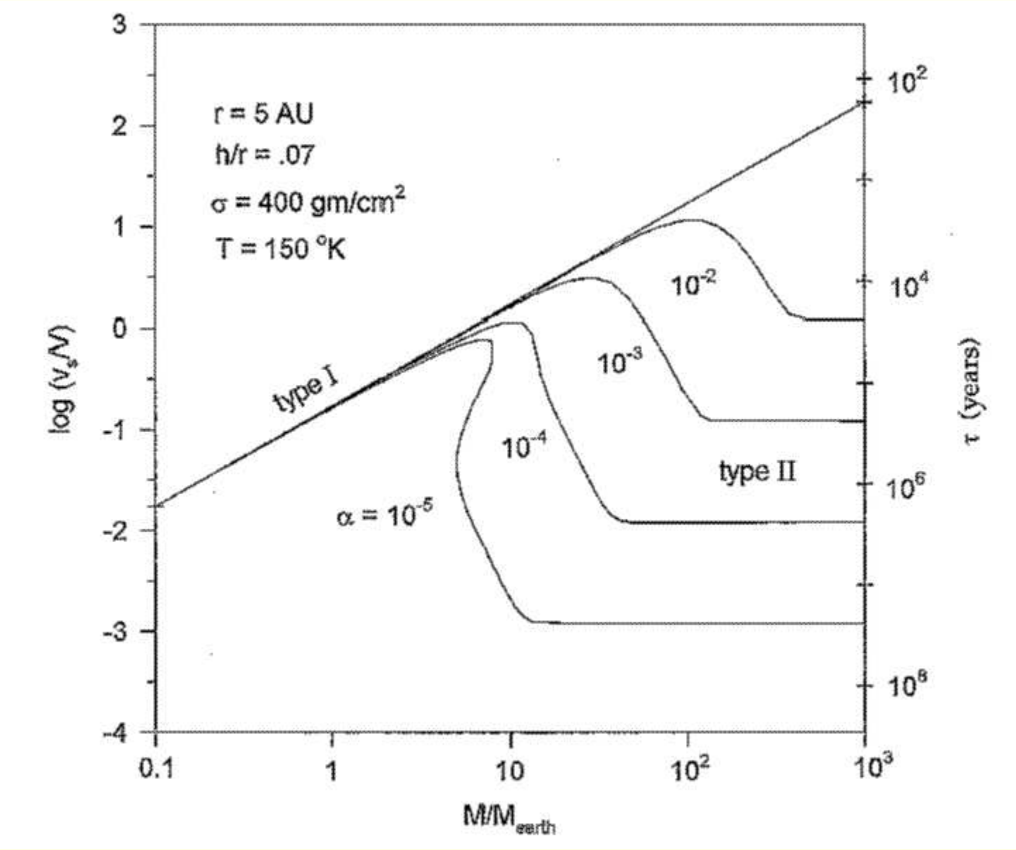
Lin & Papaloizou 1986.

Note that a high α disk can destroy its protoplanets!

Migration timescales

Observations of circumstellar disks indicate disk lifetimes of $\boldsymbol{\tau_{disk} \sim 10^{6\ to\ 7}}$ years. Bill’s diagram then suggests that:

- giant planets form & survive only in rather inviscid disks, $\boldsymbol{\alpha} \leq$a few$\times\mathbf{10^{-4}}$
- however disk accretion luminosities suggest $\boldsymbol{\alpha \geq 10^{-2}}$ (Stepinski 1998, Hartmann 1998)
- perhaps giant planets shouldn’t exist? they all spiral into their Suns?
- or perhaps disks spawn many generations of planets, and the timing of disk dispersal selects the surviving generation (Trilling et al. 1998)



from Ward (1997)

- disk lifetimes/migration timescale an issue for satellite formation (Canup & Ward 2002)

The planet–parking problem

The 1995 discovery of hot Jupiters with $r \sim 0.05$ AU made planet migration (and planet–migration theorists) quite popular.

Q: How does a Jupiter–class planet,
which may have formed at $r \sim 5$ AU ice boundary,
migrate 99% that distance and park itself at $r \sim 0.05$ AU?

- LBR (1996): stellar magnetosphere maintains hole in inner disk, so migrating planet gets parked just inside the disk's donut hole
- Trilling et al. (1998): luck
  disk disperses before final plunge
- Jumpin' Jupiter Scenario (Weidenschillin & Mazarie 1996, Rasio & Ford 1996): planet–scattering + tidal circularization & no disk

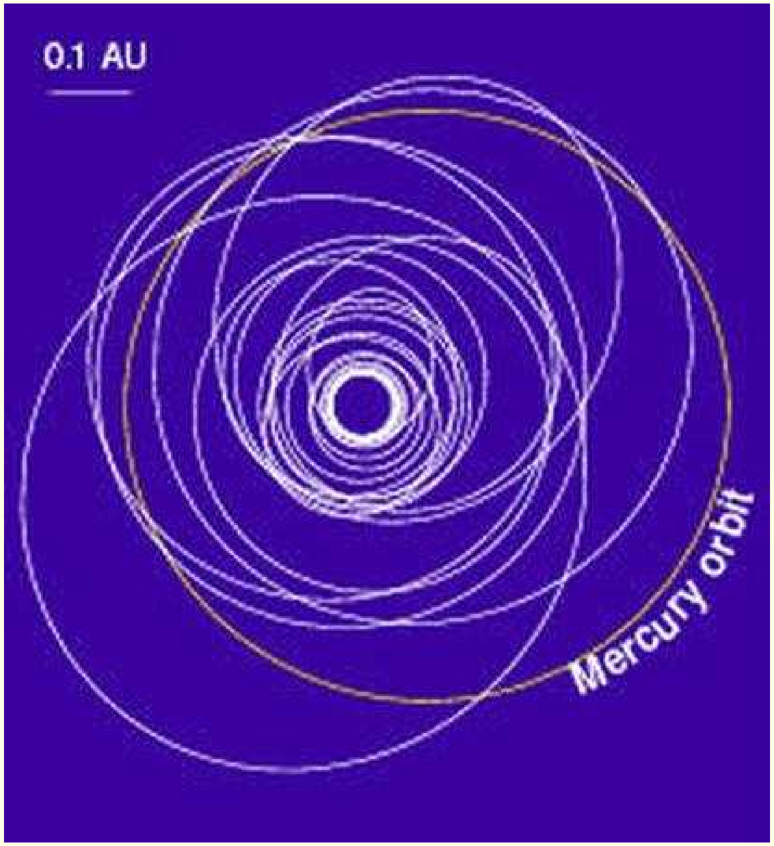


figure by Stepinski

But are extrasolar planets actually planets?

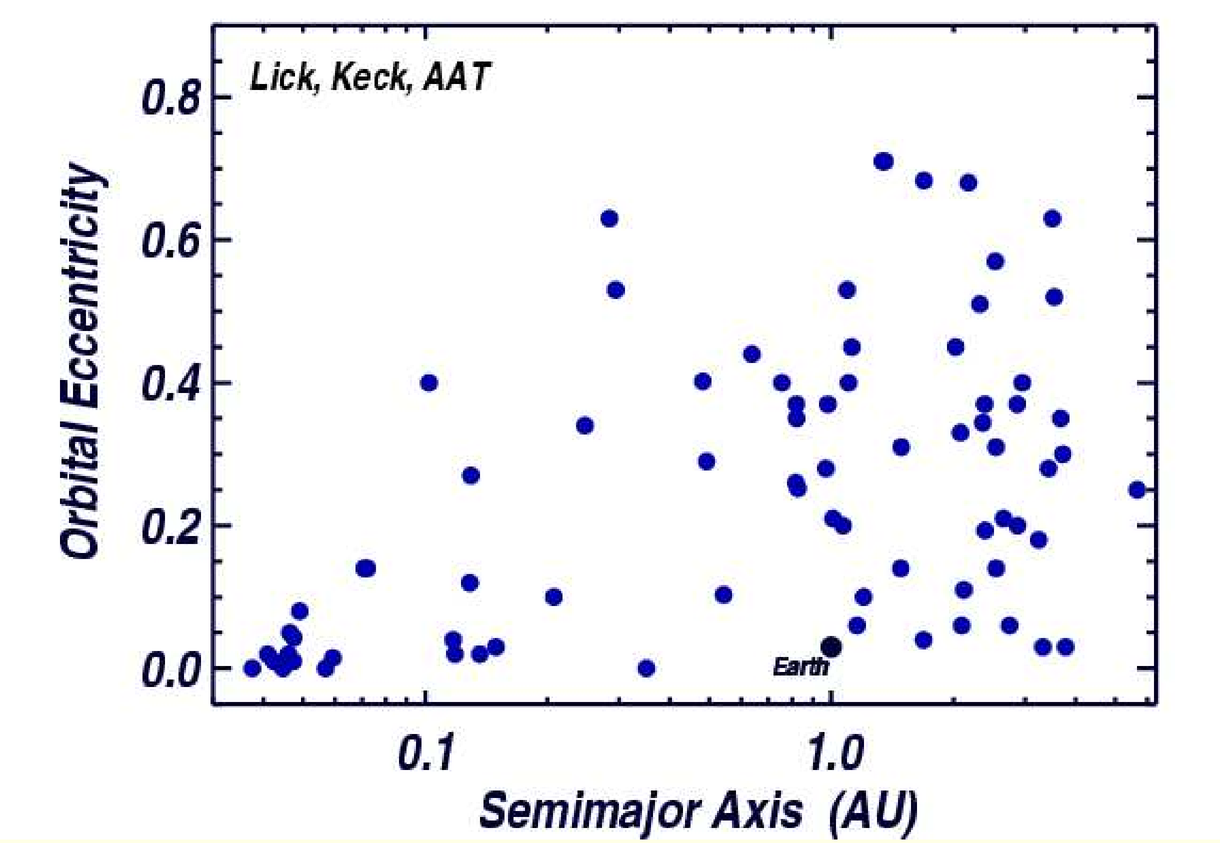
Stepinski & Black (2000, 2001) compare the extrasolar planets' (ESP's) orbits to stellar binary stars, and conclude that:

- ESPs' orbits are statistically indistinct from binary stars,
- suggests that binary stars & ESPs have common origin→common orbits

What do I conclude from this?

- first I remind myself that I really don't understand:
  - how planets form
  - how planets solve the planet–parking problem
  - how stars form

- given this similarity, I cautiously conclude that the ESP are either
  - unusual high–mass planets
  - unusual low–mass stars



from http://exoplanets.org

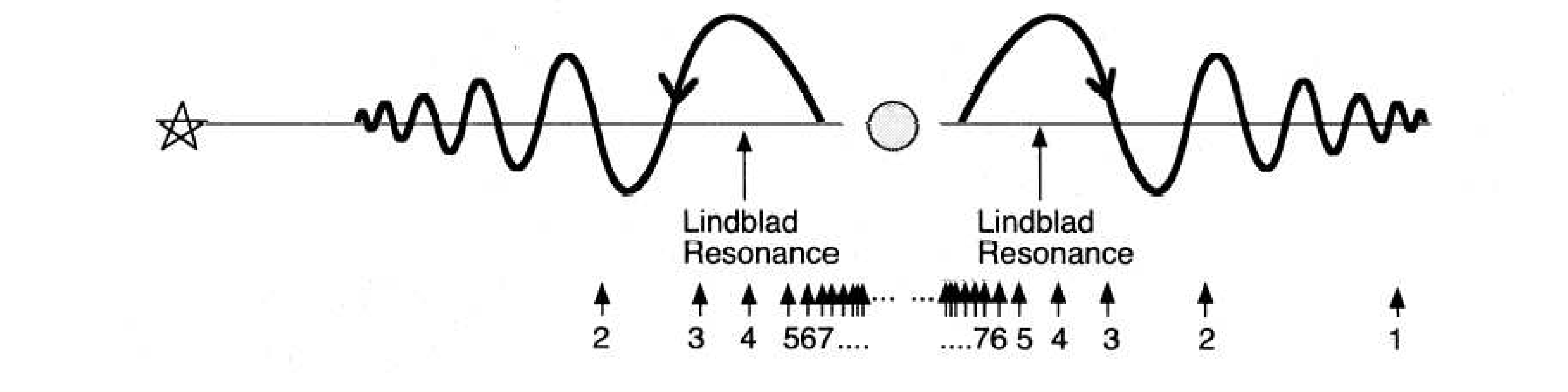
So what might we conclude from disk–planet interactions?

- disk–planet systems are seemingly unstable,
  since type I & II migration can easily drive young protoplanets into star

- solar systems like ours will be rare
  since a Jupiter at $r = 5$ AU requires $\alpha \lesssim 10^{-4}$ to avoid death by type II
  while disk observations suggest $\alpha \gtrsim 10^{-2}$

- hot Jupiter are either:
  - remarkable planets that solved their ‘planet–parking problem’
    after having migrating 99% of the way in
  - or they are remarkable planets that formed in situ at $r \sim 0.05$ AU
  - or, as their orbits suggest, are remarkably low–mass stellar companions.

what about that $m = 1$ ILR?



from Takeuchi *et al.* 1996.

The $m = 1$ ILR is a secular resonance, which is a site in the disk where a particle's precession rate matches some disturbing frequency g_i or s_i for slow horizontal/vertical perturbations:

- $\dot{\tilde{\omega}}_{particle} = g_i \simeq \dot{\tilde{\omega}}_{planet} \Rightarrow$ large e excitation
- $\dot{\Omega}_{particle} = s_i \simeq \dot{\Omega}_{planet} \Rightarrow$ large i excitation

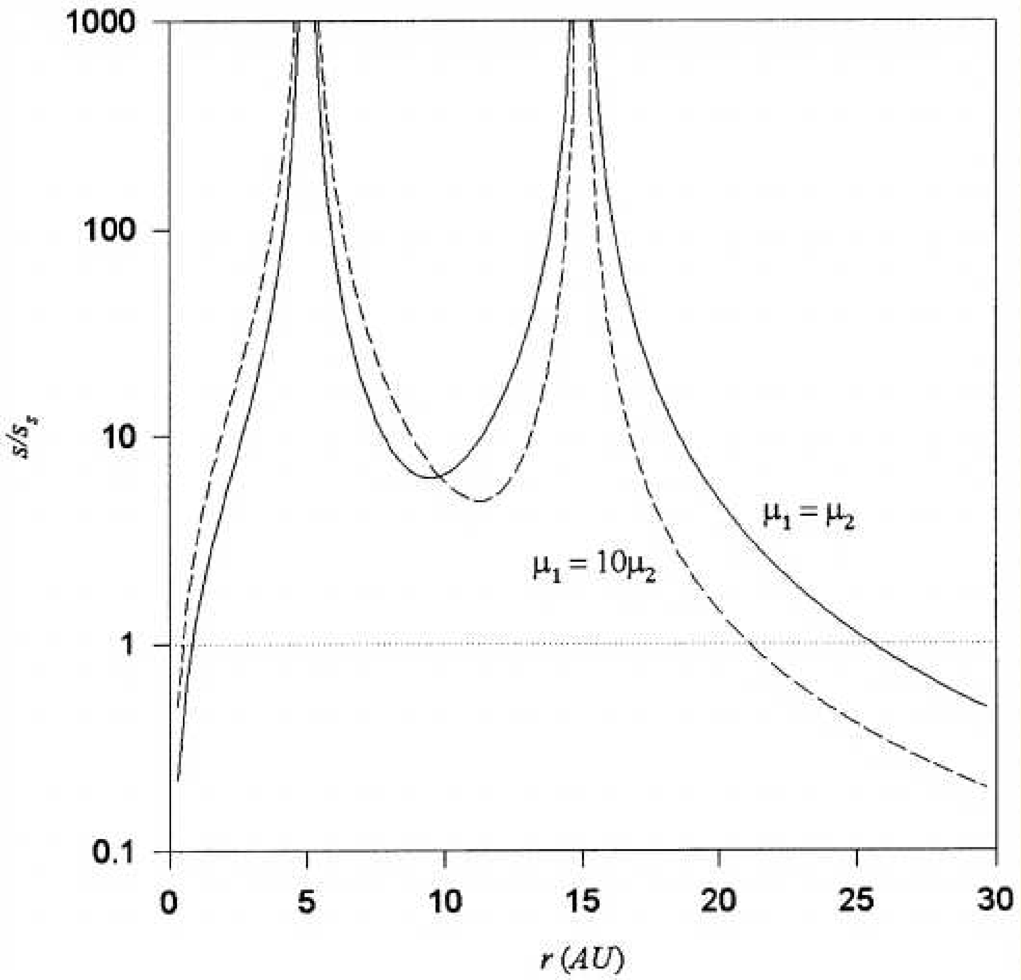
In a gravitating particle disk, these secular resonances can launch long–wavelength $m = 1$–armed spiral density and spiral bending waves (Ward & Hahn 1998, 2003)

Where is the secular resonance?

- resonance location depends in detail on the configuration of the disk–planet system

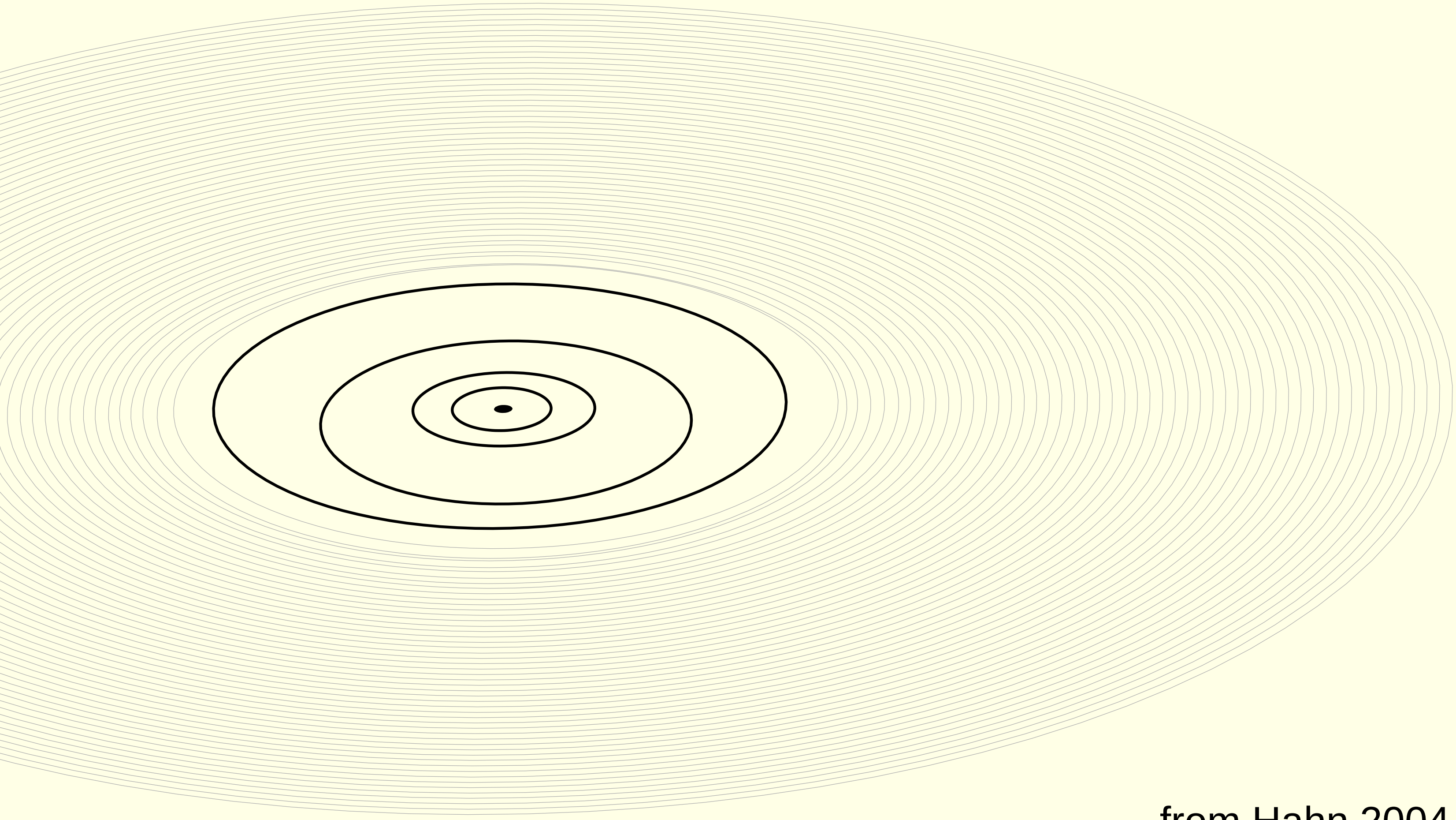
- figure is for a two–planet system embedded a low–mass particle disk'

- two resonances lie where $\dot{\Omega}_{particle} = \dot{\Omega}_{planet}$



from Ward & Hahn 2003

Animation of apsidal waves using ‘rings model’



from Hahn 2004

Waves are launched by giant planets orbiting interior to a $M = \mathbf{10} M_{\oplus}$ Kuiper Belt

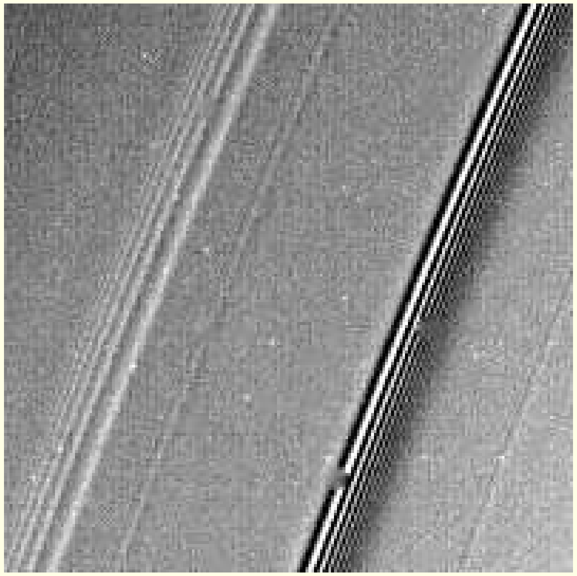
Why such long apsidal wavelengths?

- in Saturn's rings, $\lambda \sim 10^{-4} r$
  in Kuiper Belt, $\lambda \sim 0.1 r$

- due to waves radial velocity

$$v_{group} \sim \left(\frac{M_{disk}}{M_{primary}}\right) V_{kepler} \sim \frac{\lambda}{T_{pattern}}$$

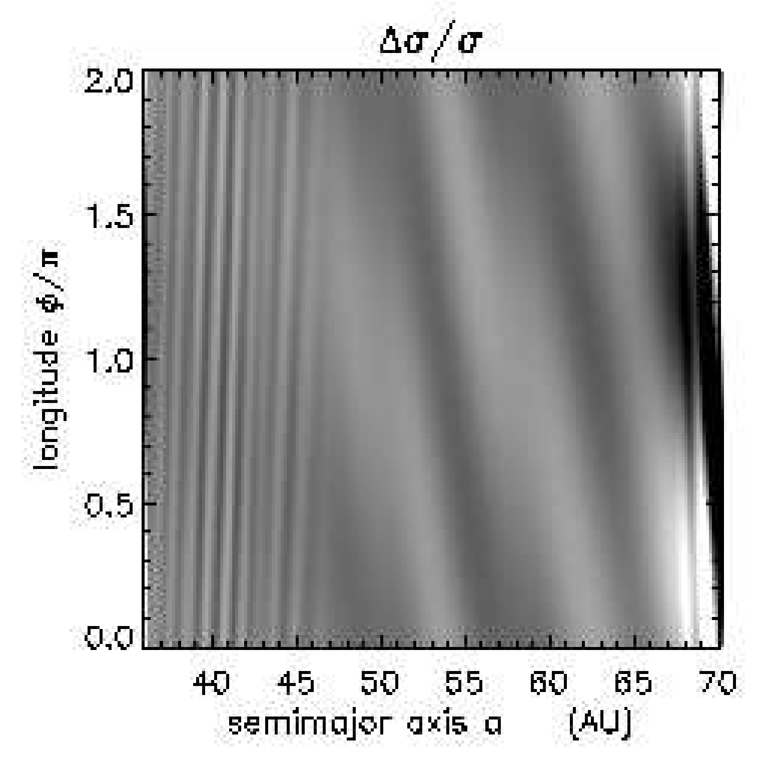
$$\Rightarrow \lambda \sim \left(\frac{M_{disk}}{M_{primary}}\right) V_{kepler} T_{pattern}$$



Showalter/PDS Rings Node

- where T the spiral pattern's rotational period is

$$\begin{aligned} T_{pattern}(m \neq 1 \text{ ILR}) &\sim T_{orbit} \\ T_{pattern}(m = \text{ ILR}) &\sim T_{precession} \\ &\gg T_{orbit} \\ \Rightarrow \lambda(m = 1 \text{ ILR}) &\gg \lambda(m \neq 1 \text{ ILR}) \end{aligned}$$



Hahn 2004

How does the planet react to apsidal and nodal waves?

These waves are a *secular* perturbation, and do not effect semimajor axes a.

However launching apsidal density waves does affect e_{planet} (Ward & Hahn 1998), and launching nodal bending waves alters i_{planet} (Ward & Hahn 2003)

- this study was motivated by G&T (1980), which showed that:
  - density wave–action at certain external LR's tends to pump up e_{planet}
- and BGT (1984):
  - bending waves at external vertical resonances pumps up i_{planet}
- our main interest was to see how apsidal/nodal waves alters e_{planet} & i_{planet} when a growing protoplanet accretes a gap in a (gas–free) planetesimal disk
  - relevant to planetary accretion, since
    - e–pumping can drive protoplanets into crossing orbits—and aid accretion
    - while i–pumping might defeat accretion, or form non–coplanar planets

- Ward & Hahn (1998, 2003) main results:
  - apsidal and nodal waves tend to *damp* e_{planet} and i_{planet}
    - however the net outcome is uncertain since it depends on the gap width Δr
    - if the gap Δr is wide enough to shut off the nearby external resonances (which want to pump e_p up), then $e_{planet} \rightarrow 0$ and $i_{planet} \rightarrow 0$

- of course, all this is true provided the disk is *cool enough to sustain spiral waves:*
  - wave–action @ external resonances is shut off when Toomre's stability parameter $Q \gg 1 \Rightarrow$ the disk is too hot (Toomre 1969)
  - Q is a measure of the planetesimals' $v_{dispersion}(R)$ which depends on size R;

    since $v_{dispersion} \sim v_{escape}$ in a planetesimal disk,
    a minimum–mass planetesimal disk has $Q \sim (R/1$ km$)$,
    $\Rightarrow$ waves at external resonances shut off when planetesimals are $R \gg 1$ km
  - however apsidal/nodal waves shut off when $Q(R) \sim |\Omega/\dot{\tilde{\omega}}| \sim 1000$ (Hahn 2004)
    $\Rightarrow$ apsidal/nodal waves can persist in hotter disks having larger R
  - mass erosion (which may have occurred in the Kuiper Belt)
    also raises the disk's Q and also shuts off apsidal/nodal waves

- short summary:

  - a protoplanet in a quiescent, low–Q, gas–free planetesimal disk (perhaps in terrestrial zone?) will excite spiral waves at its external resonances and secular resonances in the disk

  - wave–action results in an exchange of angular momentum between the protoplanet and the disk;
    the fate of e_{planet} and i_{planet} depends upon the competition between the external and secular resonances, and the width Δr of any gap

  - as planetesimals grow and raise the disk's Q,
    wave–action at the external resonances gets shut off first
    which may allow the secular resonances to damp e_{planet} and i_{planet}

  - these effects may (or may not) alter the protoplanet's orbital as well as its growth history—this depends in detail upon how vigorously wave–action alters the planet's orbit, and how quickly the planetesimal disk gets stirred–up by other large bodies growing within

- but what about a gas–giant orbiting in the solar nebula gas disk?
  See Lubow & Ogilvie (2001) and Goldreich & Sari (2003)