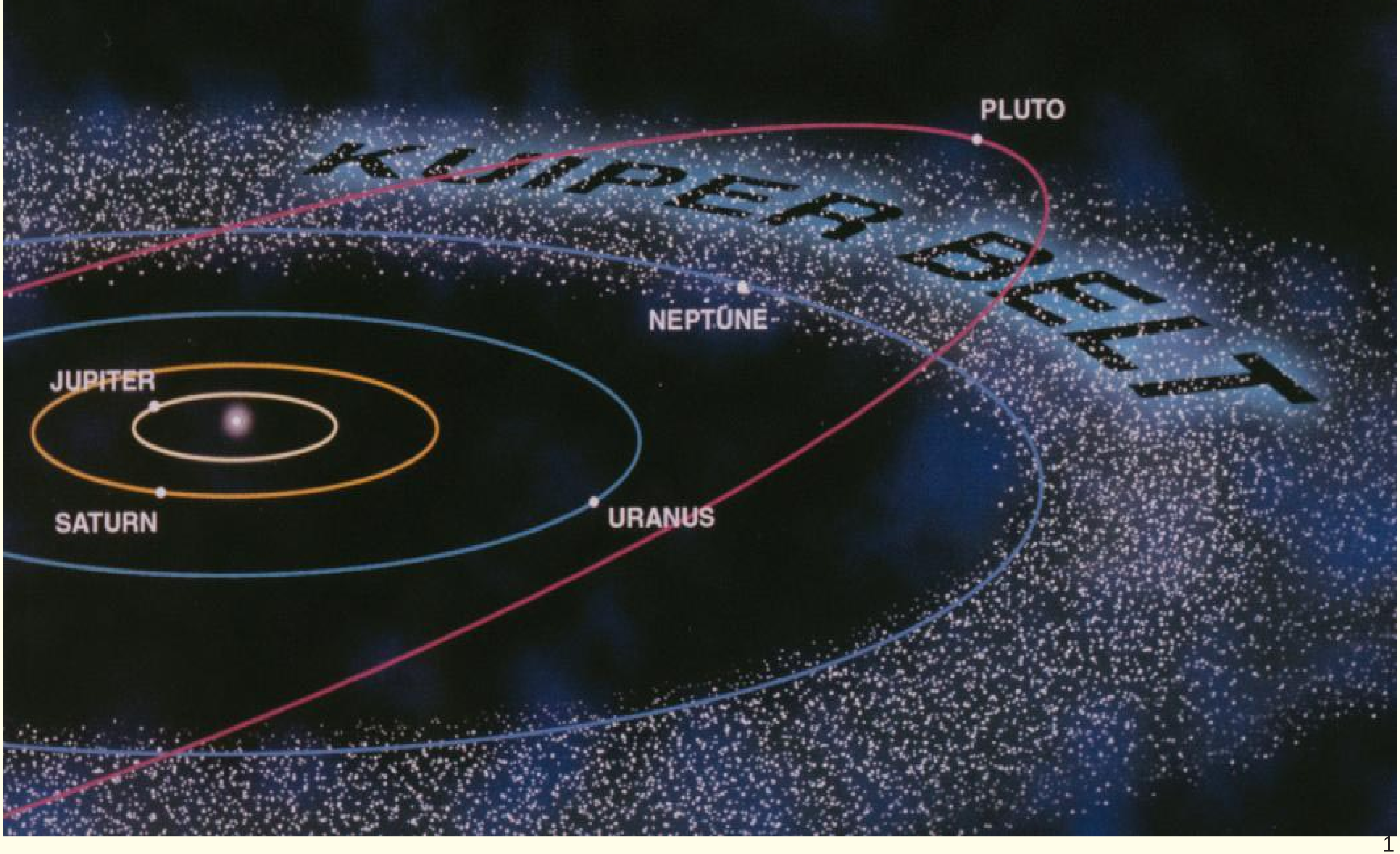
THE SECULAR EVOLUTION OF THE PRIMORDIAL KUIPER BELT

— *Joseph M. Hahn (LPI)*



Outline

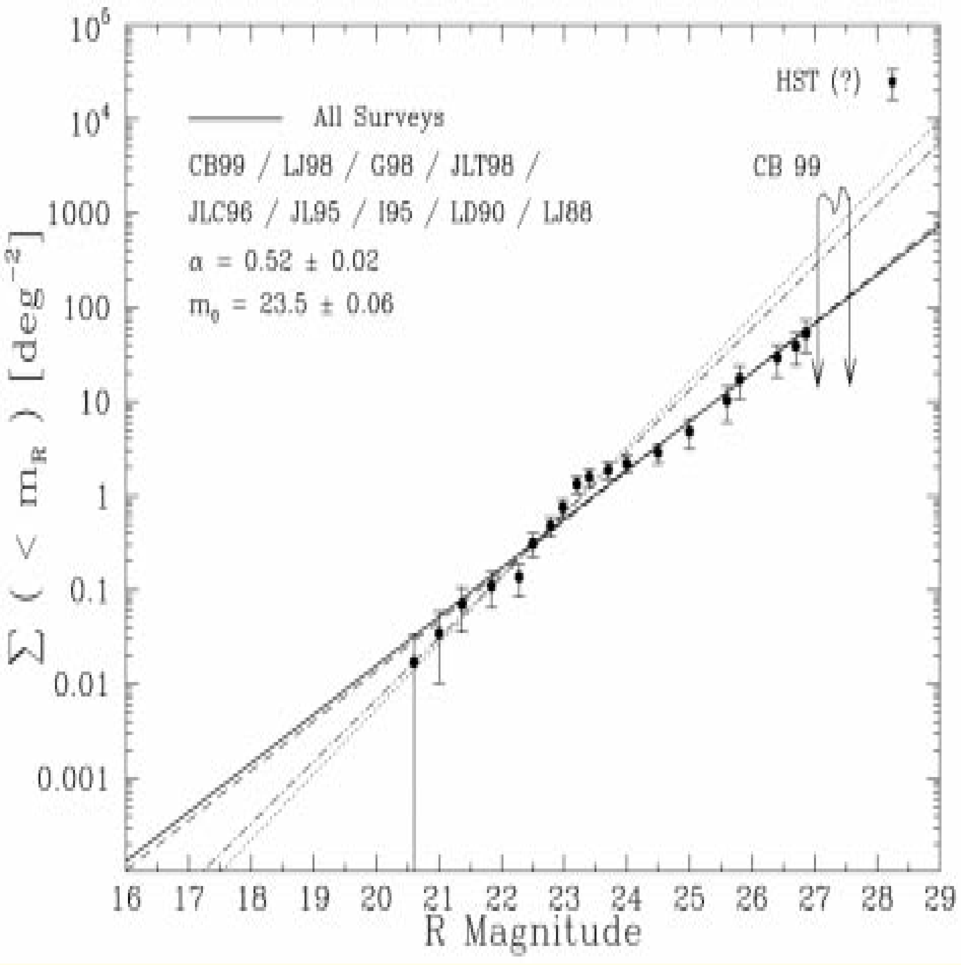
- Kuiper Belt review:
  - observed properties of the Kuiper Belt
  - KBO accretion theory
  - the Belt’s dynamical properties
  - planet migration and its effects in the Kuiper Belt
  - Gomes (2003) ‘invasion’ hypothesis

- secular evolution of the Kuiper Belt
  - rings model
  - results

- future applications

Kuiper Belt Statistics

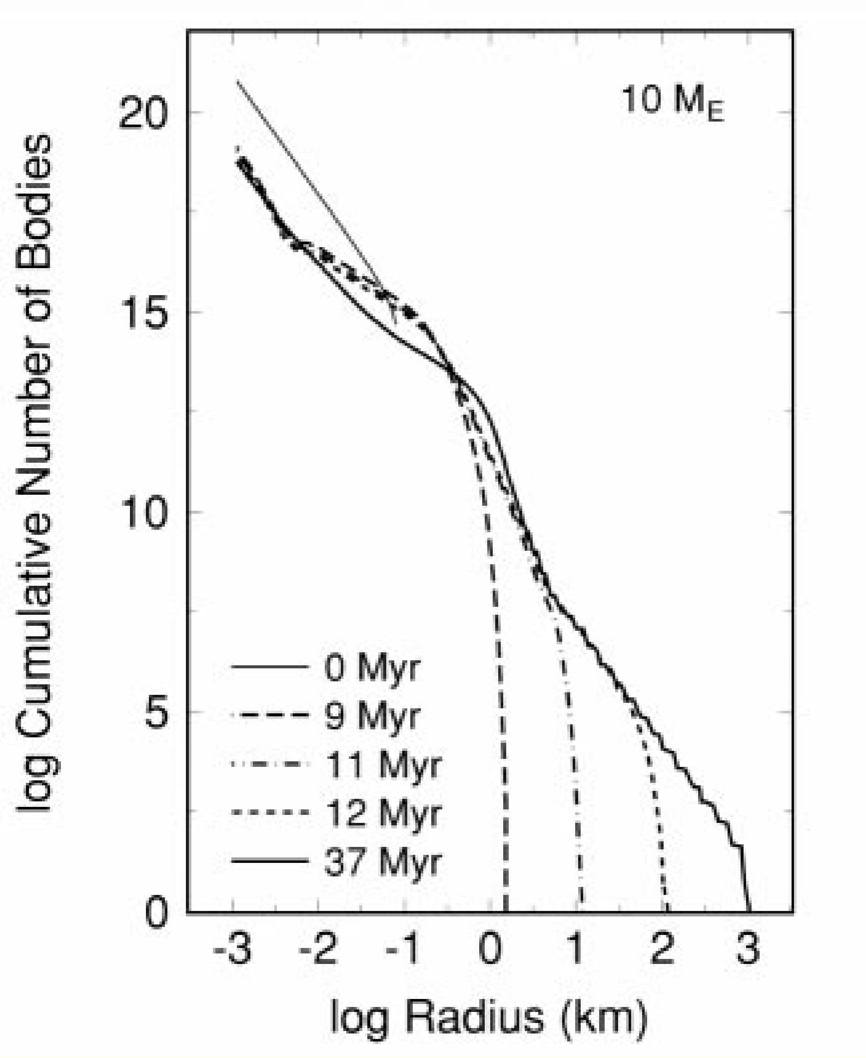
- The Kuiper Belt's observed luminosity function $\Sigma(m)$ provides:
  - population estimate $N(R > 50 \text{ km}) \sim 10^5$
  - size distribution $dN(R)/dR \propto R^{-q}$ with $q = 3.6$
    - since $q < 4$, the Belt's mass is determined by the largest KBOs
  - Belt's total mass is $M_{KB} \sim 0.2$ M$_\oplus$ (Jewitt & Luu 1996, Chiang and Brown 1999)
  - compare to the asteroid belt:
    - $N_{KB} \sim 100 \times N_{AB}$
    - $M_{KB} \sim 100 \times M_{AB}$



Chiang and Brown (1999)

Accretion in the Kuiper Belt

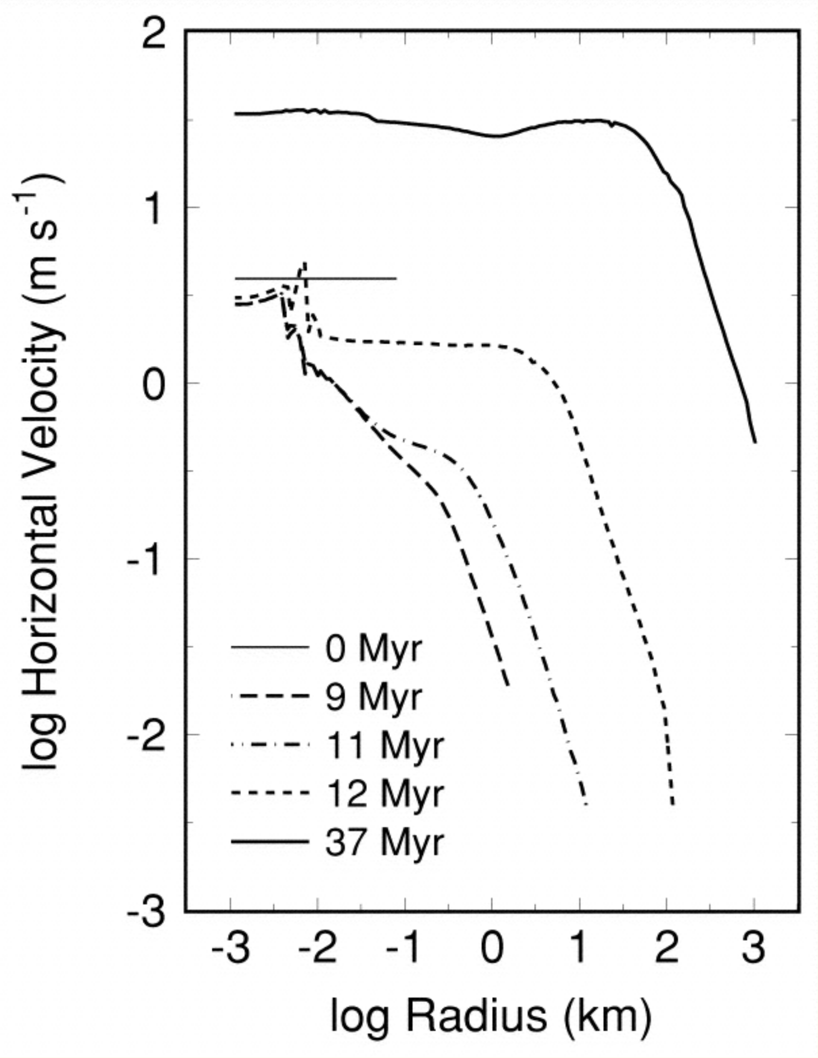
- Monte Carlo simulations of accretion show:
  - $R \sim 100$ km KBOs form via runaway growth in $\tau \sim 10^7$ years
  - a few $R \sim 1000$ km Plutos form in $\tau \sim 4 \times 10^7$ years
- this requires $M_{KB} \sim 30$ M$_\oplus$ in the $30 < a < 50$ AU zone
  - enough mass to form 1 or 2 Neptunes!
  - the primordial KB was $\sim 150\times$ more massive than the present Belt



Kenyon and Luu (1999)

So Where are the Other Neptunes?

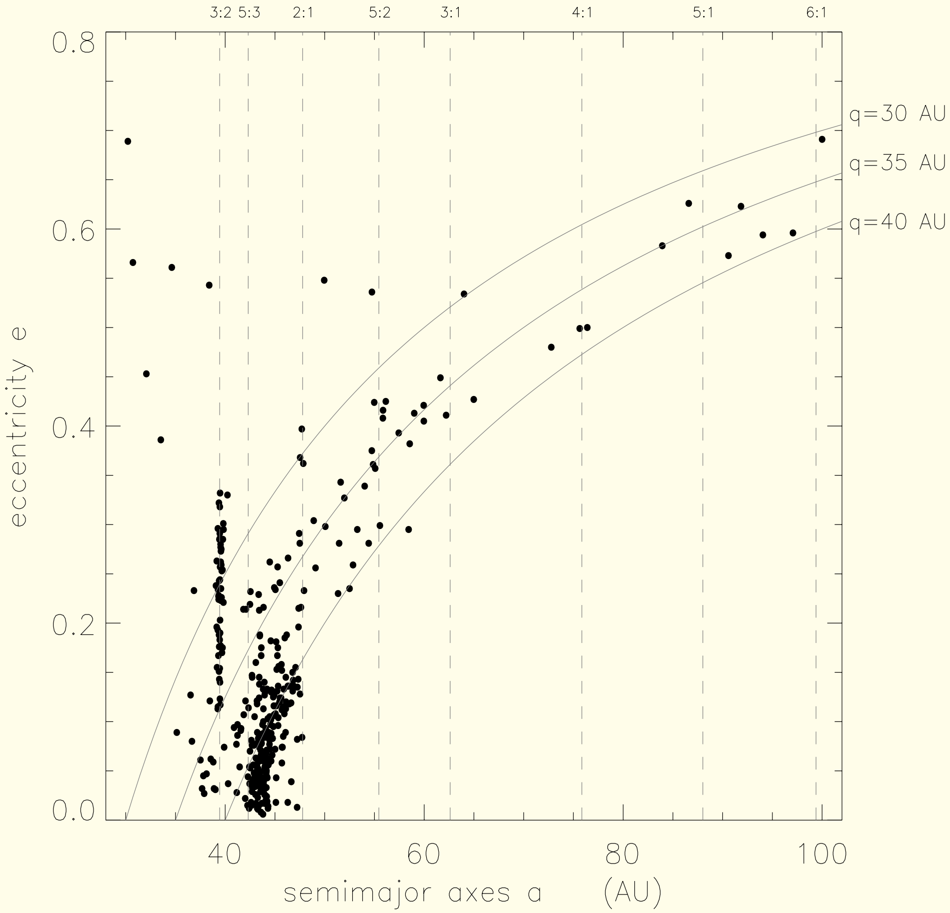
- accretion simulations also show:
  - when $R \sim 100$ km KBOs form, their mutual gravitational stirring raises the KBOs' random velocities above the shattering threshold of $v \sim 10 - 100$ m/sec for small $R \sim 1$ km KBOs
  - this halts further growth & initiates erosion
    - $*$ bodies smaller than $R \sim 1 - 10$ km are ground down to dust over the next $\tau_{\text{erode}} \sim 500 \times 10^6$ years
      - $\cdot$ this dust is removed by PR drag or radiation pressure
    - $*$ bodies with $R \gtrsim 10$ km survive intact



Kenyon and Luu (1999, 2001)

Divining the History of the Kuiper Belt History from its Orbit Elements

- the KB has 3 principal dynamical classes:
  - Scattered KBOs have perihelia $30 \lesssim q \lesssim 40$ AU
    - these eccentric bodies likely had close approaches to Neptune
  - the Main Belt KBOs reside between Neptune's 3:2 and 2:1 resonances at $40 < a < 48$ AU
  - the Plutinos inhabit Neptune 3:2 resonance at $a = 40$ AU
    - these are regarded as evidence that Neptune's orbit had migrated outwards $\Delta a_N \sim 8$ AU

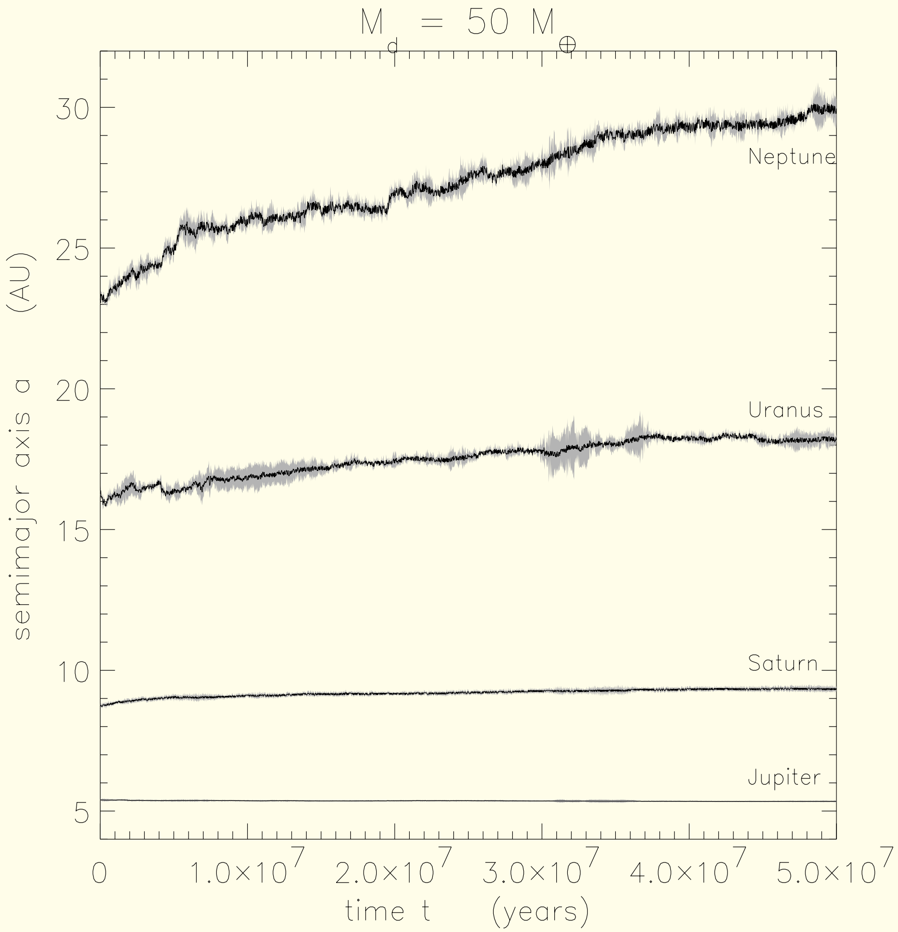


orbits from the Minor Planet Center

Why Would Planets Migrate?

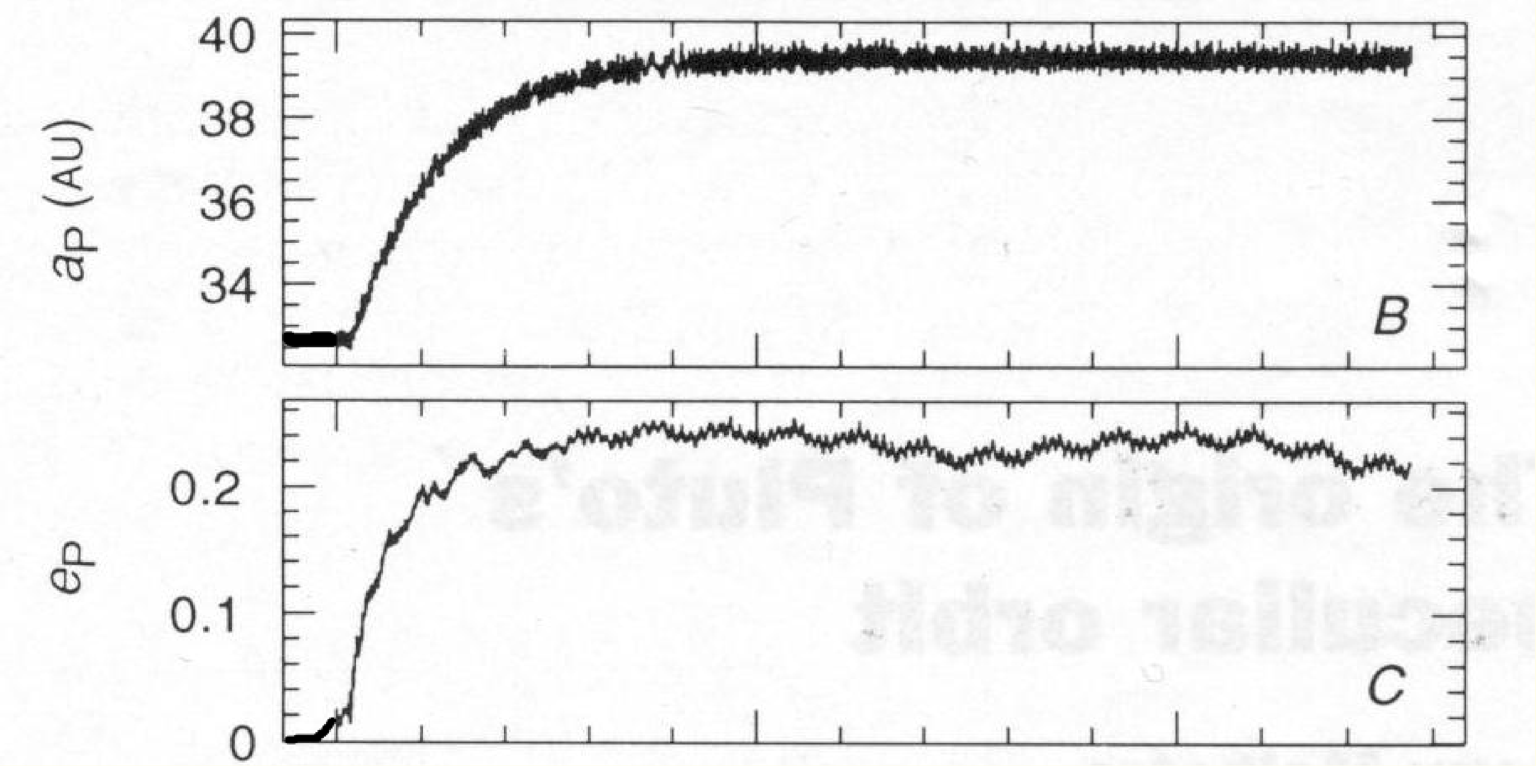
- due to an exchange of angular momentum between a debris disk and the recently–formed planets

- N–body simulations show that a $\boldsymbol{M}_{\text{disk}} \sim \mathbf{50}$ $\text{M}_\odot$ debris disk causes Neptune's orbit to expand $\boldsymbol{\Delta a_N} \sim \mathbf{7}$ AU over $\boldsymbol{\tau}_{\text{migrate}} \sim \mathbf{10^7}$ year timescale



Hahn and Malhotra (1999)

Trapping KBOs at Neptune’s Sweeping Resonances



- Malhotra (1993) recognized that this early episode of migration could explain Pluto’s unusual orbit having $e = 0.25$ at 3:2 resonance with Neptune.

- Had Neptune’s orbit expanded by $\Delta a_N \geq 5$ AU, Pluto can get trapped in the advancing 3:2 resonance and have its e pumped up to 0.25

- Planet migration & resonance trapping can also explain the swarm of KBOs orbiting at Neptune’s 3:2 resonance.

Inferring Neptune's Migration from the Plutinos

Brouwer (1963) showed that object orbiting at an $\boldsymbol{m+1:m}$ resonance obeys the integral

$$\boldsymbol{\beta \equiv a(t)[(m+1)\sqrt{1-e(t)^2}-m]^2.}$$

This is preserved even when shepherded outwards a distance $\boldsymbol{\Delta a}$ by a migrating planet (Yu and Tremaine 1999):

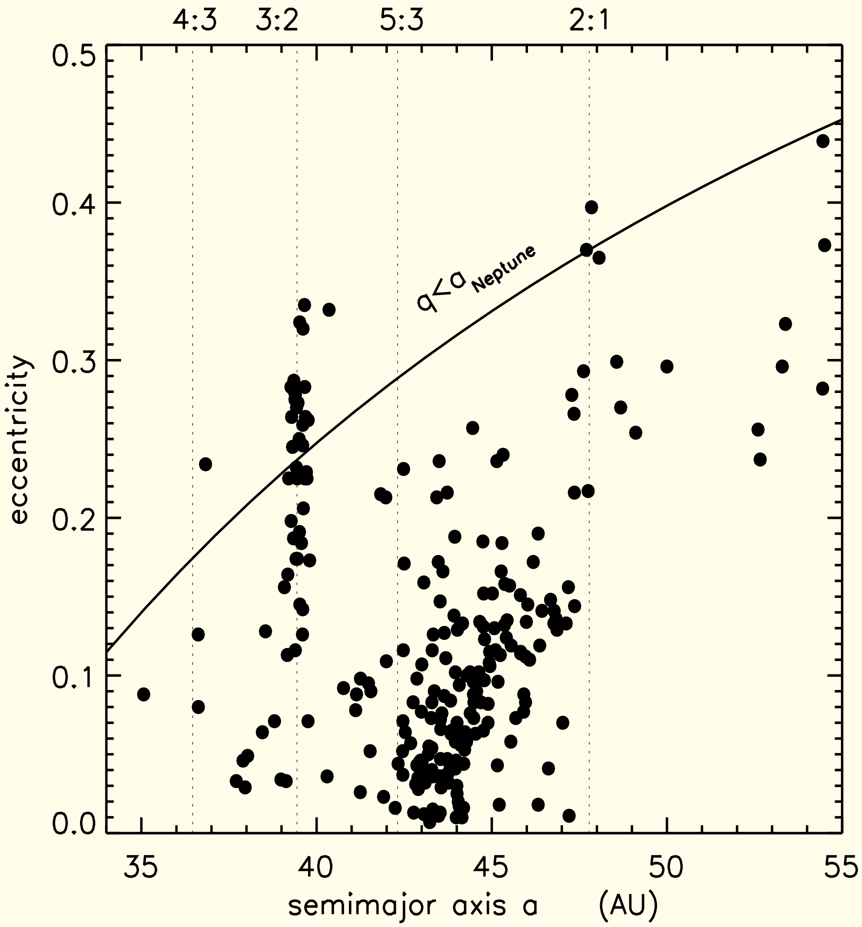
$$\boldsymbol{\frac{\Delta a}{a_f} = 1 - \left[(m+1)\sqrt{1-e_f^2} - m\right]^2 .}$$

For $m = 2$, $a_f = 39.5$ AU, $e_f = 0.3$,

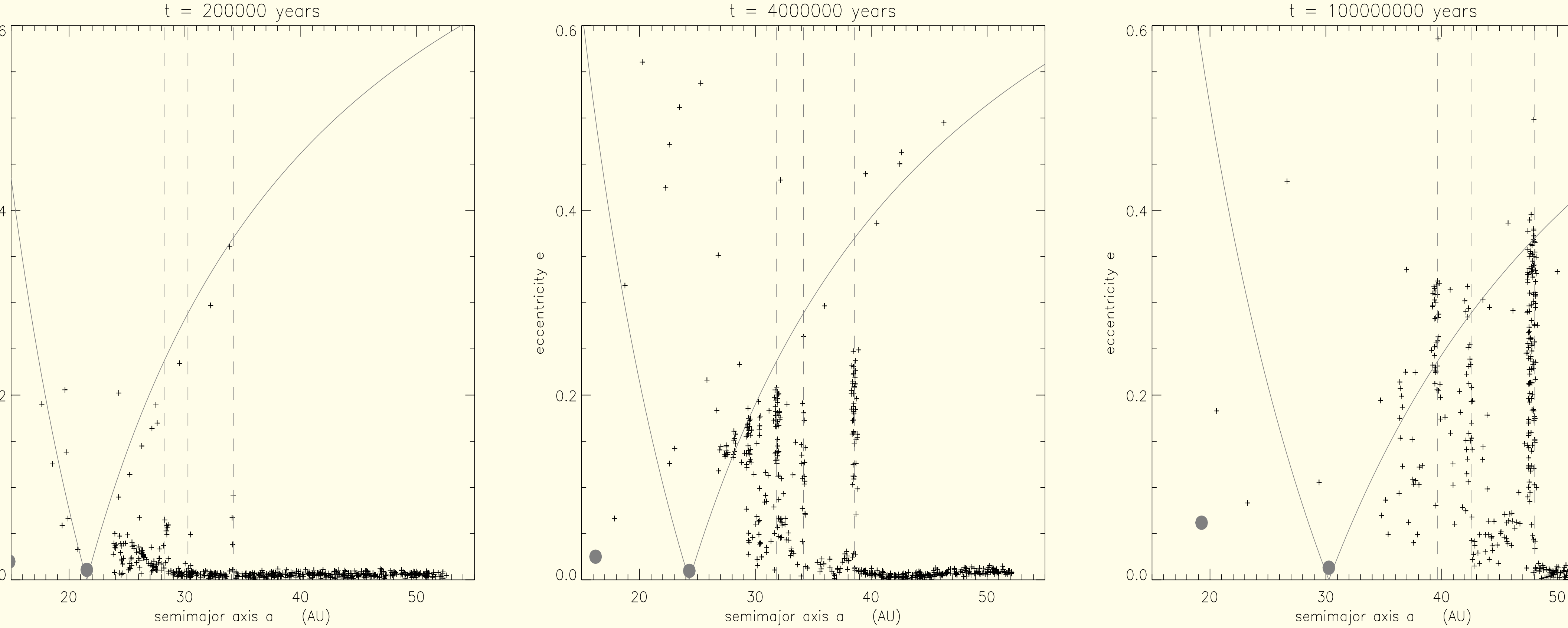
$$\boldsymbol{\Rightarrow \Delta a = 10} \quad \text{AU and}$$

$$\boldsymbol{\Delta a_N = (1+1/m)^{-2/3}\Delta a = 8} \text{ AU}$$

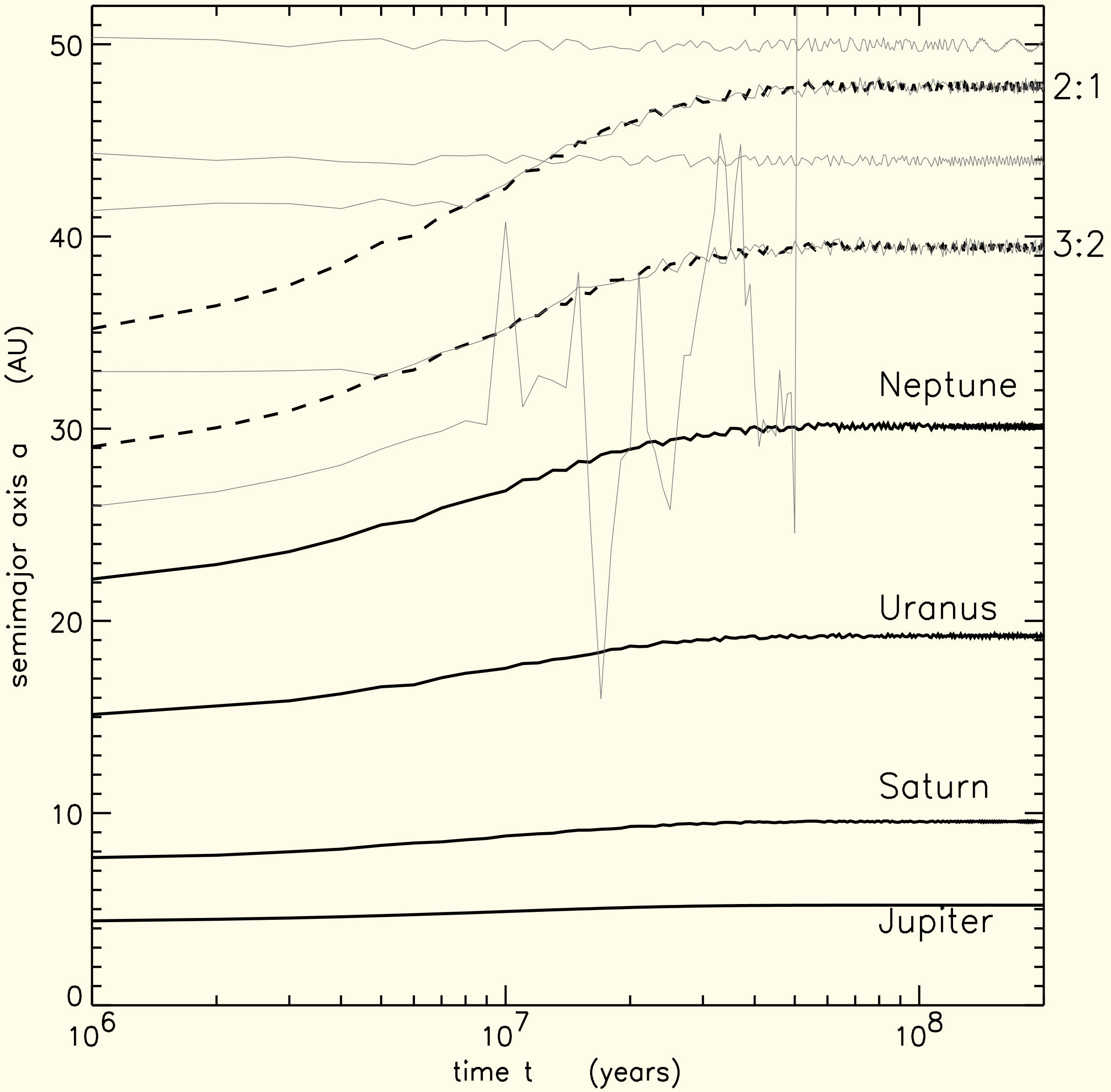
The early planetary system expanded $\sim 35\%$.



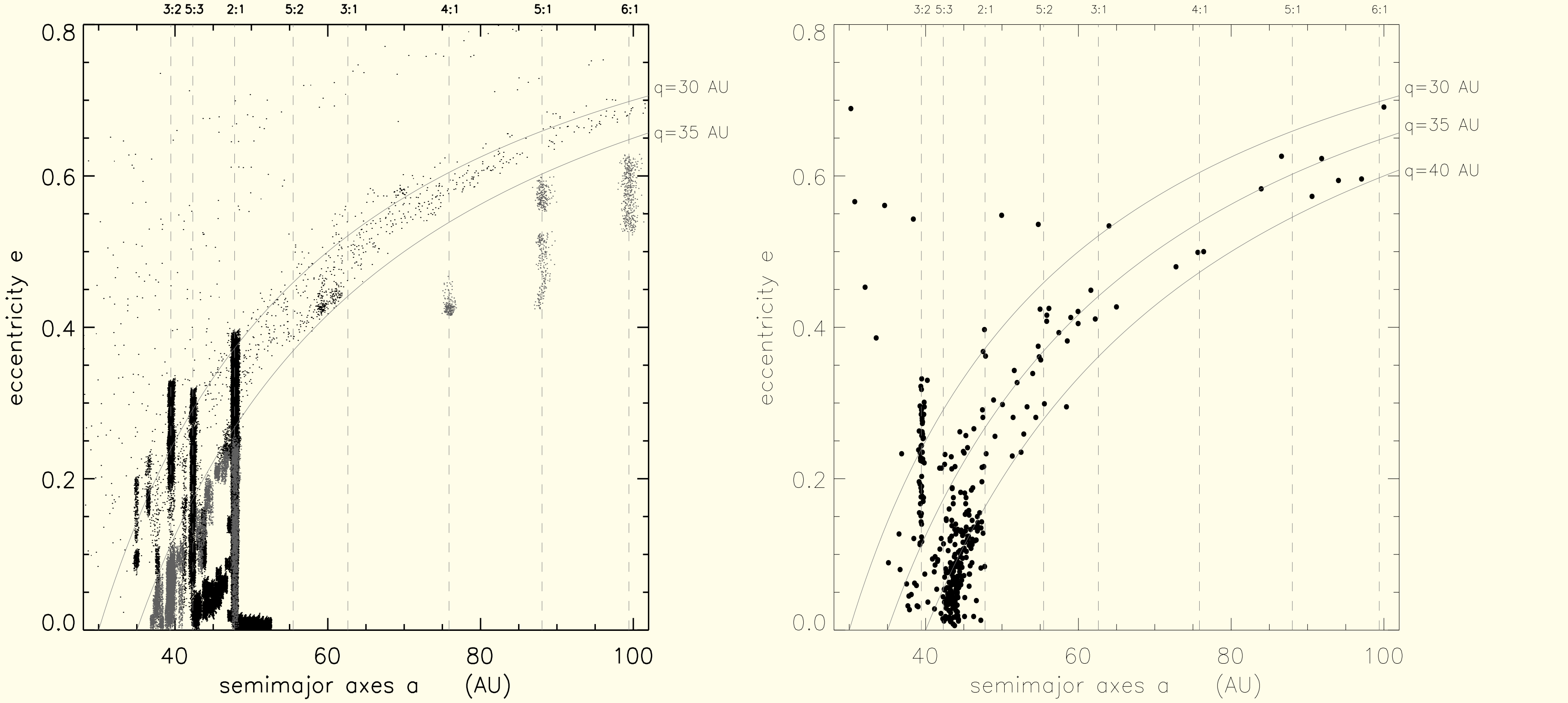
Snapshots of Planet Migration



Orbital Outcomes as Neptune Migrates



Compare Simulated and Observed Endstates

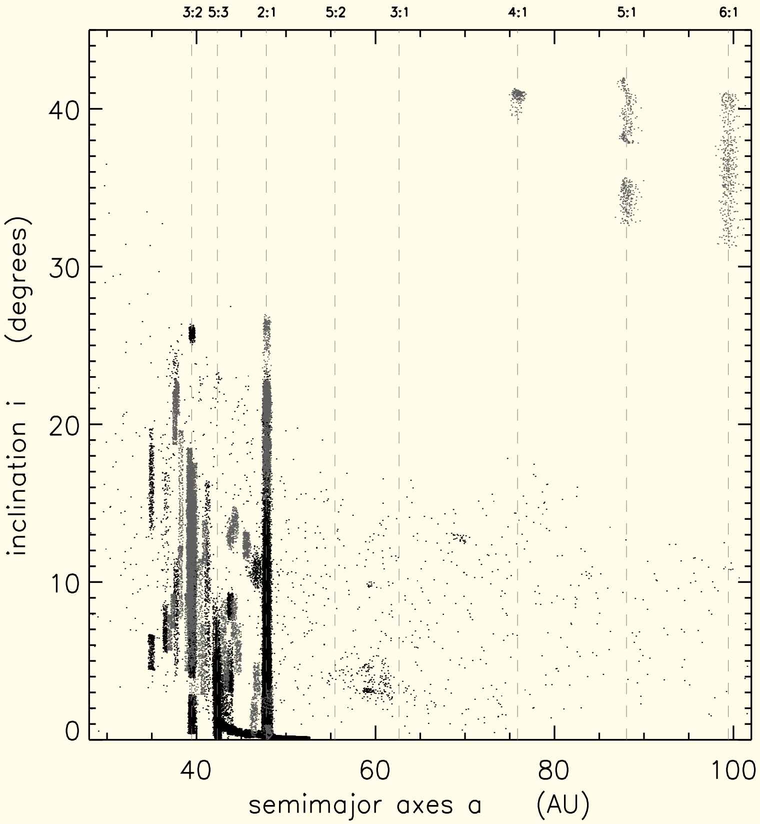


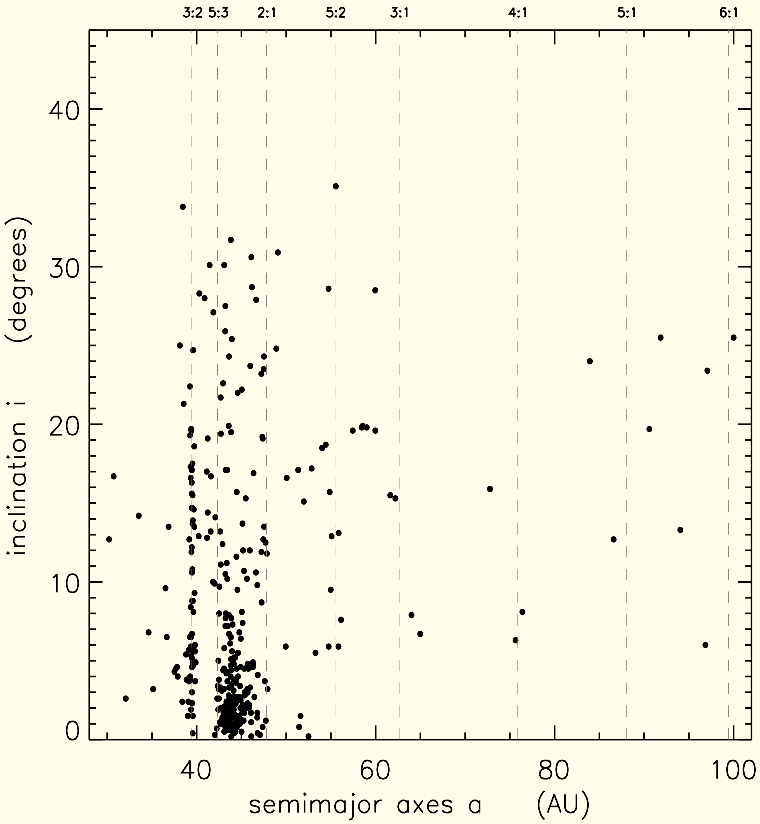
- planet migration & resonance trapping neatly explains the Plutino population

- however...
  - the model 2:1 resonance is overpopulated
    (but this may in part be due to telescopic selection effects)

  - simulated Scattered Objects have $30 < q < 35$ AU while $30 < q_{\text{obs}} < 40$ AU

The Invasion of the Kuiper Belt?





- Gomes (2003) has suggested that high i Scattered KBOs can 'invade' the Main Belt via mean–motion and Kozai resonances

- this might explain the Main Belt's bimodal i–distribution (Brown 2001)
  - the KBOs with $i \sim 2^\circ$ are 'native' to $a \sim 45$ AU
  - KBOs with $i \sim 20^\circ$ are invaders originally from $a \sim 30$ AU

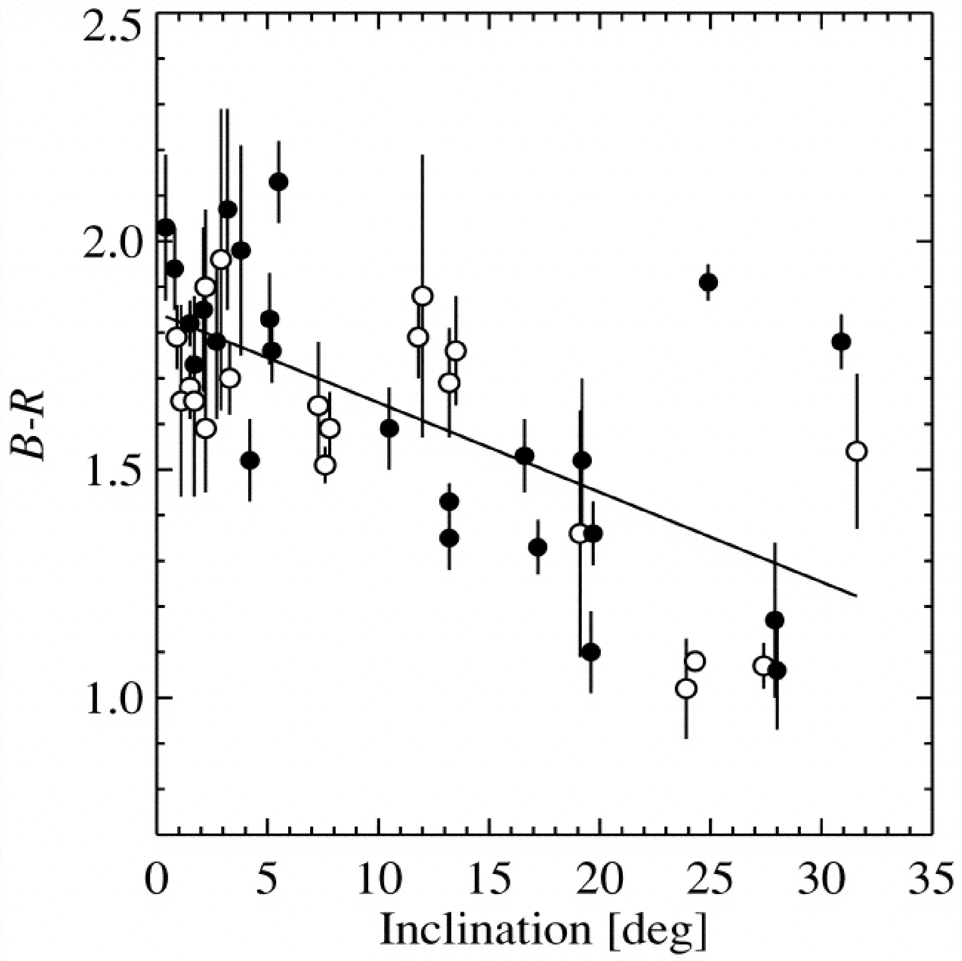
- however this invasion mechanism is very inefficient, $\varepsilon \sim 0.1\%$

Are the KBO Colors Evidence of an Invasion?

- Trujillo and Brown (2002) show that lower–i KBOs in the Main Belt are redder than higher–i KBOs

- Gomes' invasion scenario might account for these colors:
  - paint the low–i natives at $a \sim 45$ AU red
  - paint the invaders originating at $a \sim 30$ AU blue (or grey?), and then let Neptune toss these high–i KBOs into the Main Belt at $a \sim 45$ AU

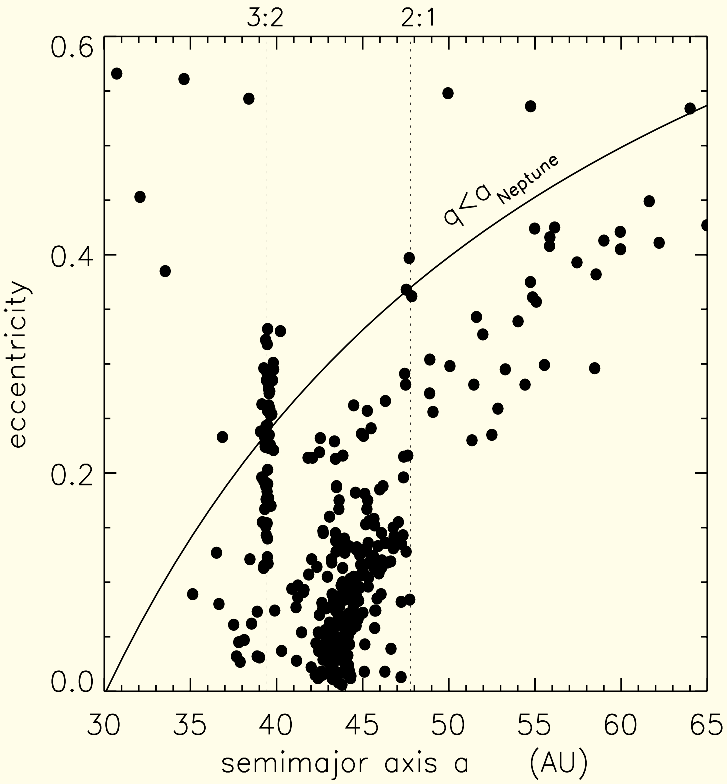
- these colors are presumably due to variations in surface composition
  - so why would more distant KBOs have redder surfaces?

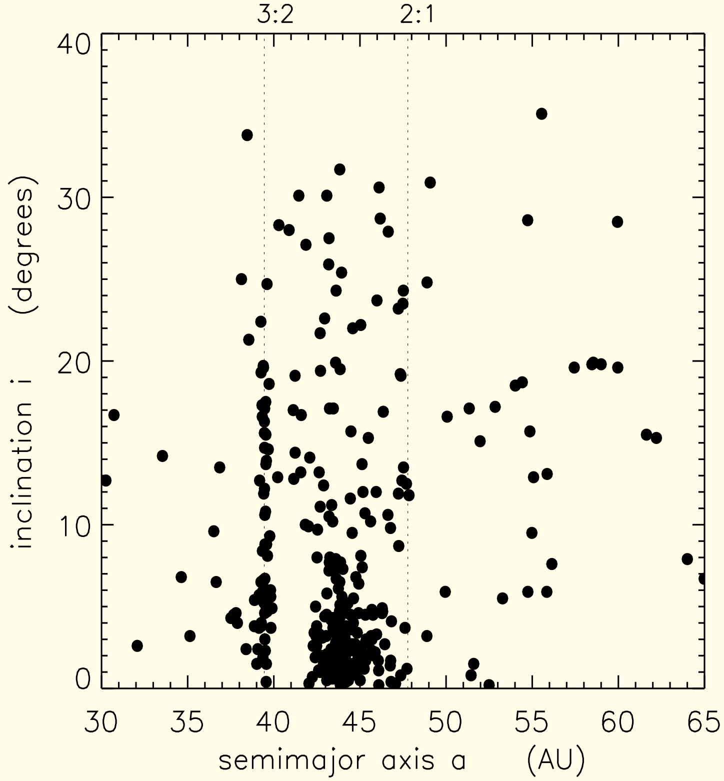


Trujillo and Brown (2002)

Invasion Hypothesis Might Explain:

- Scattered KBOs low perihelia $\mathbf{30 < q < 40}$ AU
- the Main Belt's low $\boldsymbol{i \sim 2^\circ}$ natives and high $\boldsymbol{i \sim 20^\circ}$ invaders
- Main Belt's color–$\boldsymbol{i}$ correlation
- however this invasion mechanism is very inefficient $\boldsymbol{\varepsilon \sim 0.1\%}$
- I shall explore another mechanism that might be quite (or too?) efficient at exciting KBO $\boldsymbol{e}$'s and $\boldsymbol{i}$'s





Secular Evolution of the Kuiper Belt

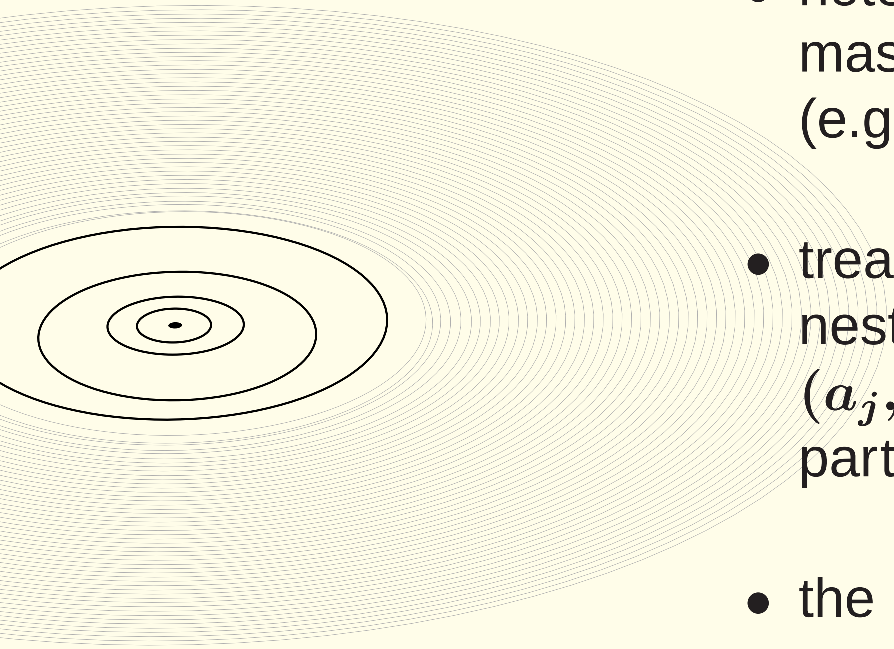
- secular perturbations are the constant or low–frequency gravitational forces exerted by a perturber

- of particular interest are secular resonances, which are sites where a perturber's precession rate matches a small body's:
  - large e's are excited where $\dot{\tilde{\omega}}_{\text{particle}} = \dot{\tilde{\omega}}_{\text{perturber}}$
  - large i's are excited where $\dot{\Omega}_{\text{particle}} = \dot{\Omega}_{\text{perturber}}$

- in a gravitating disk, this e–disturbance can propagate away from resonance as a spiral density wave [aka, apsidal wave (Ward and Hahn 1998)].

- the i–disturbance can propagate away from resonance as a spiral bending (or nodal) wave (Ward and Hahn 2003).

The Rings Model



- note that the secular evolution of a system of point-masses is identical to that of gravitating rings (e.g., Murray and Dermott 1999).
- treat a disk of numerous small bodies as a nested set of interacting rings of mass m_j, orbits $(a_j, e_j, i_j, \tilde{\omega}_j, \Omega_j)$ and thickness h_j due to their particles dispersion velocities c_j.
- the planets are thin $h_j = 0$ rings.

- evolve the system as per the Lagrange planetary equations
  - apply the well-known Laplace–Lagrange solution to obtain the system's secular evolution
  - note, however, that the rings' finite thickness h softens their gravity, which in turn requires softening the solution's Laplace coefficients over the scale h/a.

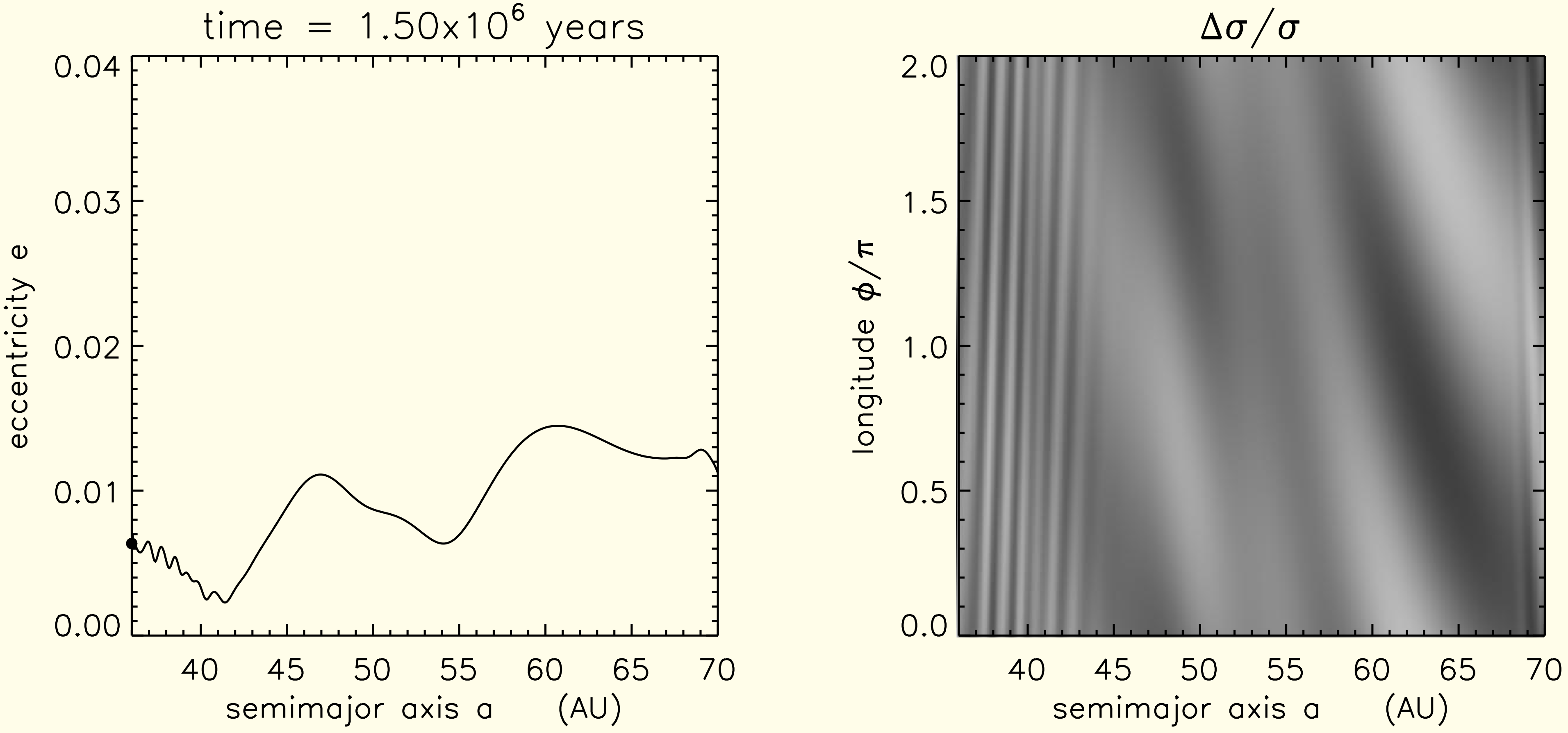
WKB Analysis

- a WKB solution (e.g., an approximate solution) to the planetary equations yields the properties of these waves
  - two types of apsidal density waves:
    - long waves with wavelength $\lambda_L \propto M_{KB}$
    - short waves with wavelength $\lambda_S \lesssim 10h$
  - there are only long nodal bending waves with wavelength $\lambda_L \propto M_{KB}$
- apsidal density waves propagate propagate between a resonance and the Q–barrier, which lies where h exceeds the threshold

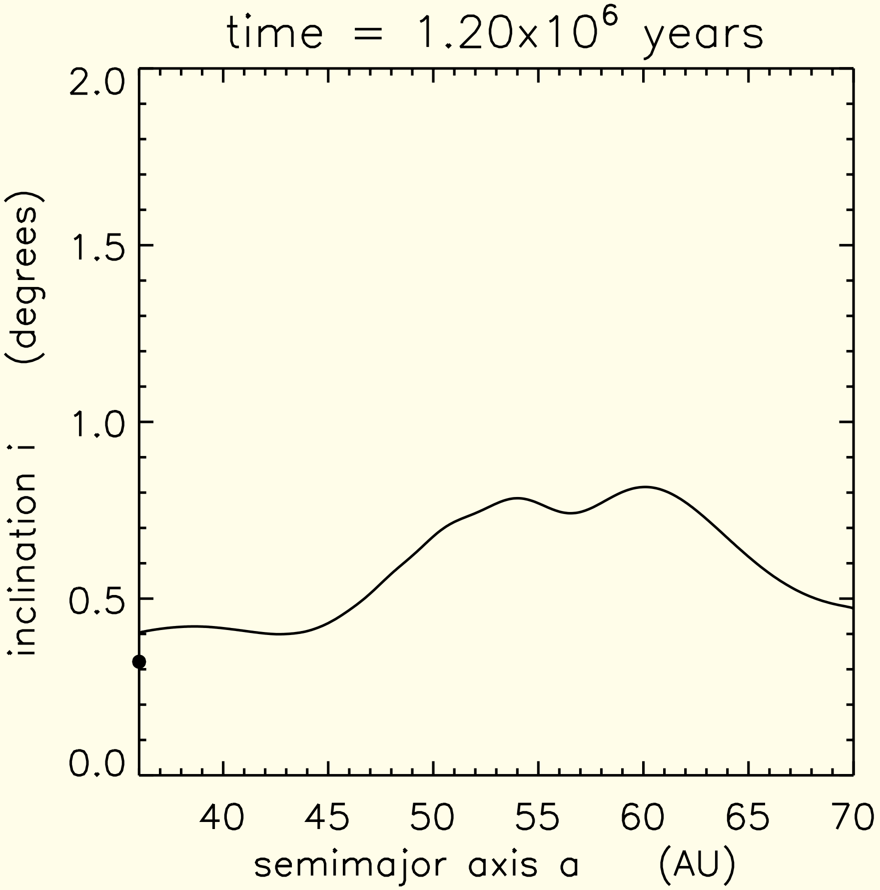
$$h_Q \simeq 0.3 \frac{M_{KB}}{M_{\text{Sun}}} \left| \frac{n}{\Omega_{\text{pattern}}} \right| a$$

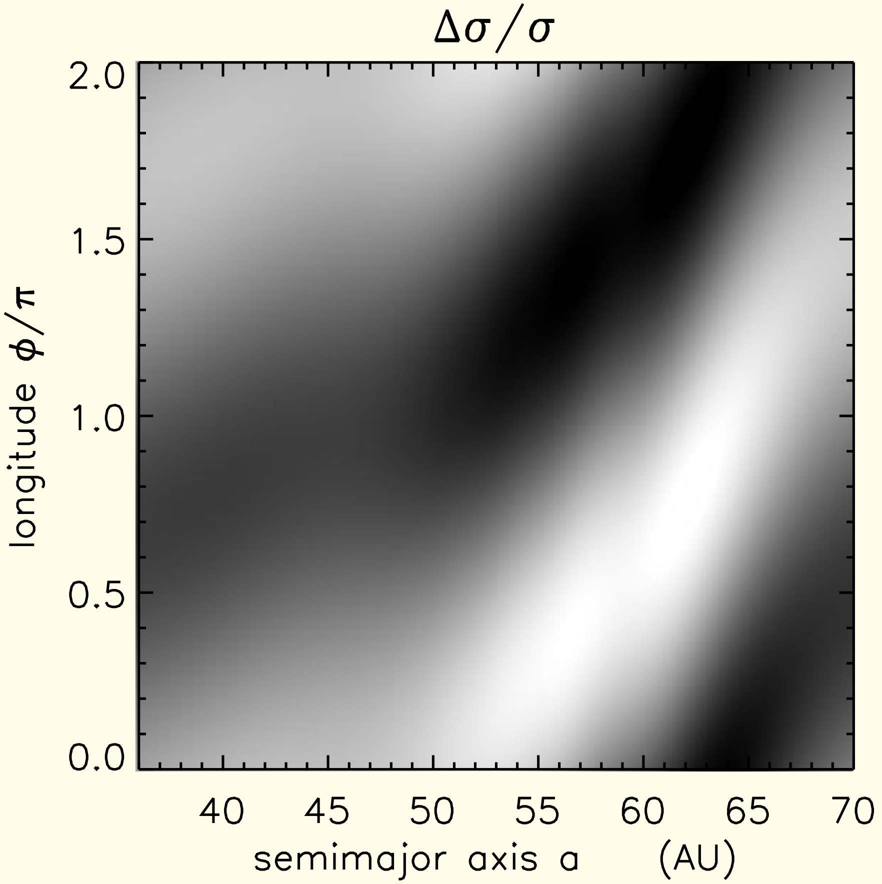
- if long density waves encounter a disk edge or a Q–barrier, they reflect as short density waves
  - Q–barrier is a low–pass filter, *ie.*, $\Omega_{\text{pattern}} < \Omega_Q$
- nodal bending waves propagate between resonance and the disk edge, or else they *stall* where $h \simeq 3h_Q \Leftarrow$ New!

Simulation of Apsidal Density Waves
in a $M_{KB} = 10$ M$_\oplus$ Kuiper Belt with $h = 0.01a$



Simulation of Nodal Bending Waves
in a $M_{KB} = 10$ M$_{\oplus}$ Kuiper Belt with $h = 0.01a$



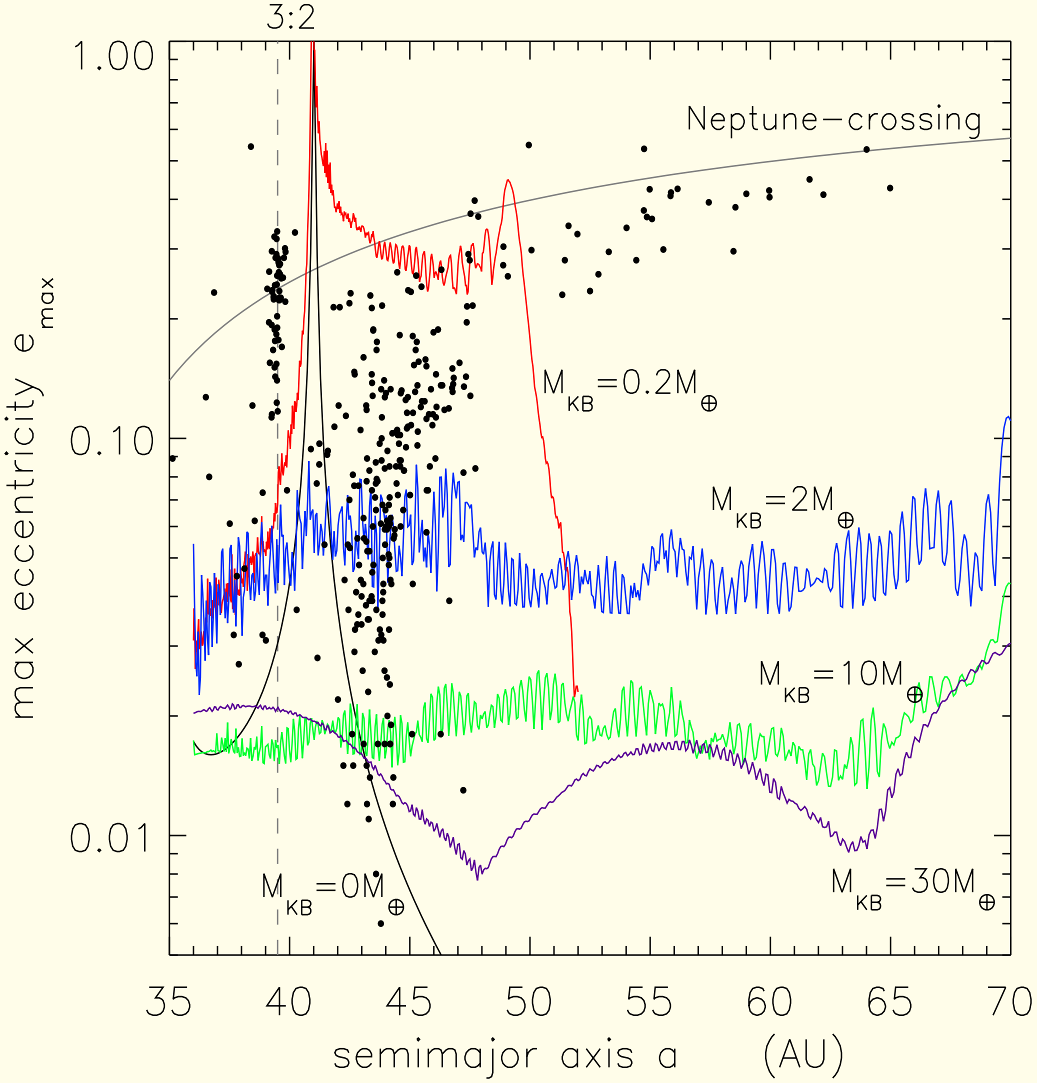


Summary of Apsidal Density Waves in the KB

- simulated Belt's have masses $M_{KB} = 30$ to 0.2 M$_\oplus$ (e.g., the Belt's primordial mass to its current, eroded mass) and $h = 0.002a$

- density waves reflect at the disk edge at 70 AU or at a Q–barrier.
  - reflected short waves are nonlinear, ie., $\Delta\sigma/\sigma \sim 1$

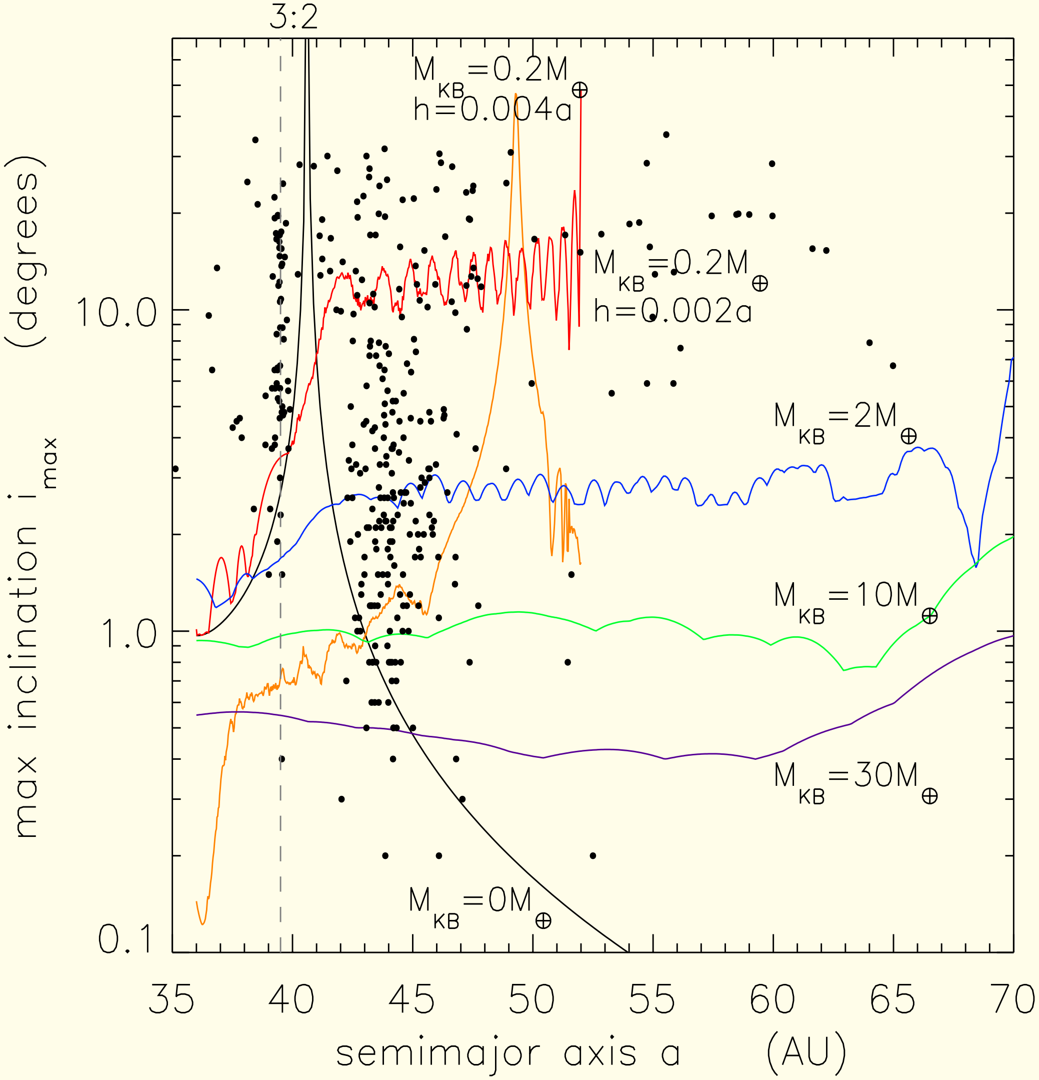
- the giant planets deposit $\sim 0.5\%$ of their e–AMD into the disk in the form of spiral density waves.
  - consequently, larger e's get excited in lower–mass disks
  - waves excite large e's in low–mass disks, $e \sim 0.3$ for $M_{KB} \sim 0.2$ M$_\oplus$
  - but this requires a very thin disk, $h \sim 0.002a$



Summary of Nodal Bending Waves in the KB

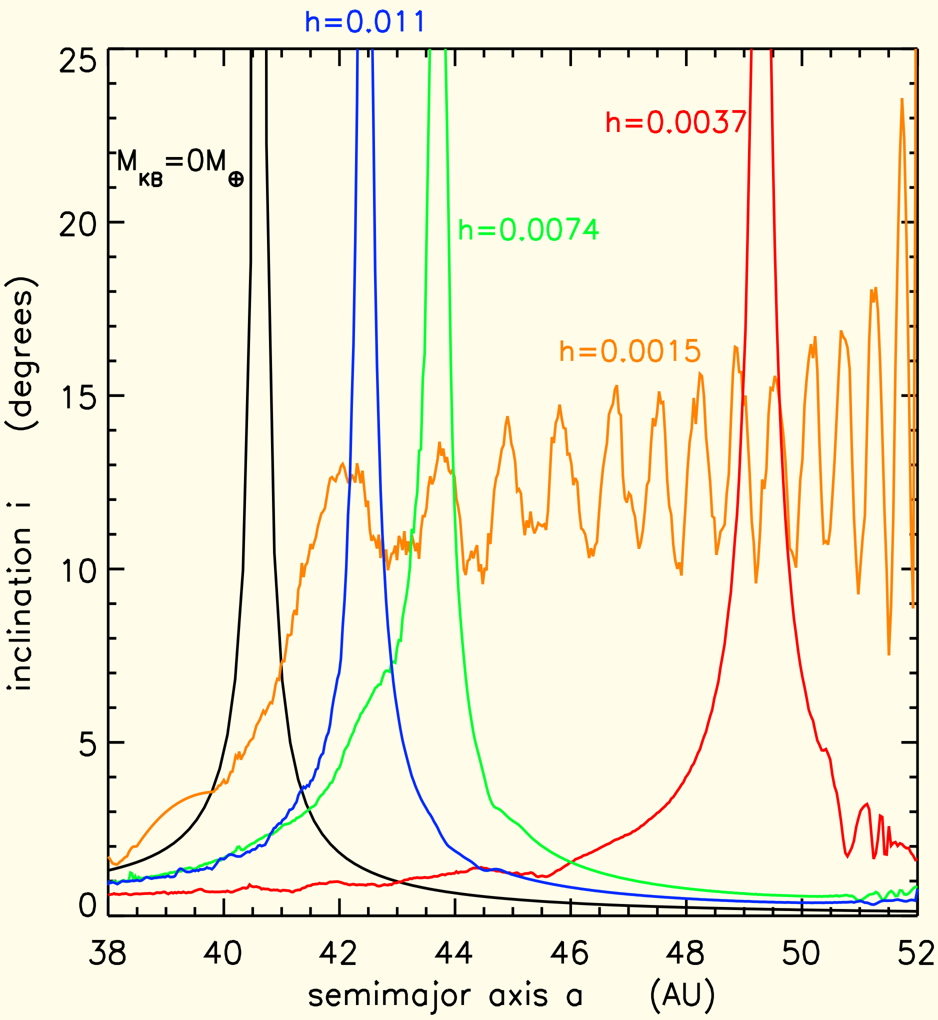
- the giant planets deposit $\sim \mathbf{10\%}$ of their $\boldsymbol{i}$–AMD into the disk in the form of spiral bending waves.
  - again, larger $\boldsymbol{i}$'s get excited in lower–mass disks

- bending waves also reflect at the disk edge at 70 AU or else they stall where $\boldsymbol{h \gtrsim 3h_Q}$
  - note the low $\boldsymbol{i}$'s interior to the stall–zone



Examples of Stalled Bending Waves

- simulations of a Kuiper Belt with $M_{KB} = 0.2$ M$_\oplus$ and thickness $h = 0.0015a$ to $0.011a$
  - bending waves stall when $h > 3h_Q \sim 0.003a$
  - wave reflect in the thinnest disk (orange curve)
  - the stall–zone moves inwards as h increases
  - large i result as waves dump their angular momentum into a narrow annulus in the disk
- increasing h draws the Q–barrier & stall–zone inwards towards the wave–launch site



- eventually wave–action is shut off when $h \gg h_Q$ (perhaps due to grav' stirring?) and the disk behaves as if it were non–gravitating (e.g., the black $M_{KB} = 0$ curve)

Waves & Their Implications for the Primordial Kuiper Belt

- when the KB was still young and quite massive, $M_{KB} \sim 30$ $\text{M}_\oplus$, then low–amplitude apsidal density waves ($e_{\text{max}} \sim 0.02$) and nodal bending waves ($i_{\text{max}} \sim 0.5^\circ$) were sloshing about the KB.
  - wave propagation times were short,

$$T_{\text{prop}} \sim 10^6 \left(\frac{\Delta a}{30 \text{ AU}}\right) \left(\frac{M_{KB}}{30 \text{ M}_\oplus}\right)^{-1} \text{ years}$$

  - the density waves eventually reflect and return as nonlinear short waves having $\Delta\sigma/\sigma \sim 1$ which dominate the Belt's surface density structure
  - wave–action keeps the disk dynamically cool by smearing the planets' gravitational disturbances across the disk
    - ∗ there is no localized heating of the disk at secular resonances.
    - ∗ N–body simulations that treat the disk as massless would fail to resolve this phenomena

Implications for the Current Kuiper Belt

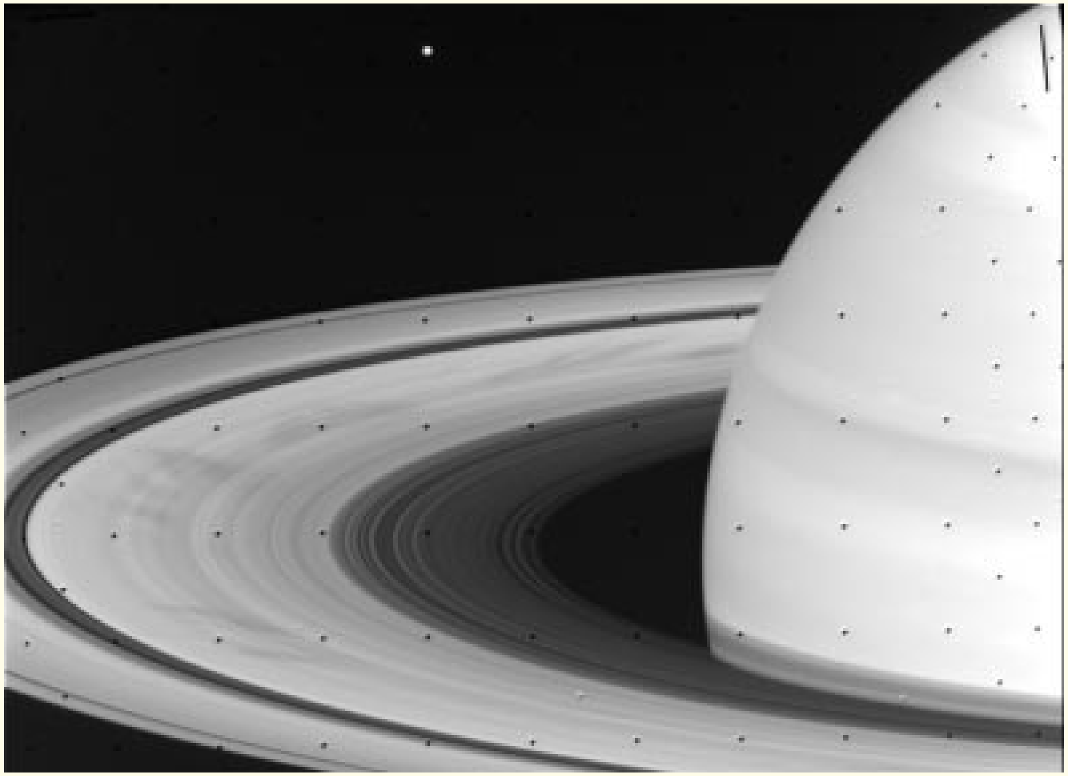
- over time, gravitational stirring by large KBOs increased the disk thickness h while collisional erosion decreased $M_{KB} \rightarrow 0.2$ $M_\oplus$
  - stirring/erosion draws the Q-barrier and the stall–zone inwards to the secular resonances at ~ 40 AU which ultimately shuts off wave action

- this epoch of wave propagation in the Kuiper likely lasted for the first
  - $\tau_{\text{form}} \sim 10^7$ years
    (when the large $R \sim 100$ km KBOs formed and started stirring things up)
  - $\tau_{\text{erode}} \sim 5 \times 10^8$ years
    (when collisions eroded 99% of the KB's mass away)

- gravitational stirring and collisional erosion prevented apsidal and nodal waves from stirring up the Kuiper Belt.

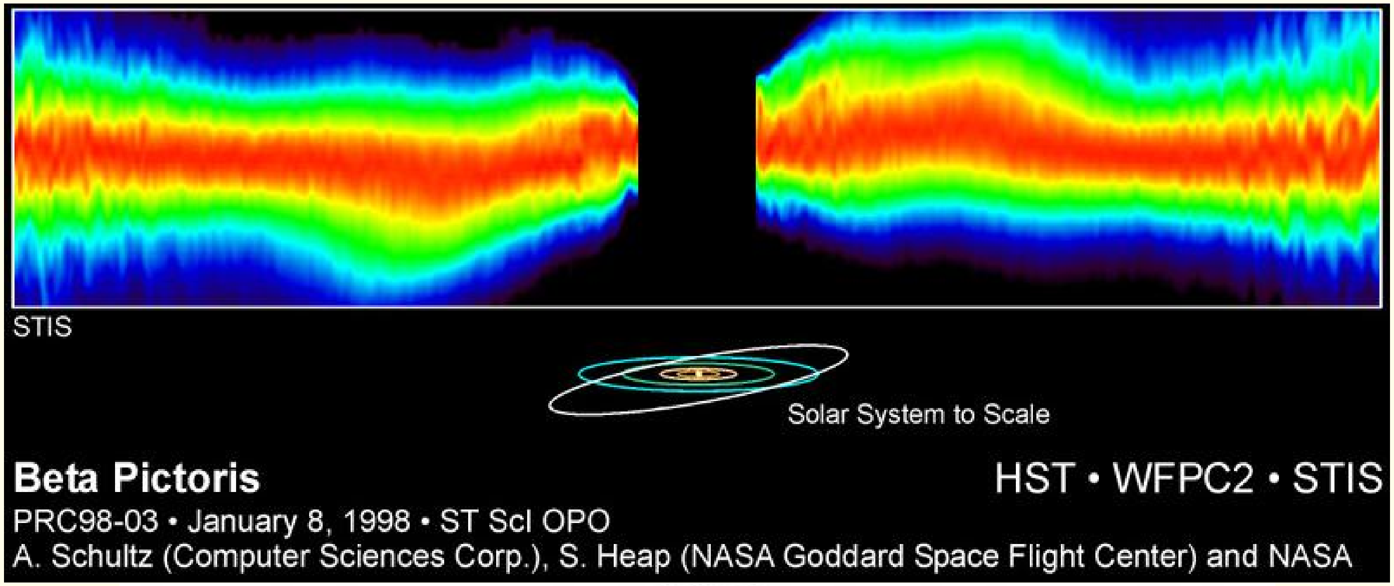
Other Applications of the Rings Model: Saturn's Rings

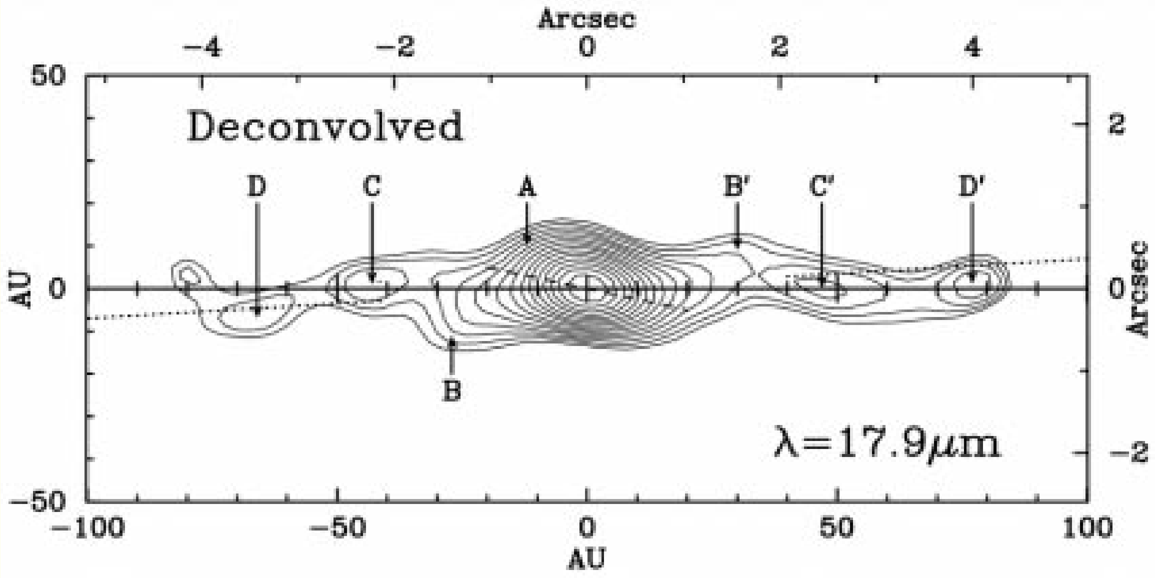
- apsidal & nodal waves propagate in thin disks
- short density waves with $\lambda_S \sim 10h \propto v_{\text{dispersion}}$ are of particular interest since:
  - $v_{\text{dispersion}}$ is an important parameter in ring dynamics, but is not well–constrained at Saturn
  - these waves can be nonlinear, *ie.*, $\Delta\sigma/\sigma > 1$, which would make their detection easier
- however Saturn's oblateness might defeat this type of wave–action



Voyager 2/Planetary Rings Node

Other Application: Circumstellar Dust Disk at β Pictoris





Wahhaj *et al.* (2003)

- warps & tilted dust rings are attributed to perturbations from unseen planets; this rings code can rapidly explore the available range of planetary parameters