

Joseph M. Hahn

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The Dynamics of Planetary Systems and Astrophysical Disks



THE DYNAMICS OF PLANETARY SYSTEMS AND ASTROPHYSICAL DISKS

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